Leptons e quarks com spin 3/2

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Introdução

I Espinores contravariantes

$$\overset{\circ}{\psi} \equiv (\psi^\alpha) = \begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix} \qquad \alpha = 1, 2$$

espaço de definição das matrizes

$$A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \equiv \begin{pmatrix} A^1{}_1 & A^1{}_2 \\ A^2{}_1 & A^2{}_2 \end{pmatrix}$$

$$\det A = 1 \quad \begin{cases} \mathrm{Re} \det A = 1 \\ \mathrm{Im} \det A = 0 \end{cases}$$

do grupo $SL(2, C)$:

$$\boxed{\overset{\circ}{\psi}{}' = A \overset{\circ}{\psi}} : \quad \psi'^\alpha = A^\alpha{}_\beta \psi^\beta$$

Espinores covariantes

$$\underset{\circ}{\psi} = (\psi_\alpha) = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

componentes são tais que

$$\psi_1 = -\psi^2, \quad \psi_2 = \psi^1$$

pois para dois espinores ψ e φ vale:

$$\begin{pmatrix} \psi'^1 & \varphi'^1 \\ \psi'^2 & \varphi'^2 \end{pmatrix} = \begin{pmatrix} A^1{}_1 & A^1{}_2 \\ A^2{}_1 & A^2{}_2 \end{pmatrix} \begin{pmatrix} \psi^1 & \varphi^1 \\ \psi^2 & \varphi^2 \end{pmatrix}$$

$$\Downarrow$$

$$\psi'^1 \varphi'^2 - \psi'^2 \varphi'^1 = \psi^1 \varphi^2 - \psi^2 \varphi^1$$

$=$ invariante em rel. a $SL(2,C)$

Logo o covariante surge de

$$\psi^1 \chi_1 + \psi^2 \chi_2 = \text{invar.}$$

onde

$$\chi_2 = \chi^1 \qquad \chi_1 = -\chi^2$$

Assim:

$$\psi^s = -C^{sr}\psi_r = \psi_r C^{rs}$$

$$(C^{sr}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -(C^{rs})$$

De: $r,s = 1,2$

$$C^{rs} C_{st} = -\delta^r{}_t$$

vem:

$$\psi_t = -\psi^s C_{st} = C_{ts}\psi^s$$

$$(C_{st}) = -(C_{ts}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = (C^{st})$$

$$C^2 = -I, \quad C^{-1} = -C$$

$${}^tC = -C$$

$$C^{+} = C^{-1}$$

De $\psi'^r = A^r{}_s \psi^s$

$\downarrow$

$$\psi'_t = C_{tr}\,\psi'^r \equiv -C_{tr}\,A^r{}_s\,C^{su}\,\psi_u$$

logo:

$$\psi'_t = A_t{}^u\,\psi_u$$

$$A_t{}^u = -C_{tr}\,A^r{}_s\,C^{su}$$

Encontra-se

$$\left(A_t{}^u\right) = \begin{pmatrix} A_1{}^1 & A_1{}^2 \\ A_2{}^1 & A_2{}^2 \end{pmatrix} \equiv {}^t\!\left(A^{-1}\right)$$

logo $\boxed{\underset{\circ}{\psi}' = {}^t\!\left(A^{-1}\right)\underset{\circ}{\psi}}$

Temos ainda:

$$\dot{\psi} \equiv (\psi^{\dot{\alpha}}) \equiv \begin{pmatrix} \psi^{\dot{1}} \\ \psi^{\dot{2}} \end{pmatrix}$$

tal que:

$$\dot{\psi}' = A^{*} \dot{\psi}$$

E:

$$\underset{\cdot}{\psi} \equiv (\psi_{\dot{\alpha}})$$

tal que:

$$\boxed{\underset{\cdot}{\psi}' = (A^{+})^{-1} \underset{\cdot}{\psi}}$$

Assim espinores de Weyl

$$\overset{\circ}{\psi}, \quad \underset{\circ}{\psi}, \quad \dot{\psi}, \quad \underset{\cdot}{\psi}$$

$$\boxed{\begin{array}{l} \overset{\circ}{\psi}' = A \overset{\circ}{\psi} \; ; \quad \underset{\circ}{\psi}' = {}^{t}(A^{-1}) \underset{\circ}{\psi} \\ \dot{\psi}' = A^{*} \dot{\psi} \; ; \quad \underset{\cdot}{\psi}' = (A^{+})^{-1} \underset{\cdot}{\psi} \end{array}}$$

Espinor de ordem $n+m$:

$$\psi^{r_1 \dots r_n \dot{t}_1 \dots \dot{t}_m}$$

elem. de esp. di repr. do grupo $SL(2,\mathbb{C})$ que é represent. a dois valores do grupo de Lorentz próprio ortocrônico $L_+^\uparrow$

x x x

Dado $\psi^{r\dot{s}}$ pode-se escolher $\psi^{1\dot{1}}$ e $\psi^{2\dot{2}}$ reais e $\psi^{1\dot{2}} = (\psi^{2\dot{1}})^*$ e escrever

$$\psi^{1\dot{1}} = x^0 + x^3$$

$$\psi^{2\dot{2}} = x^0 - x^3$$

$$\psi^{1\dot{2}} = x^1 - ix^2$$

$$\psi^{2\dot{1}} = x^1 + ix^2$$

donde a definição das matrizes de Pauli

$$\psi^{r\dot{s}} = (\sigma_\alpha)^{r\dot{s}} x^\alpha$$

$$\left((\sigma_0)^{r\dot{s}}\right) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\left((\sigma_1)^{r\dot{s}}\right) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\left((\sigma_2)^{r\dot{s}}\right) = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\left((\sigma_3)^{r\dot{s}}\right) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

A importancia de tudo isso é que as transform. do espaço de espinores sob o grupo $SL(2,C)$ acarreta a transformação dos x^μ pelo grupo $L_{+\uparrow}$ de Lorentz proprio ortocrono.

$$\mathring{\psi}' = A\mathring{\psi} \Rightarrow (\psi'^{rs}) = (A\psi A^+)^{rs} \Rightarrow$$

$$\Rightarrow x'^\mu = \ell^\mu{}_\nu x^\nu \quad \text{com} \quad \ell^\mu{}_\nu = \frac{1}{2} Tr(\sigma_\mu A \sigma_\nu A^+)$$

Ha um homomorfismo entre $L_{+\uparrow}$ e $SL(2,C)$

$A \to L$, $L \to \pm A$, represent. a dois valores.

Assim os espinores de Weyl são talvez os elementos geométricos fundamentais pois que as coordenadas cartesianas de um ponto do espaço se obtêm a partir de um espinor de 2ª ordem misto. Como os espinores tais como ψ^0 e $\dot{\chi}$ estão associados à descrição de partículas com spin ½, o spin ½ parece ser um elemento geométrico num nível mais profundo da geometria.

$$\text{De } \ell^{\mu}{}_{\nu} = \frac{1}{2}\left(\sigma_{\mu} A \sigma_{\nu} A^{+}\right)$$

resulta:

$$\det \ell = 1$$

$$\ell^{0}{}_{0} \geq 1$$

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Para uma partícula de spin 1/2 e massa m cujo op. impulsão satisfaz

$$P_\alpha P^\alpha = m^2$$

a equação de Dirac com um par de espinores de Weyl é:

$$\begin{cases} (\sigma_\alpha P^\alpha)^{rs} \chi_s = m \varphi^r \\ (\sigma_\alpha P^\alpha)_{sr} \varphi^r = m \chi_s \end{cases}$$

onde
$$(\sigma_0)_{sr} = (\sigma_0)^{rs}$$
$$(\sigma_k)_{sr} = -(\sigma_k)^{rs}$$

Para $m = 0$

$$(\sigma_\alpha P^\alpha)^{rs} \chi_s = 0 \quad \text{neutrino levógiro}$$

$$(\sigma_\alpha P^\alpha)_{sr} \varphi^r = 0 \quad \text{neutrino dextrógiro}$$

Eq. de Dirac na represent. de Weyl é pois:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

onde:

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad 4 \times 4$$

$$\gamma^k = \begin{pmatrix} 0 & -\sigma^k \\ \sigma^k & 0 \end{pmatrix}$$

$$\vec{\gamma} = \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

e $\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$ ou $\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \varphi^1 \\ \varphi^2 \\ \chi_{\dot{1}} \\ \chi_{\dot{2}} \end{pmatrix}$

Equ. acima $(\gamma^\mu P_\mu - m)\psi = 0$

equiv. a:

$$(\sigma_0 p^0 - \vec{\sigma}\cdot\vec{p})\varphi = m\chi$$

$$(\sigma_0 P^0 + \vec{\sigma}\cdot\vec{p})\chi = m\varphi$$

e: $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$

Tem-se $x' = Lx \Longrightarrow \psi' = \begin{pmatrix} A & 0 \\ 0 & (A^+)^{-1} \end{pmatrix}\psi$

Spin 1 e a eletrodin. de Majorana

$$\Psi \equiv \begin{pmatrix} \varphi^{rs} \\ \chi^{r}{}_{\dot s} \\ \omega_{\dot s}{}^{r} \\ \eta_{\dot r \dot s} \end{pmatrix} \qquad r, s, \dot r, \dot s = 1, 2$$

onde $\varphi^{rs} = \varphi^{sr}$, $\eta_{\dot r \dot s} = \eta_{\dot s \dot r}$

Dez componentes

A la Dirac:

$$\psi_{ab} = \psi_{ba} \qquad a, b = 1, 2, 3, 4$$

Rel. entre ψ de Dirac e Ψ de Weyl:

$$\varphi^{rs} \longrightarrow \tfrac{1}{2}(I+\gamma^5)\tfrac{1}{2}(1+\gamma^5)\psi = \begin{pmatrix} \begin{matrix} \psi_{11} & \psi_{12} \\ \psi_{12} & \psi_{22} \end{matrix} & 0 \\ 0 & 0 \end{pmatrix}$$

$$\eta_{\dot r \dot s} \longrightarrow \tfrac{1}{2}(1-\gamma^5)\tfrac{1}{2}(1-\gamma^5)\psi = \begin{pmatrix} 0 & 0 \\ 0 & \begin{matrix} \psi_{33} & \psi_{34} \\ \psi_{34} & \psi_{44} \end{matrix} \end{pmatrix}$$

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$$\chi^{r}{}_{s} \to \frac{1}{2}(1+\gamma^5)\frac{1}{2}(1-\gamma^5)\,\psi$$

$$\omega_{\dot{s}}{}^{\dot{r}} \to \frac{1}{2}(1-\gamma^5)\frac{1}{2}(1+\gamma^5)\,\psi$$

$$\omega_{\dot{s}}{}^{r} = {}^{t}(\chi^{r}{}_{s})$$

Equs. de Weyl para partic. com spin 1:

$$\begin{cases} i(\sigma^{\alpha}\partial_{\alpha})_{\dot{s}r}\,\varphi^{rt} = m\,\omega_{\dot{s}}{}^{t} \\ \frac{1}{2}\left\{i(\sigma^{\alpha}\partial_{\alpha})^{r\dot{s}}\,\omega_{\dot{s}}{}^{t} + i(\sigma^{\alpha}\partial_{\alpha})^{t\dot{s}}\,\omega_{\dot{s}}{}^{r}\right\} = \\ \qquad = m\,\varphi^{rt} \end{cases}$$

$r, s, \dot{r}, \dot{s} = 1, 2$

$$\begin{cases} \frac{1}{2}\left\{i(\sigma.\partial)_{\dot{s}r}\,\chi^{r}{}_{\dot{t}} + i(\sigma.\partial)_{\dot{t}r}\,\chi^{r}{}_{\dot{s}}\right\} = m\,\eta_{\dot{s}\dot{t}} \\ i(\sigma.\partial)^{r\dot{s}}\,\eta_{\dot{s}\dot{t}} = m\,\chi^{r}{}_{\dot{t}} \end{cases}$$

No esp. das matrizes 2x2, uma base é $I, \vec{\sigma}$; outra é

$$C, \quad \vec{\sigma} C$$

onde ${}^{t}C = -C, \quad C^{+} = C^{-1}$

$${}^{t}\vec{\sigma} = -C^{-1}\vec{\sigma}\,C$$

logo ${}^{t}(\vec{\sigma} C) = (\vec{\sigma} C)$

${}^{t}(\sigma_k)^{rs} = -(C^{-1})_{ru}(\sigma_k)^{uv} \cdot C_{vs}$

Como pois escrever:

$$\Rightarrow \quad \varphi^{rs} = \vec{F}(x)\cdot(\vec{\sigma} C)^{rs}$$

$$\Rightarrow \quad \omega_s \overset{t}{=} \{ f^{0}(x)\,C - \vec{f}(x)\cdot(\vec{\sigma} C)\}_s^{\,t}$$

Obtemos: $i\,\vec{\nabla}\cdot\vec{F} + m^2 f^{0} = 0$

$$i\,\partial_0 \vec{F} - \vec{\nabla}\times\vec{F} - m^2 \vec{f} = 0$$

$$\vec{F} = i\,(\partial^{0}\vec{f} + \vec{\nabla} f^{0}) + \vec{\nabla}\times\vec{f}$$

Ponha:
$$\vec{\tilde{F}} = \vec{B} - i\vec{E} \qquad \vec{B}, \vec{E} \text{ reais}$$

obtemos Proca-Maxwell

$$\begin{cases} \vec{E} = -\partial^0 \vec{f} - \vec{\nabla} f^0 \\ \vec{B} = \vec{\nabla} \times \vec{f} \\ \vec{\nabla}\cdot\vec{E} + m^2 f^0 = 0 \\ \partial_0 \vec{E} - \vec{\nabla}\times\vec{B} - m^2 \vec{f} = 0 \end{cases}$$

O outro par de Weyl dá o primeiro por refl. espacial - o que quer dizer que podemos considerar formalmente
$\vec{E}$ vect. polar, $\vec{B}$ axial
ou:
$\vec{E}$ axial e $\vec{B}$ polar

Para o campo eletromagn.
considero o foton como um neutrino de Weyl com spin 1:

$$i(\sigma\cdot\partial)_{sr}\,\varphi^{rt}=0$$

$$\varphi^{rt}=\varphi^{tr}$$

Faço

$$\varphi^{rt}=\vec{F}\cdot(\vec{\sigma}C)^{rt}$$

então formando traços de eq. de Weyl obtenho:

$$\vec{\nabla}\cdot\vec{F}=0 \quad ; \quad i\partial_0\vec{F}-\vec{\nabla}\times\vec{F}=0$$

Se $\vec{F}=\vec{B}-i\vec{E}$

então:

$$\vec{\nabla}\cdot\vec{E}=0 \qquad \vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\times\vec{E}+\partial^0\vec{B}=0$$

$$\vec{\nabla}\times\vec{B}-\partial_0\vec{E}=0$$

Majorana foi primeiro a descrever eletrodin. com um tal $\vec{F}$ complexo

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A eq. $\vec{\nabla}\cdot\vec{F}=0$

no esp. dos mom. é $\vec{k}\cdot\vec{F}(\vec{k})=0$

logo $\vec{F}(\vec{k})$ é transversal.

$$\vec{F}(\vec{k}) = \frac{1}{\sqrt{2}}(F_1 - iF_2)\vec{e}_+(\vec{k}) +$$

$$+ \frac{1}{\sqrt{2}}(F_1 + iF_2)\vec{e}_-(\vec{k})$$

$$e_\pm(\vec{k}) = \frac{1}{\sqrt{2}}(\vec{e}_1(\vec{k}) \pm i\,\vec{e}_2(\vec{k}))$$

$$\vec{k}\cdot\vec{e}_\pm(\vec{k}) = 0\,, \quad \vec{e}_j(\vec{k})\cdot\vec{e}_k(\vec{k}) = \delta_{jk}$$

logo em geral

$$\varphi^{\mathrm{ext}} = \vec{F}\cdot(\vec{\sigma}C)^{\mathrm{ext}}$$

se decompõe em

$$\varphi = \frac{1}{\sqrt{2}}(F_1 - iF_2)(\vec{\sigma}_+ C) +$$

$$+ \frac{1}{\sqrt{2}}(F_1 + iF_2)(\sigma_- C)$$

Mas:

$$\sigma_3(\sigma_1 \pm i\sigma_2) = \pm(\sigma_1 \pm i\sigma_2)$$

e como o spin do campo é

$$\vec{\Sigma} = \frac{1}{2}\{\vec{\sigma}\otimes I + I\otimes\vec{\sigma}\}$$

15/ vemos que o fóton polarizado à direita
é representado por:

$$\varphi_R = f(k)(\sigma_+ c)$$

e o fóton levógiro é:

$$\varphi_L = g(k)(\sigma_- c)$$

Sob reflex. espac. $\vec{F} \to \vec{F}^*$;

$\varphi \to \varphi^c$, conj. na carga defin.

por:

$$\varphi^c = (C \otimes C)\varphi^*$$

Se equ. de φ_R é:

$$(\sigma^0 k^0 - \vec{\sigma} \cdot \vec{k})\varphi_R = 0$$

a de $(\varphi_R)^c$ é

$$(\sigma^0 k^0 + \vec{\sigma} \cdot \vec{k})(\varphi_R)^c = 0$$

logo $(\varphi_R)^c$ é um φ_L.

Na represent. de Dirac, spin 1 é descrito por espinor de Dirac de 2ª ordem $\psi_{ab} = \psi_{ba}$. Como as matrizes da base $4 \otimes 4$ se dividem em 10 simétricas e 6 antissim.:

$$^{t}(\gamma^{\mu}) = -\, C^{-1} \gamma^{\mu} C$$

onde $^{t}C = -C, \quad C^{+} = C^{-1}$

então as sim. são:

$$(\gamma^{\mu} C) \quad e \quad (\sigma_{\mu\nu} C)$$

as antissim:

$$C, \quad \gamma^{5} C, \quad \gamma^{\mu} \gamma^{5} C$$

pode-se escrever:

$$\psi_{ab} = \phi^{\mu} (\gamma_{\mu} C)_{ab} + \frac{1}{2m} G^{\mu\nu} (\sigma_{\mu\nu} C)_{ab}$$

Portanto obtêm-se as equ. de Proca a partir das de ψ_{ab}, para ϕ^{μ} e $G^{\mu\nu}$, i.é:

$$G_{\mu\nu} = \partial_{\nu} \phi_{\mu} - \partial_{\mu} \phi_{\nu}$$

$$\partial_{\nu} G^{\mu\nu} + m^{2} \phi^{\mu} = 0$$

Spin 3/2

Um tensor de terceira ordem forma:

$$\psi = \begin{pmatrix} \varphi^{abc} \\ \chi^{ab}{}_{\dot{c}} \\ \omega^{a}{}_{\dot{b}\dot{c}} \\ \eta_{\dot{a}\dot{b}\dot{c}} \end{pmatrix} \qquad a, b, c = 1, 2$$

φ^{abc} e $\eta_{\dot{a}\dot{b}\dot{c}}$ simétricos.

$$\chi^{ab}{}_{\dot{c}} = \chi^{ba}{}_{\dot{c}}$$

$$\omega^{a}{}_{\dot{b}\dot{c}} = \omega^{a}{}_{\dot{c}\dot{b}}$$

dá 20 componentes

$$\varphi^{abc} \to \varphi^{111}, \varphi^{112}, \varphi^{122}, \varphi^{222}$$

$$\eta_{\dot{a}\dot{b}\dot{c}} \to \eta_{\dot{1}\dot{1}\dot{1}}, \eta_{\dot{1}\dot{1}\dot{2}}, \eta_{\dot{1}\dot{2}\dot{2}}, \eta_{\dot{2}\dot{2}\dot{2}}$$

$$\omega^{a}{}_{\dot{b}\dot{c}} \to \begin{matrix} \omega^{1}{}_{\dot{1}\dot{1}} & \omega^{1}{}_{\dot{1}\dot{2}} & \omega^{1}{}_{\dot{2}\dot{2}} \\ \omega^{2}{}_{\dot{1}\dot{1}} & \omega^{2}{}_{\dot{1}\dot{2}} & \omega^{2}{}_{\dot{2}\dot{2}} \end{matrix}$$

$$\chi^{ab}{}_{\dot{c}} \to \begin{matrix} \chi^{11}{}_{\dot{1}}, \chi^{12}{}_{\dot{1}}, \chi^{22}{}_{\dot{1}} \\ \chi^{11}{}_{\dot{2}}, \chi^{12}{}_{\dot{2}}, \chi^{22}{}_{\dot{2}} \end{matrix}$$

Tem-se

$$\varphi^{abc} \to \tfrac{1}{2}(1+\gamma^5) \otimes \tfrac{1}{2}(1+\gamma^5) \otimes \tfrac{1}{2}(1+\gamma^5)\,\psi$$

$$\eta_{\dot a \dot b \dot c} \to \tfrac{1}{2}(1-\gamma^5) \otimes \tfrac{1}{2}(1-\gamma^5) \otimes \tfrac{1}{2}(1-\gamma^5)\,\psi$$

etc onde $\psi = (\psi_{\alpha\beta\gamma})$ de Dirac $\alpha, \ldots \gamma = 1, \ldots 4$ simétrica em $\alpha\beta\gamma$

Devemos ter 8 compon. indep. para spin 3/2 partic. e não 20.

Admit. eqs. de Weyl

$$i(\sigma.\partial)_{\dot a b}\, \varphi^{bcd} = m\, \chi_{\dot a}{}^{cd}$$

$$\tfrac{1}{2}\{ i(\sigma.\partial)^{b\dot a} \chi_{\dot a}{}^{cd} + i(\sigma.\partial)^{c\dot a} \chi_{\dot a}{}^{bd} \} =$$

$$= m\, \varphi^{bcd}$$

podemos pôr:

$$\varphi^{abc} = \vec F^{a} . (\sigma C)^{bc}$$

$$\chi_{\dot a}{}^{bc} = \vec G_{\dot a} . (\sigma C)^{bc}$$

o que conduz às eqns:

$$i(\sigma.\partial)_{\dot a b}\, \vec F^{b} = m\, \vec G_{\dot a}$$

$$i(\sigma.\partial)^{a\dot b}\, \vec G_{\dot b} = m\, \vec F^{a}$$

com as conds:

$$\vec\sigma . \vec F = 0$$

$$\vec\nabla . G = 0$$

20) Vemos que o sist. passa a ser descrito por 6 $\vec{F}^a$ e 6 $\vec{G}_{\dot{a}}$ submet. a 4 cond. o que dá 8 compon. indep.

Caso neutrino com spin 3/2

se postularmos:

$$i(\sigma\cdot\partial)_{\dot{a}b}\,\varphi^{bcd} = 0$$

a decomposição:

$$\varphi^{bcd} = \vec{v}^{\,b}\cdot(\vec{\sigma}C)^{cd}$$

conduz às equs.

$$i(\sigma\cdot\partial)_{\dot{a}b}\,\vec{v}^{\,b}(x) = 0$$

com as conds. subsid.

$$\vec{\sigma}\cdot\vec{v} = 0$$

$$\vec{\nabla}\cdot\vec{v}(x) = 0$$

e as 6 quant. $\vec{v}^{\,a}(x)$ se reduzem a 2 indep.

$\vec{v}(x)$ corresponde ao vector $\vec{F} = \vec{B} - i\vec{E}$ de Majorana para spin 1

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Um neutrino de spin 3/2 com helicidade + 3/2 é descr. por

$\varphi^{abc}(p; s = 3/2) =$

$= (\vec{V}(p) \cdot \vec{e}_-(p))^a \; (\vec{\sigma} \cdot \vec{e}_+(p) C)^{bc}$

onde $\vec{\sigma} \cdot \vec{e}_+(p) C$ é a generalização de $\sigma_+ C$, e $\vec{V} \cdot \vec{e}_-$ a de V_-.

Para helicidade $-3/2$

$\eta_{\dot{a}\dot{b}\dot{c}}(p; s = -3/2) = (\vec{V} \cdot \vec{e}_+(p))_{\dot{a}}$

$(\vec{\sigma} \cdot \vec{e}_-(p) C)_{\dot{b}\dot{c}}$

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O problema do lagrangeano e da equação única para campos com spin 3/2.

I. Caso do spin 1.

As duas equações de Dirac para spin 1:

$$(i\gamma\cdot\partial - m)_{aa'}\,\psi_{a'b} = 0$$
$$(i\gamma\cdot\partial - m)_{bb'}\,\psi_{ab'} = 0$$

podem deduzir-se de uma só equ.:

$$\left\{\frac{i}{2}\left(\gamma^\mu_{aa'}\,\delta_{bb'} + \delta_{aa'}\,\gamma^\mu_{bb'}\right)\partial_\mu - m\,\delta_{aa'}\,\delta_{bb'}\right\}\psi_{a'b'} = 0$$

ou

$$\left\{\frac{i}{2}\left(\gamma^\mu\otimes I + I\otimes\gamma^\mu\right)\partial_\mu - m\, I\otimes I\right\}\psi = 0$$

[Multipl. à esq. por $\frac{i}{2}(\gamma^\mu\otimes I - I\otimes\gamma^\mu)\,\partial_\mu$ e obtem-se $\frac{i}{2}\{\gamma^\mu\otimes I - I\otimes\gamma^\mu\}\partial_\mu\psi = 0$

Soma e subtr. dão as 2 equs. acima]

Definindo

$$\overline{\Psi}_{ab} = \psi^*_{\alpha\beta}\, \gamma^0_{\alpha a}\, \gamma^0_{\beta b}$$

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o lagrang. é:

$$L = \overline{\Psi}\left\{ \frac{i}{2}\left(\gamma^\mu \otimes I + I \otimes \gamma^\mu\right)\partial_\mu - m\, I \otimes I \right\}\Psi$$

ou

$$L = \overline{\Psi}_{ab}\left\{ \frac{i}{2}\left(\gamma^\mu_{aa'}\, \delta_{bb'} + \gamma^\mu_{bb'}\, \delta_{aa'}\right)\partial_\mu - m\, \delta_{aa'}\, \delta_{bb'} \right\}\Psi_{a'b'}$$

que é equivalente ao de Proca

$$L = \frac{1}{2} \mathcal{G}^*_{\mu\nu}\, \mathcal{G}^{\mu\nu} - \frac{1}{2} \mathcal{G}^*_{\mu\nu}\left(\partial^\nu \varphi^\mu - \partial^\mu \varphi^\nu\right) - \frac{1}{2}\left(\partial^\nu \varphi^{\mu *} - \partial^\mu \varphi^{\nu *}\right)\mathcal{G}_{\mu\nu} + m^2 \phi^*_\mu\, \phi^\mu$$

com as eq.s:

$$\mathcal{G}_{\mu\nu} = \partial_\nu \varphi_\mu - \partial_\mu \varphi_\nu$$

$$\partial_\nu \mathcal{G}^{\mu\nu} + m^2 \phi^\mu = 0$$

$$\Downarrow$$

$$\partial_\mu \varphi^\mu = 0$$

II. Spin 3/2: equação de Rarita-Schwinger

Neste caso, na representação de Dirac $\psi = (\psi_{abc})$, $a, b, c = 1, 2, 3, 4$ decompomos:

$$\psi_{abc} = f_a^{\mu} (\gamma_\mu C)_{bc} - \frac{1}{2m} G_a^{\mu\nu} (\sigma_{\mu\nu} C)_{bc}$$

então para que ψ_{abc} seja simétrica obtém-se:

$$G_a^{\mu\nu} = \partial^\nu f_a^\mu - \partial^\mu f_a^\nu$$

e a equação:

$$(i\gamma^\mu \partial_\mu - m) f^\alpha = 0$$

com as condições:

$$\gamma_\alpha f^\alpha = 0$$

$$\partial_\alpha f^\alpha = 0$$

Basta pois considerar o spinor-vetor f_a^α, de Rarita-Schwinger do qual graças às condições só há 8 componentes independentes.

25 Assim, em vez de considerar f^{μ}_{a} estudamos f^{μ}_{a}, $\mu = 0, 1, 2, 3$ (vetor) $a = 1, 2, 3, 4$ (spinor) tal que:

$$(i\gamma.\partial - m) f^{\alpha} = 0$$

$$\gamma_{\alpha} f^{\alpha} = 0$$

$$\partial_{\alpha} f^{\alpha} = 0$$

Para obter lagrangeano precisamos de equação única que dê estas como resultado.

Há em princípio uma classe infinita de tais equações, a saber

$$(i\gamma.\partial - m) f^{\alpha} + iA(\gamma^{\alpha}\partial^{\beta} + \gamma^{\beta}\partial^{\alpha}) f^{+}_{\beta}$$
$$+ \gamma^{\alpha}(iB\gamma.\partial + mC)(\gamma_{\mu} f^{\mu}) = 0$$

com A, B, C constantes. Para obter as 3 equações acima chega-se a:

$$A \neq 1/2$$

$$B = \frac{3}{2} A^{2} - A + \frac{1}{2}$$

$$C = 3A^{2} - 3A + 1.$$

Rarita e Schwinger tomaram originalmente:

$$A = \frac{1}{3} \Rightarrow B = C = \frac{1}{3}$$

Entretanto, inspirando-se na teoria com spin 1, pode-se impôr à equação com A, B, C, que seja invariante em relação à transformação de calibre:

$$f^{\alpha} \to f^{\alpha} + \partial^{\alpha}\chi$$

onde χ é um espinor arbitrário, quando $m=0$. Isso dá:

$$A = B = C = 1.$$

Assim a equação de Rarita-Schwinger que admitimos é:

$$\left\{ (i\gamma\cdot\partial - m) g^{\alpha\beta} - i(\gamma^{\alpha}\partial^{\beta} + \gamma^{\beta}\partial^{\alpha}) + \gamma^{\alpha}(i\gamma\cdot\partial + m)\gamma^{\beta} \right\} f_{\beta} = 0$$

Pela identidade:

$$\epsilon_{\alpha\mu\nu\beta}\gamma^{5}\gamma^{\mu} = i\left\{ \gamma_{\alpha}\gamma_{\nu}\gamma_{\beta} + g_{\alpha\beta}\gamma_{\nu} - g_{\alpha\nu}\gamma_{\beta} - g_{\nu\beta}\gamma_{\alpha} \right\}$$

$$Z^0 \to \nu_{3/2}\ \bar{\nu}_{1/2}$$

$$\frac{\Gamma(Z^0 \to \nu_{3/2}\ \bar{\nu}_{1/2})}{\Gamma(Z^0 \to e^+_{1/2}\ e^-_{1/2})} =$$

$$= \frac{1}{2(a^2+v^2)}\ \frac{(1-x)^3}{x^2(1+x)}\ (1+2x+3x^2)$$

$a^2 = 1, \quad v^2 = 1 - 4\sin^2\theta_W$

x x

Considere

$$Z^0 \longrightarrow e^+_{3/2}\ e^{-1/2}$$

$$e^+_{3/2} \longrightarrow e^+_{1/2}\ \gamma$$

Branching ratio $\sim \frac{1}{2x^2}\ \frac{(1-x)^3}{(1+x)}\ (1+2x+3x^2)$

Obtem-se

$$J^{\nu}_{\gamma} = -i\,\epsilon^{\alpha\mu\nu\beta}\,\bar{e}_{\alpha}\,\gamma^5\gamma_\mu\,e_\beta$$

$$J^{\nu}_{W^-} = -i\,\epsilon^{\alpha\mu\nu\beta}\,\bar{e}_{L\alpha}\,\gamma^5\gamma_\mu\,\nu_{L\beta}$$

$$J^{\nu}_{Z} = -i\epsilon^{\alpha\mu\nu\beta}\Big\{\bar{\nu}_{L\alpha}\,\gamma^5\gamma_\mu\,\nu_{L\beta} +$$

$$+\,\bar{e}_\alpha\,\gamma^5\gamma_\nu\left(\frac{3g'^2-g^2}{2(g^2+g'^2)}+\frac{1}{2}\gamma_5\right)e_\beta\Big\}$$

x x x

Calcularmos proc. envolvendo leptons e quarks com spin 3/2 para saber se são observados.

Caso de leptons podemos admitir um lepton carregado com spin 3/2 em cada família

Se lepton é composto de três objetos com spin 1/2 então podemos ter

C 32

no Spin 3/2 ν

	1/2	3/2
$L_e = 1$	ν_e, e	$\mathcal{E}^-\ (\mathcal{E}^+)$
$L_\mu = 1$	ν_μ, μ	$\mathcal{M}^-\ (\mathcal{M}^+)$
$L_\tau = 1$	ν_τ, τ	$\mathcal{T}^-\ (\mathcal{T}^+)$
	$\uparrow\downarrow\uparrow$	$\uparrow\uparrow\uparrow$

$$e^+ + e^- \to \gamma^* \to L^+ + L^-$$

$$R = \frac{\sigma(e^+e^- \to L^+L^-)}{\sigma(e^+e^- \to \mu^+\mu^-)} =$$

$$= \frac{|\vec{P}|}{E}\left(\frac{16}{9}\gamma^4 - \frac{8}{9}\gamma^2 - \frac{2}{9} + \gamma^2\right)$$

$$\gamma = \frac{E}{M}$$

$$E \ll 4M$$

Weak currents (Phenomenology)

Simplest: $J_1^\alpha = \bar{u}^\alpha(P)(1 + a\gamma^5)u_{\nu_e}(K)$

$$J_2^\alpha = \frac{\lambda}{M}\bar{u}^\lambda(P)\, q_\lambda \gamma^\alpha (1 + a\gamma^5) u_{\nu_e}(K)$$

$$q = K - P$$

$$L^- \to \pi^- + (\nu_L)_{L,R}$$

$$\Gamma = \frac{(G m_p^2)^2}{192\pi}\left(\frac{M}{m_p}\right)^2\left(\frac{m_\pi}{m_p}\right)^2 M \cdot$$

$$\cdot\left(1-\frac{m_\pi^2}{M^2}\right)^4$$

τ		M (GeV)
	1.19×10^{-12}	5
	1.86×10^{-14}	20
	3.54×10^{-16}	75
	1.49×10^{-16}	100

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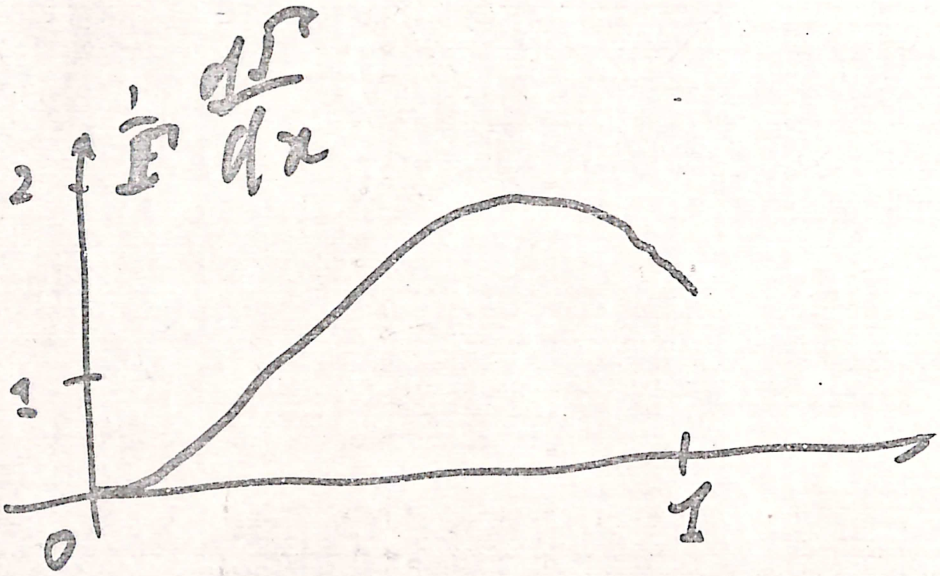
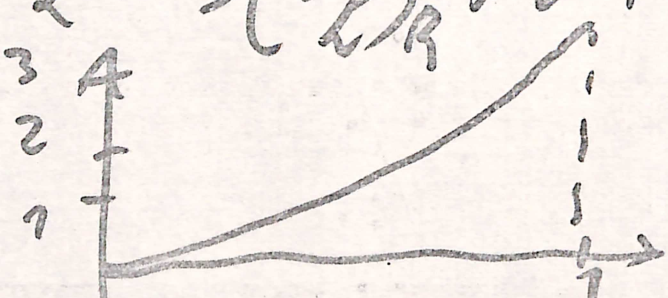
$$L^- \to (\nu_L)_L + l^- + \bar{\nu}_l$$

$$\frac{d\Gamma}{dx} = \frac{G m_p^2}{288\pi^3}\left(\frac{M}{m_p}\right)^4 M \, \frac{x^2\left(\frac{5}{4}x^2 - 3x + 3\right)}{\left(1 - \frac{1}{2} x \, M^2/m_W^2\right)^2}$$

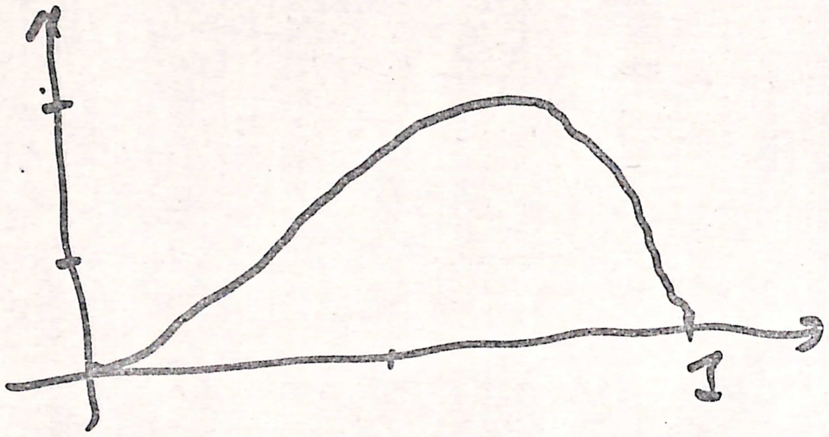
$$x = \frac{2E_l}{M}, \quad 0 \le x < 1$$



$$L^- \to (\nu_L)_R + l^- + \nu_l$$



$$L^{+} \to (\nu_\ell)_L + \ell^{+} + (\nu_e)_L$$



[illegible] possibili doublets:

$\nu_e \quad e$ $\qquad\qquad$ $\nu_\Sigma \quad \Sigma$

$\nu_\mu \quad \mu$ $\qquad\qquad$ $\nu_M \quad M$

$\nu_\tau \quad \tau$ $\qquad\qquad$ $\tau_T \quad T$

$$\nu_\mu + e \to \nu_M + e$$

$$M^{-} \to \mu^{-} + e^{-} + e^{+}$$

$$M^{-} \to \mu^{-} + \mu^{-} + \mu^{+}$$

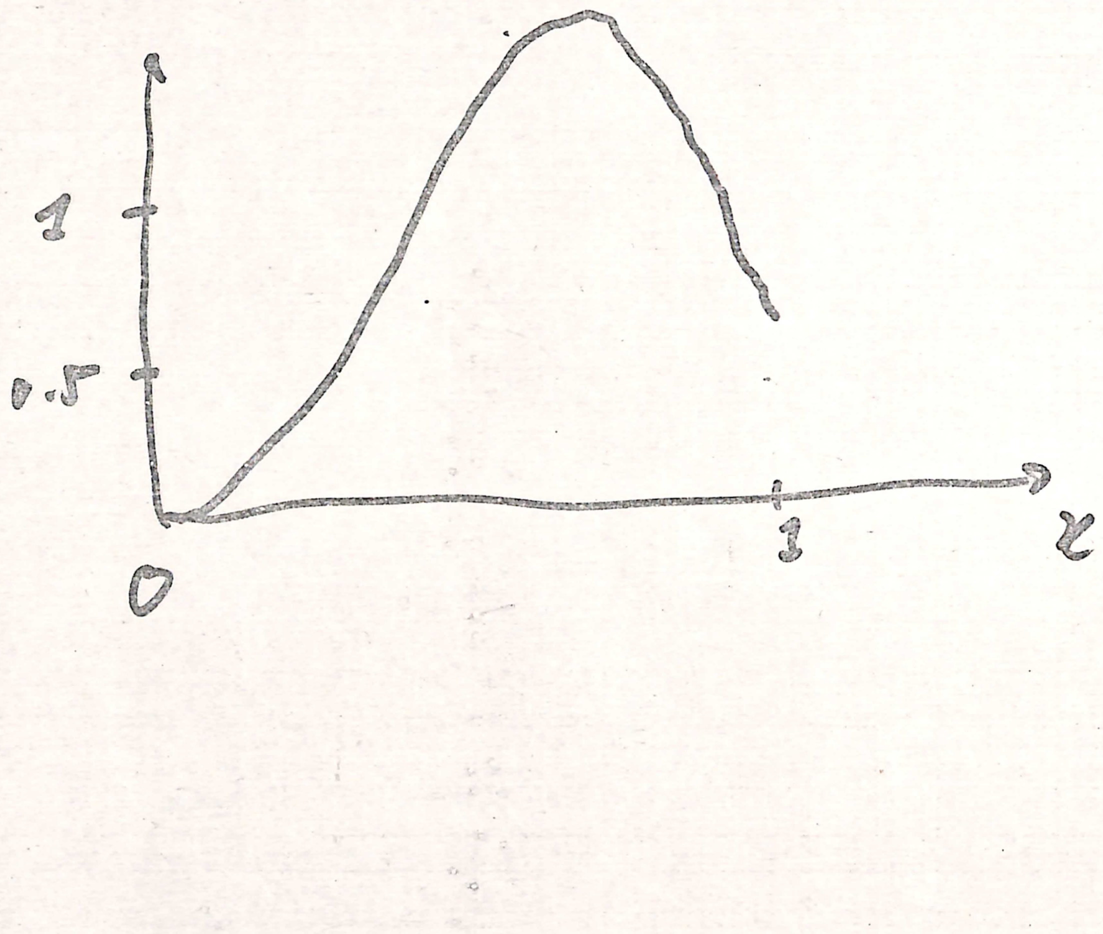
$$L \to \nu_L + \pi$$

$$L \to \nu_L + e^{-} + \bar{\nu}_e$$

$$\eta^- \rightarrow \mu^- + e^- + e^+$$

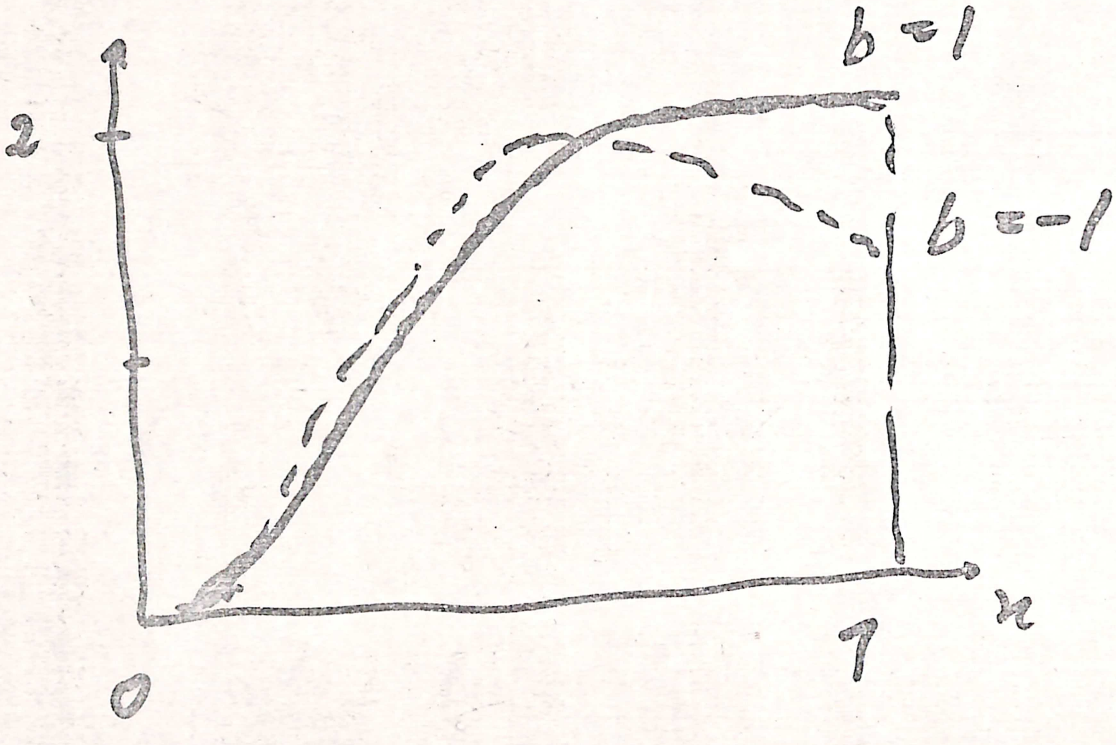
$$M = \frac{f}{\sqrt{2}} \left[\bar{u}(p_\mu)(1+b\gamma^5)u_\alpha(p_\eta)\right] \cdot$$

$$\left[\bar{u}(p_e')\gamma^\alpha(1+a\gamma^5)v(p_e)\right]$$



$\eta^- \to \mu^- + \mu^- + \mu^+$

η = previous antisym. in two μ's.



Em relação às experiências recentes no CERN

$$Z \to e^+ e^- \gamma$$

$$p\bar{p} \longrightarrow e^+ + e^- + \gamma + X$$

onde γ é mais energético que bremsstrahlung na teoria de Salam-Weinberg, podemos ter duas possibilidades

L^- pode ser lepton pesado com massa ~ 50 GeV segundo Cabibbo, Maiani e Srivastava com interação de Pauli

$$F^{\mu\nu} \bar{e}\, \sigma_{\mu\nu} L$$

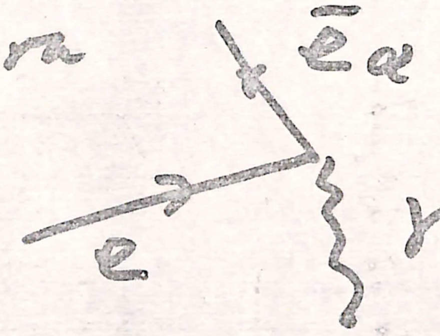
ou pode ser devido à criação de lépton com spin 3/2 como L, graças à interação:

$$L = -\kappa\, \epsilon^{\mu\nu\alpha\beta}\, \bar{L}_\mu \gamma^5 \gamma_\nu D_\alpha D_\beta L$$

que dá lugar a interação tipo Pauli

$$\kappa\, \epsilon^{\alpha\mu\nu\beta}(\bar{e}_\alpha \gamma^5 \gamma_\mu e) F_{\nu\beta}$$

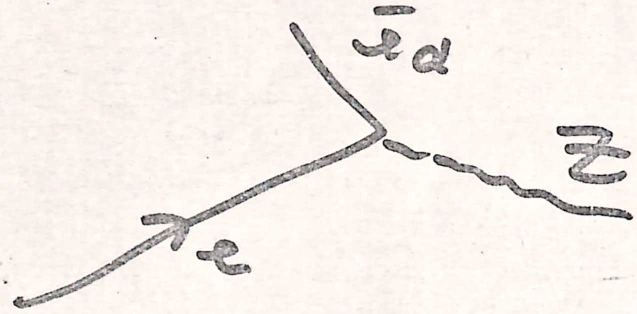
para



e

$$\kappa\, \epsilon^{\alpha\mu\nu\beta}(\bar{e}_\alpha \gamma^5 \gamma_\mu e) G_{\nu\beta} \cdot \frac{1}{4}\sqrt{g^2+g'^2}\ ; \quad G_{\nu\beta} \sim \partial_\beta Z_\nu - \partial_\nu Z_\beta$$

para o vértice



com acoplam. axial

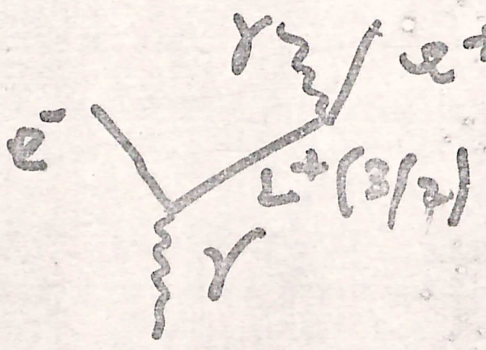
e:

$$K \frac{3g'^2 - g^2}{4\sqrt{g^2+g'^2}} \cdot$$

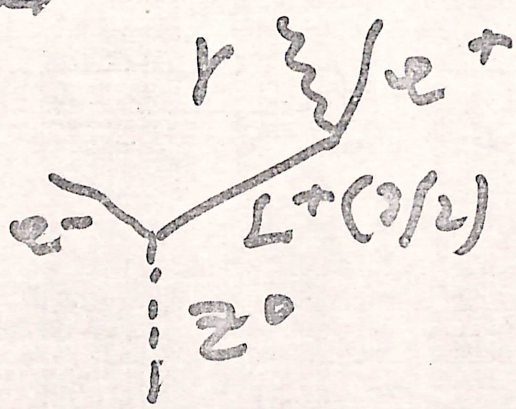
$$\cdot\, \epsilon^{\alpha\mu\nu\beta} (\bar{e}_\alpha \gamma^5_\mu e)\, g_{\nu\beta}$$

para acopl. vectorial.

Assim podemos ter



e



Desintegrações possíveis:

$$Z^0 \to e^+_{3/2}\, e^-_{1/2}$$

We obtain in the standard model

$$\frac{\Gamma\left(Z^0 \to e^+_{3/2}\, e^-_{1/2}\right)}{\Gamma\left(Z^0 \to e^+_{1/2}\, e^-_{1/2}\right)} = \frac{1}{4x^2}\,\frac{(1-x)^3}{(1+x)}\left(1+2x+3x^2\right)$$

$$x = \left(\frac{M_{3/2}}{M_Z}\right)^2$$

× ×

$$W^\pm \to e^+_{3/2}\, \nu_{1/2}$$

$$W^\pm \to e^+_{1/2}\, N_{3/2}$$

$$\frac{\Gamma\left(W^\pm \to E^+_{3/2}\, \nu_{1/2}\right)}{\Gamma\left(W^\pm \to e^+_{1/2}\, \nu_{1/2}\right)} = \frac{1}{4y^2}\,\frac{(1-y)^3}{1+y}\left(1+2y+3y^2\right)$$

$$y = \left(\frac{M_{3/2}}{M_W}\right)^2$$