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Pathaways of the electron in the hydrogen atom.  
A generalization of the relativistic equivalen-  
ce principle

by

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I n t r o d u c t i o n

In previous papers (Rocha e Silva, 1978,79) we have suggested a geometrical pattern to account for the emission and absorption of radiation by the hydrogen atom, in the well known model introduced by Bohr in 1913-15, and developed by Broglie (1925,27), Schrödinger (1925,27), Heisenberg (1930) and all those who have developed the wave-particle pattern for the electron, associating a quantum of energy to a particle of frequency ( $\nu$ ) and energy ( $h\nu$ ), according to the basic principles, formulated by Planck (1901) and Einstein (1905, 52). None-the-less, the fundamental structure of matter as a remanescent of classical Newtonian views, continued to infiltrate into the new model as far as the relations between matter and gravitational or electrostatic field are concerned. Great changes have been introduced by quantum mechanics in the relationship between matter and space, but the possibility of considering relativistic time as a parameter to define the stationnary orbit, to my kwoledge, has never been considered. Also the possibility of considering a "time mass", i.e. an inertial coefficient ( $m_{44}$ ) significant only along the time (ict) parameter remained as a theoretical possibility in the



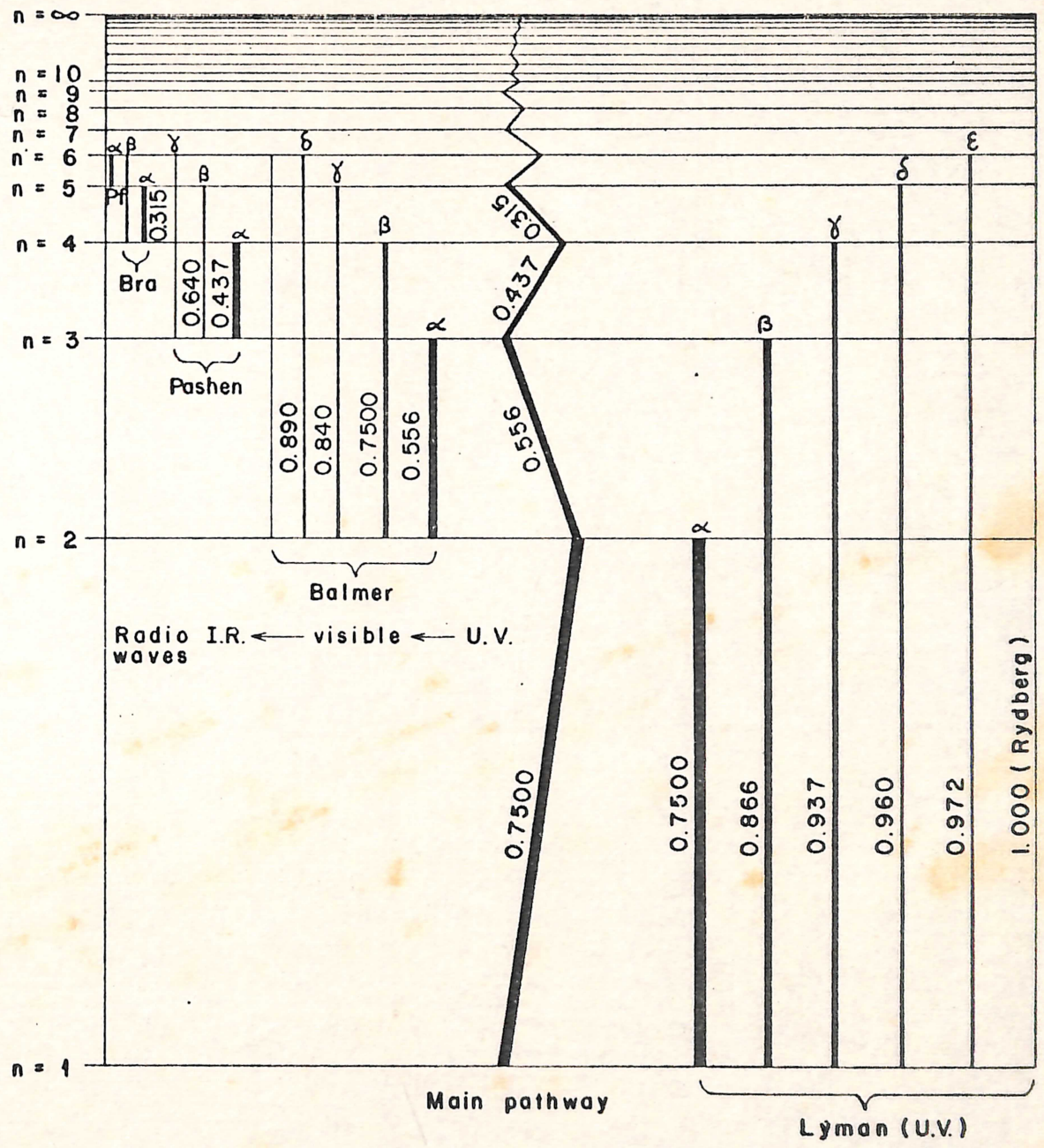


Fig 1. Pathways of the electron inside the Hydrogen atom.



Table I

Coefficients of "Red-shift" ( $\Delta f/f$ ) of spontaneous emissions in the hydrogen atom

| Frequencies ( $f_n$ )<br>$\times 10^{15}$<br>( $Rc/n^2$ ) | I               |         | II              |         | III             |         | IV              |         |
|---|-----------------|---------|-----------------|---------|-----------------|---------|-----------------|---------|
|   | Single jumps    |         | Double jumps    |         | Triple jumps    |         | Quadruple jump  |         |
|   | (Found)         | (Calc.) | (Found)         | (Calc.) | (Found)         | (Calc.) | (Found)         | (Calc.) |
| $f_1 = Rc = 3.287,870$                                    |                 |         |                 |         |                 |         |                 |         |
| $f_2 = 0.821,973$   | (1→2)<br>0.7500 | 0.750   | (1→3)<br>0.864  | 0.866   | (1→4)<br>0.896  | 0.910   | (1→5)<br>0.926  | 0.930   |
| $f_3 = 0.365,318$   | (2→3)<br>0.556  | 0.562   | (2→4)<br>0.7500 | 0.7499  | (2→5)<br>0.840  | 0.828   | (2→6)<br>0.889  | 0.865   |
| $f_4 = 0.205,491$   | (3→4)<br>0.437  | 0.437   | (3→5)<br>0.640  | 0.649   | (3→6)<br>0.750  | 0.753   | (3→7)<br>0.816  | 0.804   |
| $f_5 = 0.131,352$   | (4→5)<br>0.315  | 0.316   | (4→6)<br>0.555  | 0.562   | (4→7)<br>0.673  | 0.689   | (4→8)<br>0.750  | 0.748   |
| $f_6 = 0.091,330$   | (5→6)<br>0.236  | 0.237   | (5→7)<br>0.489  | 0.487   | (5→8)<br>0.609  | 0.624   | (5→9)<br>0.691  | 0.696   |
| $f_7 = 0.067,099$   | (6→7)<br>0.165  | 0.178   | (6→8)<br>0.437  | 0.422   | (6→9)<br>0.556  | 0.568   | (6→10)<br>0.691 | 0.696   |
| $f_8 = 0.051,375$   | (7→8)<br>0.096  | 0.133   | (7→9)<br>0.334  | 0.366   | (7→10)<br>0.510 | 0.529   | (7→11)<br>0.595 | 0.602   |
| $f_9 = 0.040,591$   | -               | -       | (8→10)<br>0.320 | 0.316   | (8→11)<br>0.470 | 0.470   | (8→12)<br>0.555 | 0.559   |
| $f_{10} = 0.038,787$                                      | -               | -       | -               | -       | -               | -       | -               | -       |
| $f_{11} = 0.022,832$                                      | -               | -       | -               | -       | -               | -       | -               | -       |
| $f_{12} = 0.022,832$                                      | -               | -       | -               | -       | -               | -       | -               | -       |

Note - That the ratios of column I appear with decreasing frequency in the multiple jumps.



formulation of relativity theory and Minkowski four dimensional continuum.

The main concern in the following report is to show that inside the hydrogen atom, the pathways of the electron, when jumping from orbit to orbit (k to i, or vice-versa), are regulated by changes in frequency, each orbit being characterized by a time ( $f_i^{-1} = T_i$ ) parameter that behaves as a relativistic "local time", strictly deterministic, dependent upon the powerful electrostatic field existing inside of the hydrogen atom.

As a basic assumption, when the electron jumps from orbit k to orbit i with emission of a strictly monochromatic frequency ( $f_{ik}$ ) it follows the rules of a parallel displacement in a Weyl geometric continuum along the relativistic time ( $y^4 = ict$ ) coordinate. The coefficient of homothety in such non-euclidian unidimensional ( $f_i^{-1} = \text{"local time"}$ ) coordinate, representing changes of "gauge", could be identified to a coefficient of relativistic "red-shift" ( $\Delta f/f$ )<sup>n</sup>, where n is the principal quantum number. According to the theory, the values of such coefficients regulated by the local intensities of the electrostatic field would be able to produce "red-shifts" of the order of  $0.75^n$  to 1.00, to compare with the order of  $10^{-28}$  as observed in the surface of the earth (See Einstein, 1915,17).

One of the consequences of the theory, if confirmed by experiment, would be the possibility of shifting the frequency of a laser ray from high frequency (in the U.V. or blue) to the red or even to lower frequencies (I.R. and radio waves) by exposing it to electrostatic fields of the order of  $10^{28}$  e.s.u. or to fields  $10^{48}$  times that of gravity in the surface of the earth, as shown in continuation. If technically realisable,



such an experiment will cause the marvel of transforming, for instance, a blue laser of frequency  $0.605 \times 10^{15}$  Hertz ( $H\beta$ ) to one in the red, of frequency  $0.455 \times 10^{15}$  ( $H\alpha$ ), with emission of an I.R. quantum of frequency  $0.165 \times 10^{15}$  Hertz in the Paschen series. However, since a laser beam is already free from its constraints in the stationary orbits, continuous variations of the electrostatic field, may produce small or desirable changes of frequency, in a process of "modulation". Such a possibility which has the optimistic consequence of providing the means to "modulate" masers and lasers, may be useful for their main application in communication operations, as suggested by (1972).

Another question that may receive a fresh apportion of light, is that concerned with the time during which the electron stays in different orbits. We quote Silfvast (1973): "Electrons at certain levels decay (fall to a lower state) more easily than electrons at other levels. Each excited electronic state of the atom has a characteristic lifetime that indicates the average time an electron takes to fall to a lower level and thus radiate a photon. Most excited states have lifetimes of about  $10^{-8}$  second... There are some excited states or levels in all atoms in which an electron cannot decay easily by giving up a photon... Electrons in energy states of this kind tend to stay there for relatively long periods of time (0.001 second or more) and are referred to as being in metastable states. They play an important role in storing energy... which can then be employed in the excitation process of a metal-vapor laser... The normal radioactive decay from a higher to a



lower state is termed spontaneous emission... the stimulated electron can move to a lower level, provided such a level exists and that the difference between the two levels correspond exactly to the energy of the colliding photon. This is the process of stimulated emission." Those are essentially the basis to produce lasers and masers, according to W.T. Silfvast(1973).

### The new pattern of the electron

The pattern suggested in our previous papers was to consider the electron as formed of a stable core surrounded by a halo formed by "one Rydberg", namely by a quantum of light or photon in the U.V. with a basic frequency represented by the constant value.

$$R_{BR} \text{ or } R_c = 3.287,870 \times 10^{15} \text{Hertz}$$

or cycles/sec. It is that part of the electron that determines the multiple pathways of the particle when jumping from one orbit i to k or vice-versa, according to the equation

$$f_{ik} = \pm (f_k - f_i) \quad (1)$$

or, in terms of energy levels:

$$h.v = \pm (E_k - E_i) \quad (2)$$

Since the parameters of the orbits, as shown in (1) only involve the values of the frequencies  $f_i$  and  $f_k$ , we may assume that what determines the choice of one or the other orbit (i or k) is a relativistic time parameter ( $f_i = T_i^{-1}$ ), therefore the reciprocal of a frequency ( $T_i = f_i^{-1}$ ). Based on such a postulate



the jumps have been understood as resulting from parallel displacements along the relativistic time parameter, according to the rules of Weyl's geometry, i.e. with adaptation of metric or "gauge". See Weyl (1922-52).

As the geometry of the jump can be considered independent of the "space coordinates", for commodity, we have defined a component of mass, the "time mass" ( $m_{44} = -f/c^2$ ) which would play the role of the inertial mass along the time ( $y^4 = ict$ ) coordinate. It is easy to see that by defining "time mass" in terms of  $-f/c^2$  (frequency/square of light velocity), it would play a similar role as "space mass" ( $m_{ii}$ ) in terms of energy ( $E = hf$ ) in the known formula  $m_{ii} = E/c^2$ . In that situation, Planck's constant ( $h = h/2\pi$ ) would play the role of a factor of proportionality transforming "time mass" into "space mass":

$$m_{ii} = \frac{h}{2\pi} \cdot m_{44} \quad (3)$$

Emission or absorption of a photon of light with a frequency  $f_{ik}$ , will denote a reduction of "time mass" ( $m_{44}$ ):

$$\Delta m_{44} = -\Delta g_{44} = f_{ik}/c^2 = \pm (f_i - f_k)/c^2 \quad (4)$$

This is the proposed law of emission or absorption of radiation in the hydrogen atom. In many equations, we may drop the coefficient  $1/c^2$  and use in a colloquial way

$\Delta m_{44} = f_{ik} = (f_k - f_i)$ . From equation (4) we may deduce directly a coefficient of "red-shift"  $\Delta f/f$ , which from depend all frequencies of radiation spontaneously emitted by the hydrogen atom, according to the model of Bohr, as shown in continuation.



### Transits inside the hydrogen atom

In its transits inside the hydrogen atom, from orbit k to i or vice-versa, the electron obeys the following rules:

a) If k is above the i level in terms of frequency or energy ( $f_k > f_i$ ) or ( $-E_i > -E_k$ ), the transit from i to k (ascensional) is one in which an exact amount of frequency ( $f_i - f_k$ ) or energy  $-(E_k - E_i)$  must be added to the electron by way of external force or energy, in the form of heat, for instance, or collision with other atoms, or by an apport of a quantum exactly equal to the differences  $\hbar (f_i - f_k) = -(E_k - E_i)$ . Any excess above the maximum ( $E_m - E_0$ ) will appear in the form of kinetic energy, according to the theory developed for the photo-electric effect by Einstein (1905, 52). Therefore, if we consider the electron in its ground state, the frequency or energy needed by the electron to attain its highest level (1) of frequency, or lowest level of potential energy (-13.6 e.Volts), involves absorption of an U.V. quantum of energy represented by Rydberg constant ( $Rc = 3.287,870$  Hertz). In that sense, Rydberg constant which plays such a fundamental role in all our calculations, could also be represented in terms of energy, as

$$\text{"One Rydberg"} = -13.6 \text{ e.Volts} = \text{ergs.}$$

For the unitage in which Rydberg constant should be given, see Rocha e Silva (1979). We prefer to use the expression  $Rc = cR$  i.e.  $R_H = 109,677.459 \times 2.99 \times 10^{10} = 3.287,870$  cycles/sec.

If i is below the k level, then transition from k to i involves emission of exactly the same quantum as that required to produce the inverse transition (by absorption) of a quantum,



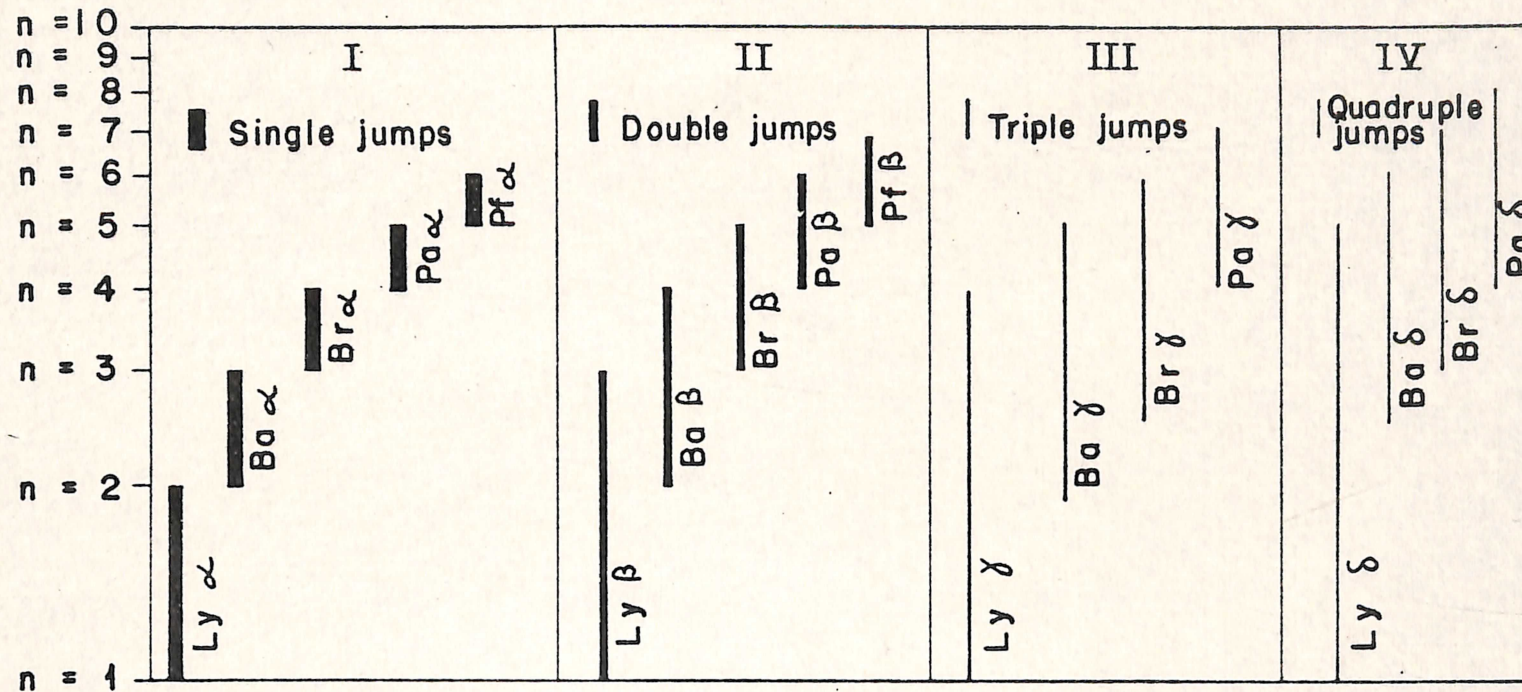
that is a square fraction  $R_c(\frac{1}{i^2} - \frac{1}{k^2})$  of Rydberg constant (Rc)

### The stationary orbits

b) What makes the idea of the red-shift plausible is the unavoidable circumstance that any "spontaneous event" occurring in the hydrogen atom, is the emission of quanta cascading from the high U.V. level to low U.V., to blue, to red, to I.R. to radio waves. It is a law of nature of the same generality as the equivalence of inertial mass and weight, so profitably used by Einstein in his formulation of the general Relativity Theory (Einstein, 1917, 52).

Under such general formulation we can consider the different states (stationary orbits) of the electron inside the hydrogen atom as represented in Fig 1 and 2 and Table I. We can consider two main directions, one so to speak strained, the "ascensional" one, as studied in the previous paragraph (a) that requires enormous amounts of energy apported to "ascend" from the ground state (0 frequency and  $-\infty$  energy to any of the levels indicated (from  $-\infty$  energy to ... 3, 2, 1 and to the free electron). For instance, it has been calculated that to ascend from ground state to level 2 it is necessary to apport an energy corresponding to 5,000 degrees  $^{\circ}\text{K}$ , as calculated by applying Boltzmann distribution law (see Eisberg, 1961) a temperature that can be found in some hot stars (Novae). This indicates that we are going uphill doing enormous work that





Classes of radiation ( $\alpha$ . $\beta$ . $\gamma$ . $\delta$ .....)

Fig. 2. Schematic representation of the spectral lines of the hydrogen atom. Note that the lines are assembled in classes, not in series, see Text.



is reverted when the electron jumps from a level 2 to  $\infty$  (ground state). To raise an electron from its ground state to level 1, we need a still higher temperature, and finally to eject the electron from its ground state to a free state, we would need a level of temperature of the order of

$$3.73 \times 10^5 \text{ degrees Kelvin}$$

i.e. of the order of 373 thousand degrees Kelvin! a temperature observed in the hottest Novae stars, when photographed with an U.V. sensitive cameras, or from Satelites above the ionosphere. This amount of energy from level 1 to ground state, is emitted spontaneously in the form of a cascade of quanta from U.V. to violet, to blue, to red to I.R. to Radio waves, and so forth, in the form indicated in Fig.2, I, by quanta of single jumps. This sequence of single jumps may be considered the one of highest probability, being the central pathway of Fig. 1, and was called the "fundamental class" as represented in Fig. 2, I. What characterizes such single jump lines is that they are the broadest of the spectrum of emission of the hydrogen atom, and therefore that of highest probability as shown in Fig. 2. We are going to see that they constitute the "main gate" from the highest excited state (1) of the electron to its ground state, all depending upon a shift to the red ("red-shift") of  $(\Delta f/f)^n = (0.7500)^n$ . The sequence of powers of 0.75 as given in Table I, col. I, represent all frequencies emitted by single jumpers, as deduced directly from spectroscopical data given in col. I (found) or Fig. 2 (I).



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All other lines of the hydrogen spectrum depend on double, triples, quadruple jumps represented by thinner and thinner lines of the spectrum, as seen in Fig. 2. Such is the classical indication that these jumps are of less and less probability. However, we are going to indicate that it is not only by chance that such a phenomenon occurs. The other explanation for this decreasing intensity (lower and lower probability) is the fact shown in Table I, that the distribution of the fundamental "red-shift"  $(0.7500)^n$  appears with a half frequency in the sequence of double jumps, with one third of frequency in the triple jumps, a fourth of frequency in the quadruple jumps, and so-forth. Note that any of the values of powers of  $0.75^n$  (such as 0.556, or 0.437, or 0.315 and so-forth) correspond to the same "red-shift" coefficient  $(0.7500)^n$ , for increasing values of  $n$ . Furthermore it is also to be noted that the first horizontal line of Table I, corresponds to the observed (found) lines of the Lyman series in the U.V. region of the hydrogen spectrum; the second line corresponds to the visible Balmer series ( $H_\alpha$   $H_\beta$   $H_\gamma$   $H_\delta$  and so-forth), as shown in Fig. 1. The following horizontal lines correspond to the Paschen, Brackett and Pfund series in the I.R. and in further shorter waves (radio waves). It is to be reminded that to raise the electron energy to such a potential difference (level 1) in relation to the ground state, we would need the enormous amount of energy of -13.6 e.V, that corresponds to the ionization energy of the hydrogen atom. If one calculates the temperature to attain such a degree of ionization, we would need a level of temperature of the order of  $3.73 \times 10^5$  degrees  $^\circ\text{K}$ , as seen above i.e. of the order

(3)



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of 373 thousand degrees Kelvin! It is obvious that such levels can be attained by absorption of a quantum in the U.V., as Lyman  $\alpha, \beta, \gamma$  and so-forth, and the discharge in a hydrogen filled tube may knock out the electron giving to it a surplus of energy, that is transformed in kinetic energy of the free electron. A pictorial view of the phenomenon is given in Fig. 5, left band.

The electrostatic field as the origin of the "red-shift"

c) This preliminary discussion of items a and b, brings us to the most important phase of our story, namely to the discussion about wherefrom arises such an enormous force opposing the "ascension" of the electron inside the hydrogen atom. It is certainly not depending upon anything resembling a gravitational force operating upon the extremely small weights of the proton-electron system of the order of  $10^{-24}$  g or  $10^{-28}$  g an infinitesimal acceleration to the particles. This acceleration of the gravitational field inside of the hydrogen atom at a distance  $r_0 = 0.53 \times 10^{-8}$  (the classical Bohr's radius), can be easily calculated by the formula

$$F_g = \frac{G \cdot m_e \cdot m_p}{r_0^2} \quad \text{dynes / cm} \quad (4)$$

where  $G = 6.664 \times 10^{-8}$ , the constant of gravity;  $m_e = 9.104 \times 10^{-28}$  g the mass of the electron;  $m_p = 1.67 \times 10^{-24}$  g, the mass of the proton; and  $r_0^2 = 0.281 \times 10^{-16}$  cm<sup>2</sup>, the square of Bohr's radius, in its ground state. Substituting all such values in the above formula

$$F_g = 3.605 \times 10^{-42} \quad (4a)$$

as the strength of the gravitational field created inside of the



atom by the two elementary particles, or else the force exerted by the earth gravitational field (of constant G) upon the particles with the indicated inertial masses ( $m_e$  and  $m_p$ ).

If the electrostatic force is calculated by the formula:

$$F_e = \frac{e^2}{(4\pi\epsilon_0) \cdot r_0^2} \quad (5)$$

where  $e = 4.80 \times 10^{-10}$  and  $e^2 = 2.304 \times 10^{-19}$ , the square of the charge of the electron:  $4\pi\epsilon_0 = 3.33 \times 10^{-10}$ ,  $\epsilon_0$  being the coefficient of permmissivity of the electrostatic field, the electrostatic force comes to

$$F_e = 2.462 \times 10^7 \text{ e.s.u} \quad (5a)$$

The quotient of both values will give the relative potential  $\phi_{rel.} = F_e/F_g = 6.827 \times 10^{48}$  (See Table II, Fig. 3.) (6)

This value of  $\phi_{e/g}$  will enter the formula of the red-shift

$$\frac{\Delta f}{f} = \frac{\phi_{rel}}{c^2} = \frac{0.827 \times 10^{48}}{2.99 \times 10^{20}} = 0.7586 \times 10^{28} \quad (7)$$

This value is to compare with the red-shift observed in the surface of the earth, of the order of  $0.741 \times 10^{-28}$ . Such a coincidence indicates that a red-shift produced by such strong accelerating field, brings almost exactly the value of the intra-atomic red-shift to the observed 0.7500 as indicated in Table I.

The other excellent approximation is to the value found in Table I, for single jumpers:

$$\left( \frac{f_m - f_{m-1}}{f_m} \right)^n = (0.7500)^n \quad (8)$$



As seen above, such a value can be obtained by inserting in equation (5) the value of  $4\pi\epsilon_0 = 3.33 \times 10^{-10}$ , giving in a first approximation, a "red-shift of

$$\frac{\Delta f}{f} = 0.758 \quad (8a)$$

It is clear that the value given to  $\epsilon_0$ , and therefore to  $4\pi\epsilon_0$  will change the value of the relative potential  $(\phi_{rel}) = F_e/F_g$  and therefore the nominator in the ratio giving the relativistic "red-shift"

$$\frac{\Delta F}{f} = \frac{\phi_{rel}}{c^2} = \frac{F_e}{F_g \cdot c^2} \quad (9)$$

and indicated in Table II and Fig 3. If small variations in the value of  $4\pi\epsilon_0$  are introduced in formula (5) for the calculation of the electrostatic field we can regulate the values of the "red-shifts" in the ultra-violet (U.V.) region of the line spectrum, from 0.750 to 1.00 accounting for the lines in the Lyman series, as indicated in the first horizontal line of Table I and in Fig. 1. It is easy to see that "red-shift" values from 0.750 (Lyman Ly<sub>α</sub>) to 1.00 (Rc constant) in the highest U.V. regions of the spectrum, can be calculated by inserting values of  $4\pi\epsilon_0$  comprised between  $3.32 \times 10^{-10}$  to  $2.5 \times 10^{-10}$ , and all the possible U.V. lines will be emitted by values of  $\phi_{rel}$  going up from  $6.77 \times 10^{48}$  to  $9.08 \times 10^{48}$ , with a range of "red-shifts" from 0.7500 to 1.00, as seen in Table II. In summary, the value of  $\epsilon_0$  or of  $4\pi\epsilon_0$  is the key to calculate the value of the ratio  $\phi_{rel} = \text{electrostatic potential} / \text{gravidic potential} = \frac{F_e}{F_g}$ . The magnitude of such a ratio is of the order  $5.44$  to  $9.084 \times 10^{48}$ , giving coefficients of "red-shift" of the order of  $10^{28}$ , covering the whole line spectrum of the hydrogen atom.



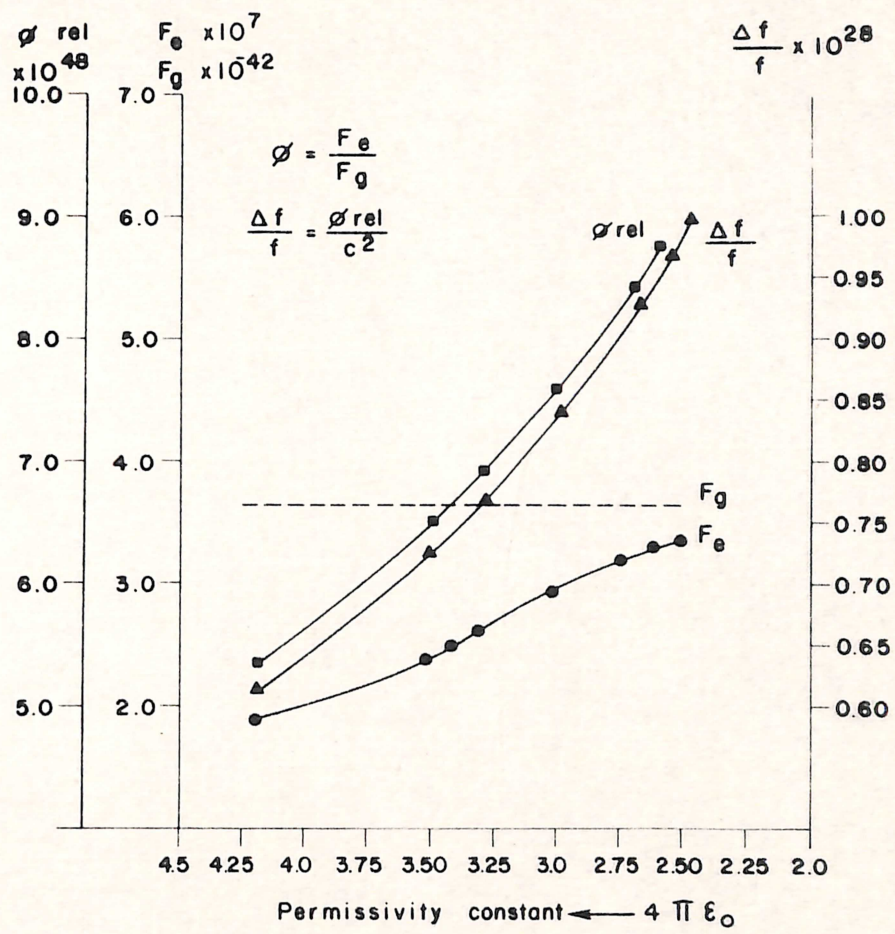




Table II

Values of the ratio  $\phi_{rel} = \frac{F_e}{F_g}$ , in function of the values of  $4\pi\Sigma_0$  adopted ( $r_0 = 0,53 \times 10^{-8}$ )

| Values of $4\pi\Sigma_0$ | Values of $F_g$ (const.) at $r = r_0$ | Values of $F_e$ at $r = r_0$ | Relat. Potential $\phi \frac{e}{g}$ | Relativistic Red-shift $\frac{\Delta f}{f} = \frac{\phi}{c^2}$ ( $\times 10^{28}$ ) |
|--------------------------|---------------------------------------|------------------------------|-------------------------------------|---|
| $4.18 \times 10^{-10}$   | $3.605 \times 10^{-42}$               | $1.96 \times 10^7$           | $5.44 \times 10^{48}$               | 0.605   |
| $3.50 \times 10^{-10}$   | $3.605 \times 10^{-42}$               | $2.34 \times 10^7$           | $6.498 \times 10^{48}$              | 0.722   |
| $3.36 \times 10^{-10}$   | $3.605 \times 10^{-42}$               | $2.44 \times 10^7$           | $6.767 \times 10^{48}$              | 0.752   |
| $3.34 \times 10^{-10}$   | $3.605 \times 10^{-42}$               | $2.45 \times 10^7$           | $6.810 \times 10^{48}$              | 0.756   |
| $3.33 \times 10^{-10}$   | $3.605 \times 10^{-42}$               | $2.46 \times 10^7$           | $6.827 \times 10^{48}$              | 0.758   |
| $3.30 \times 10^{-10}$   | $3.605 \times 10^{-42}$               | $2.48 \times 10^7$           | $6.892 \times 10^{48}$              | 0.766   |
| $3.20 \times 10^{-10}$   | $3.606 \times 10^{-42}$               | $2.56 \times 10^7$           | $7.107 \times 10^{48}$              | 0.789   |
| $3.00 \times 10^{-10}$   | $3.606 \times 10^{-42}$               | $2.733 \times 10^7$          | $7.581 \times 10^{48}$              | 0.842   |
| $2.70 \times 10^{-10}$   | $3.606 \times 10^{-42}$               | $3.03 \times 10^7$           | $8.420 \times 10^{48}$              | 0.939   |
| $2.60 \times 10^{-10}$   | $3.606 \times 10^{-42}$               | $3.153 \times 10^7$          | $8.740 \times 10^{48}$              | 0.972   |
| $2.50 \times 10^{-10}$   | $3.606 \times 10^{-42}$               | $3.275 \times 10^7$          | $9.084 \times 10^{48}$              | 1.009   |



The introduction of increasing values of  $\epsilon_0$  in the calculation of the electrostatic field, as shown in Table II is not so arbitrary as it might appear at once. Some assymetry of the electrostatic field inside of the hydrogen atom would be expected if the orbit is elliptic as suggested first by Sommerfeld (1916). Such a configuration of the orbit, would generate a phenomenon similar to the precession of the perihelion of the planet Mercury, as is going to be studied in the latest section of this paper. When the spontaneous emission of radiation is determined by the outer orbits, with coefficients or red-shift above 0.750, in the ultra-violet range (Lyman series, in Fig.1), coincides with the maximum excentricity of the orbit when exposed to the electrostatic field, and therefore, the movement of the perihelion may be more accentuated, giving the largest "rosettes-like patterns", and therefore coincident with some higer degree of "red-shift", as indicated by the values of the coefficients  $> 0.750$ , as shown in Table I and Fig. 1.

Such considerations help to assume that the local intensity electrostatic field may be regulated by the value of the permissivity coefficient ( $4\pi\epsilon_0$ ). Therefore, this would agree with the hypothesis formulated above that the electrostatic field intensity would be the main accelerating field responsible for the relativistic phenomenon to explain the production of the line spectrum of the excited hydrogen-atom. Such considerations may account for the observation shown in Table I, Fig. 1 and Table II, that "red-shifts" from 0.750 (Lyman  $\alpha$  to 1.00 for  $Rc = f_m$ ) in the highest U.V. series, can be calculated by inserting values of  $4\pi\epsilon_0$  comprised between  $3.32 \times 10^{-10}$  to  $2.5 \times 10^{-10}$ , a very narrow variation, indeed. It means that all possible U.V. lines will be emitted by values of  $\phi_{rel.}$  going up from  $6.77 \times 10^{48}$  to  $9.08 \times 10^{48}$  corresponding to a



range of "red-shift" coefficients, from 0.750 to 1.000 as shown in Fig.1 and Table II. Also the corrections introduced in the values below 0.7500 in previous papers (Rocha e Silva, 1978,79) may depend upon the eccentricity of the orbits, reflecting upon the value of Rydberg constant, at different levels of energy (or frequency). This possibility is being worked out in our studies, but lies beyond our present scope in this paper.

In summary, the value of  $\epsilon_0$  or of  $4\pi\epsilon_0$  is a sort of key to regulate the value of the ratio  $\phi_{rel.} = F_e/F_g$  i.e. the ratio between the electrostatic field /gravitational field. The magnitude of such a ratio is of the order of  $5.44 \times 10^{48}$  to  $9.084 \times 10^{48}$  giving coefficients of "red-shift" of the order of  $10^{28}$  times 0.75 to 1.00 covering the whole line of the spectrum of hydrogen, the only operation has been to apply Einstein's formula:

$$\frac{v_k - v_i}{v_i} = \frac{\phi}{c^2} \quad (10)$$

The treatment given to the calculus of  $\phi_{rel.} = F_e/F_g$  in this paper led to the conclusion that the enormous "red-shift" as observed of 0.750 up to 1.00 became a possibility by a factor of  $10^{28}$ , that corrects exactly for the "red-shift" as observed in earth gravitational field ( $0.75 \times 10^{-28}$ ). With such a possibility, the phenomenon of emission in the Bohr model of a multiple "red-shift" from the highest U.V. (Rc)  $\rightarrow$  to the lowest U.V.  $\rightarrow$  to Visible  $\rightarrow$  to I.R.  $\rightarrow$  to Radio waves, can be accounted for on basic principles and give a hint to solve such a riddle proposed by nature in the Bohr's model of the hydrogen atom.



## Generalization of the Equivalence Principle

7 d) It has been a great concern to physicists (see Born, 1927) that the electrostatic field is of such a magnitude, exceeding  $10^{40}$  times the earth gravitational field, becoming one of those mammoth numbers, to play a role in the mysterious structure of the universe. Actually, the measurement of such a ratio has been done in a wrong way omitting the important factor  $4\pi\epsilon_0$  to identify the field as electrostatic. We have seen above that by introducing such a factor  $4\pi\epsilon_0 = 3.33 \times 10^{-10}$  the ratio goes up to  $\phi_{rel} = 6.82 \times 10^{48}$  and then if divided by  $c^2$ , we have the unitary red-shift  $0.75 \times 10^{28}$  produced by an acceleration field of potential  $\phi = 6.82 \times 10^{48}$  times the earthly gravitational field. If accepted the idea that an electrostatic field with such a relative potential ( $\phi_{rel}$ ) produces such a reversal of the power in the basic formulas of "red-shift"

$$v_k = v_i \left( 1 + \frac{\phi}{c^2} \right) \quad (10a)$$

or

$$T_i = T_k \left( 1 + \frac{\phi}{c^2} \right) \quad (11)$$

and so-forth, all derived from the statement (Einstein, 1907,52) that the velocity of light entering such strong gravitational fields, as that of the sun, will suffer a decrement that can be calculated by the formula

$$c = c_0 \left( 1 + \frac{\phi}{c^2} \right) \quad (12)$$

according to Einstein (1907), with the added remark: "The principle of constancy of velocity of light holds good according to this theory in a different form from that which usually underlies the



ordinary theory of relativity" (Einstein, 1907).

Precession of the perihelion of Mercury in comparasion  
with that of the electron in its orbit

It was on the basis of such theoretical considerations that Einstein (1907,1915) forecast the two important possibilities which were lately confirmed by experiment or astronomical observation, namely:

a) The deflexion of a light ray when going across a strong gravitational field, such that of the sun or Sirius, for instance. Such a deflexion (a) could be calculated by the formula

$$a = \frac{2k.M}{c^2.d} \quad (13)$$

where (a) is the deflexion; k, the gravitational constant; M, the mass of the heavenly body (as the sun), and d the distance of the ray (beam of light) from the center of the body. The calculated value  $a = '83''$  of arc. (Einstein, 1907, 52). Such a deflexion was verified by observations of the total eclipse of the sun in the province of Crato, in the Estate of Ceara, Brazil, in 1919, by astronomers of the Univesity of \_\_\_\_\_, England, under the direction of Eddington (191 ). This confirmation, made Albert Einstein famous all over the world. (See \_\_\_\_\_)

b) The second astronomical event was the precession of the perihelion of the planet Mercury circling around the Sun, describing an ellipse of eccentricity e and semi-major-axis a, and time (T) of revolution in seconds. The angle of precession for one turn was calculated by the formula:



$$\Delta\psi = \frac{24 n^3 \cdot a^2}{T^2 \cdot c^2(1-e^2)} \quad (13)$$

According to Einstein (1915,52): "Calculation gives for the planet Mercury a rotation of the orbit of 43" per century, corresponding exactly to astronomical observation (Leverrier); for the astronomers have discovered in the motion of the perihelion of this planet, after allowing for disturbances by other planets, an inexplicable remainder of this magnitude!" For details see:

It is known, since the advent of Relativity Theory, that both events (a and b) are profoundly linked to the relativistic "red-shift" of a light beam going across a difference of gravitational potential measured by  $\phi_g$  as it appears in formular (10). If such a value of  $\phi_g$  is divided by  $c^2 = 9 \times 10^{20}$  we have a "red-shfit" of the order of  $10^{-28}$  using the constant of gravitation  $G = 6.67 \times 10^{-8}$  what agrees with the value obtained by Einstein  $0.187 \times 10^{-28}$ .

The second evidence (b) of the precession of the perihelion of Mercury can also be matched with the similar phenomenon occurring with the stationary orbit of the electron, known as the "rosetta-like" pattern first described by Sommerfeld (1916) and shown in Fig. 4. The main difference resides in the known fact that the time taken for Mercury to change its orbit is of the order of 43" of an arch per century! Therefore, the planet would make a full turn ( $360^\circ$  of arch) in about 3.01 millions of years ( $3.013 \times 10^6$  years) or

$$3.01 \times 10^6 \times 3.15 \times 10^7 = 9.5114 \times 10^{13} \text{ seconds}$$

what means that the planet will turn of a fraction

$$1.0513 \times 10^{-14} \text{ of circumference. per second.}$$