Part II

Generalization of the Relativistic Equivalence Principle

Is the hydrogen line-spectrum an example of a relativistic "red-shift" phenomenon?

by

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The renewed interest in the foundations of the phenomenon of emission and absorption of light (radiation) by the hydrogen atom or other hydrogenoid atoms, as evidenced by the recent publication by Hänsch et al (1979), shows that such an intriguing phenomenon is far from being explained in terms of basic physical concepts. According to Hänsch et al.: "the spectrum of the hydrogen atom has proved to be the Rosetta stone of modern physics: once this pattern of lines has been deciphered, much else could also be understood". Though such a phenomenon has been one of the cornerstones of quantum mechanics much should be understood along the lines of the Relativity Theory, especially if we assume that the localization of the electron in any stationary orbit (state) depends chiefly of the time parameter ($y^4 = -ict$) or the frequency ($f_i = T_i^{-1}$) as postulated in a previous paper (Rocha e Silva, 1978). On that occasion we have calculated a ratio $\Delta f/f = 0.75$ for the shift of frequencies corresponding to single electronic jumps, in the hydrogenoid atom, starting from the U.V. ($f_m = Rc = 3.287,870 \times 10^{15}$) to red and short waves ($f_8 = 2.51372 \times 10^{13}$).

The different levels of frequency (or energy) were calculated, as indicated by Bohr (1913, 15) using the equations:

$$f_n = Rc/n^2 \qquad n = 1, 2, 3 \dots \infty \qquad (1)$$

for the departing levels of frequencies according to the Ritz combination principle (Bohr, 1913, 15). Such departing levels would give all the emitted or absorbed frequencies appearing in the line spectrum of the hydrogen atom. The frequencies in Bohr's model of the hydrogen atom, can be deduced by the equation:

$$hf_{ik} = \pm (E_i - E_k) \qquad (2)$$

that can be simplified by dividing all terms by Planck's constant (h), into

$$f_{ik} = \pm (f_i - f_k) \qquad (2a)$$

f_{ik} indicating the monochromatic quantum emitted or absorbed when the electron jumps from orbit (i) to (k), or vice-versa.

The stationary orbit was defined as a geodesic of a Riemannian (Lobatchevski, Bolyai) space-time continuum with a negative interval ($ds^2 = -g_{ik}du^i . du^k$). For details, see Rocha e Silva (1978). Since the whole phenomenon of emission and absorption can be fully described by changes in frequency ($f_i = T^{-1}$) the jumps have been understood as resulting from parallel displacements along the relativistic time parameter, according to the rules of Weyl's geometry, i.e. with adaptation of metric or "gauge". See Weyl (1922-52).

As the geometry of the jump can be considered independent of the "space" coordinates, for commodity, we have defined a component of mass, the "time mass" ($m_{44} = -f/c^2$) which would play the role of the inertial mass along the time ($y^4 = -ict$) coordinate. It is easy to see that by defining "time mass" in terms of f/c^2 ("frequency/square of light velocity") it would play a similar role as "space mass" (m_{ii}) in terms of energy ($E = hf$) in the known formula $m_{ii} = E/c^2$. In that situation, Planck's constant ($\hbar = h/2\pi$) would play the role of a factor of proportionality transforming "time mass" into "space mass":

$$m_{ii} = \frac{h}{2\pi} m_{44} \qquad (3)$$

Emission or absorption of a photon (light quantum), with a frequency f_{ik}, will denote a reduction of "time mass" (m_{44}):

$$\Delta m_{44} = - \Delta g_{44} = f_{ik} = \pm(f_i - f_k) \qquad (4)$$

It is important to stress that any spontaneous emission by the H atom will be a directional shift from an U.V. frequency ($Rc = 3.287 \times 10^{15}$ vibrations/sec) to the red, infrared, short waves; the opposite being observed when the atom receives a quantum of radiation (or an electric discharge, or a collision with an excited particle and so-forth). About the geometrical interpretation of Rydberg's constant we quote Rocha e Silva (1978): "Under normal physical conditions of emission or absorption of the three series of bands in the infra-red, the visible and the ultra-violet regions, it is doubtful whether the whole frequency (De Broglie's frequency, for instance) of the electron participates in such changes, and it would be reasonable to suppose that the electron has a more stable core and a sort of outer layer that can be lost or rebuilt by the usual emissions and absorptions of light. The limits of this outer layer might be imposed upon it by the value of Rydberg's constant ($R_H c = 3.287 \times 10^{15}$ vibrations/sec). According to Bohr's principle, when the electron jumps from an orbit $\underline{n}$ to ∞, to orbits of lower levels of energy, it will emit a quantum of radiation represented by -Rch, as a limit. The light emitted will be one in the U.V. series with a wave length $\lambda = 0.909 \times 10^{-5}$ and a frequency (f_m) a little less than the value of $-Rc$(f_m = 3,29 x 10^{15}). According to the postulates of Weyl's geometry if we consider the parallel displacement of a vector along the time coordinate ($y^4 = -ict$), it must adjust itself to the metric (gauge) of the orbit, in such a way that the length ($ds = L$) of a vector will undergo a reduction or expansion when the particle jumps from orbit $\underline{i}$ to $\underline{k}$ or, vice-versa from $\underline{k}$ to $\underline{i}$ ".

According to the rules of Weyl's geometry, such a metric connection is regulated by two differential forms: a quadratic (I).

$$\text{I)}\quad ds^2 = g_{ik}\, dx^i .\, dx^j$$

and a linear one (II):

$$\text{II)}\quad d\alpha = \alpha_i . dx_i$$

By changing the metric, the first is multiplied by a factor f', a positive, continuous and derivable function of the variables; the second is decreased by the differential of log f':

$$\overline{g}_{ik} = f' . g_{ik} \quad \text{and} \quad \overline{\alpha} = \alpha_i - \frac{df'}{f' . dy^i} \qquad (5)$$

Now, we can define the transport of a vector or tensor along the geodesic of a Weyl's space along the time (y^4) coordinate, in conditions in which the variation of length is completely accounted for by changes of metric, i.e. by setting equal to zero the fundamental equations:

$$\frac{1}{g_{22}} \quad \frac{\Delta g_{22}}{\Delta y^4} + \frac{\Delta f}{f.\Delta y^4} = 0 \qquad (6)$$

and

$$\frac{\Delta\alpha}{\Delta y^4} - \frac{\Delta f}{f.\Delta y^4} = 0 \qquad (7)$$

wherefrom

$$\Delta g_{22} = - g_{22} \frac{\Delta f}{f} \qquad (6a)$$

and

$$\Delta\alpha = \frac{\Delta f}{f} = d(\log f) \qquad (7a)$$

The ratio $\Delta f/f$, by which one should multiply "space mass" (g_{22}) and "time mass" (g_{44}) to obtain the values of the frequencies (f_n) on each level of energy along the time coordinate, may be taken as the value of the "red-shift" occurring when the electron jumps from the ultra-violet, to the extreme infra-red.

We have shown, that to obtain the frequencies (or energy levels) for single jumps, the first level of energy may be calculated by multiplying the maximum (f_m = Rc) by a constant ratio 0.75 = $\Delta f/f$; the second level of energy by multiplying the maximum (f_m) by the square of that ratio $(\Delta f/f)^2$, and so-forth, as indicated in Table I.

TABLE I

Values of the differential $\Delta f/f)^n = \alpha^n$ for single jumps taking f_m = Rc, to calculate the "red-shift" in the hydrogenoid atom.

I Frequencies $x10^{15}$ f_m/n^2	II Differences $(f_i-f_k) \times 10^{15}$	III(*) Ratios $\frac{(f_i-f_k)}{f_i}$ (corrected)	IV Calc. values $\left(\frac{\Delta f}{f}\right)^n$ (red-shift)
f_1 = 3.287,870 (Rc)			

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f_1 = 3.287,870 (Rc)				
f_2 = 0.821,975	2.465,895	0.7472	0.7500	0.7500
f_3 = 0.365,318	0.456,657	0.5444	0.556	0.5625
f_4 = 0.205,491	0.159,828	0.4125	0.4375	0.4218
f_5 = 0.131,352	0.074,139	0.3164	0.3155**	0.3164
f_6 = 0.091,330	0.040,019	0.2353	0.2361**	0.2373
f_7 = 0.067,099	0.024,224	0.1655	0.1653**	0.1780

(*) The values in column III were corrected by subtracting from all differences (column II), a constant value Δ = 0.00911.

(**) The same correction as indicated in (*) was applied to the three lower differences. Such corrections may indicate some small changes of Rydberg's constant for the lower levels of frequency.

If we consider this fraction $\Delta f/f = 0.75$ as a relativistic "red-shift" imposed by some strong field inside of the atom of hydrogen, to be compared with that calculated for the gravitational field of a star of mass (M_s) and radius (R_s) according to the equation:

$$f' = f\,(1 - \frac{GM_s}{R_s . c^2}) = \frac{\Delta f}{f} \qquad (8)$$

where the gravitational constant, $G = 6.67 \times 10^{-8}$ and the square of the light velocity, $c^2 = 9 \times 10^{20}$. Reducing to g/cm, the unitary "red-shift" in a gravitational field would be of the order of

$$\frac{\Delta f}{f} = \frac{G}{c^2} = 0.74 \times 10^{-28} \qquad (9)$$

indicating the smallness of the shift that could hardly be observed on the surface of our planet. Even considering the mass of the earth ($M_s = 6.978 \times 10^{27}$g) and a difference of altitude at the top of Mount Everest (6.379×10^{8} cm) it there would be a barely measurable shift of the order of $\Delta f/f = 0.9 \times 10^{-12}$. For references see v. Frish (1961, 71) and Woulfson (1968).

There are ways to increase the gravitational field on the surface of the earth as by centrifugation in the experiments by Pound and Rebka (1960) mentioned in Resnick (1971), but the shifts so far obtained do not exceed the order of magnitude of 10^{-15}. But in many of such measurements, the influence of small changes of temperature, originates a non-relativistic shift (Doppler effect, from the movement of the emitting particles) that will reduce the confidence to be attached to such measurements (see Frisch, 1971).

The main argument we wish to present is that the red-shift observed in the hydrogen atom (Balmer ladder or Bohr's pattern) may be originated by the influence of a field, presumably the electrostatic field, prevailing in the interior of the atom, enormously greater than the terrestrial gravitational field. To test this hypothesis it became momentous to calculate the ratio F_e/F_g between the "electrostatic" and the gravitational intratomic fields.

The task is easy if we calculate the electric force (F_e) in comparison with the gravitational force (F_g) using both systems, the electrostatic attraction between electron and proton of charge ($e^2/4\pi\,\varepsilon_o$), and the gravitational attraction between the masses (m_e and m_p) of the electron and the proton, both at the same distance (r_o) that may be the radius of Bohr, or multiples

(n^2) of it. To perform this task we have to divide both equations, eliminating the distance (r_o).

$$F_e = \frac{e^2}{4\pi\varepsilon_o \cdot r_o^2} \quad \text{and} \quad F_g = \frac{G \cdot m_e \cdot m_p}{r_o^2} \tag{10}$$

The ratio must give the factor by which the gravitational "red-shift" should be multiplied to have the intra-atomic "red-shift" presumably to be the cause of emission in Bohr's atom:

$$\left(\frac{\Delta f}{f}\right) = \frac{e^2}{(4\pi\varepsilon_o) \cdot G \cdot m_e \cdot m_p \cdot c^2} \tag{11}$$

Introducing the numerical values:

$e^2 = 2.304 \times 10^{-19}$; $4\pi\varepsilon_o = 3.333 \times 10^{-10}$; $G = 6.664 \times 10^{-8}$;

$m_e = 9.104 \times 10^{-28}$; $m_p = 1.67 \times 10^{-24}$ g; and $c^2 = 9 \times 10^{20}$;

we have for the coefficient of the red-shift inside of the hydrogen atom:

$$\frac{\Delta f}{f} = 0.756 \times 10^{28} \tag{12}$$

to compare with 0.74×10^{-28} that would be expected by a pure gravitational field. However, what is more remarkable is that the found "red-shift" inside of the hydrogen atom may account almost exactly for the red-shift in the emission (or absorption) spectrum of radiation in the hydrogen atom, as indicated in Table I.

$$\frac{\Delta f}{f} = 0.7500 \tag{12a}$$

for single jumps. The small difference

$$\Delta = 0.756 - 0.750 = 0.006$$

can be accounted for by the approximations used in the values of the constants inserted into formula (11). The values of such constants e, m_e, G and c, were taken from conventional sources, such as Handbook of Chemistry and Physics, the latest edition (1979) of Encyclopedia Britannica, and Physicochemical Text-Books, as indicated in the References. The most difficult one was to find an acceptable value for $4\pi\varepsilon_o$ in the C.G.S. system, departing from the conventional value

$$\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 \text{ Nt-meter}^2/\text{coulomb}$$

giving in the CGS system:

$$\frac{9 \times 10^9}{3} \quad \text{dynes-cm}^2/\text{e.s.u.}$$

or by simple inversion

$$4\pi\varepsilon_o = 3.333 \times 10^{-10}$$

the value that has been inserted into formula (11).

Such an extraordinary coincidence constitutes a strong argument of the "electrostatic origin" of the red-shift observed in the line epectrum of the hydrogen atom, from the ultraviolet, to the visible, to the infra-red, and short waves, departing from the maximum frequency in the U.V. represented by Rydberg constant ($Rc = f_m$ = 3.287,870 vibrations/sec)(*). The electrostatic nature of the field is suggested by the need to introduce the conversion factor ($\varepsilon_o \simeq 1/c$) in the above computations.

DISCUSSION

The relativistic "red-shift" in a homogeneous gravitational field (acceleration γ) was masterly defined by Einstein, in 1907, and reproduced in more recent publications (Einstein, 1952): "Let there be a stationary system of coordinate K, oriented so that the lines of force of the gravitational field run in the negative direction of the axis of Z. In a space free of gravitational fields let there be a second system of coordinates K' with uniform acceleration (γ) in the positive direction of its axis of Z... the systems K and K' are equivalent with respect to all physical processes... By assuming this to be so, we arrive at a principle which, if it is really true, has great euristic importance."

To measure what happens to a photon of frequency v_o emitted in either system, we have to provide observation-stations S_1 and S_2 with instruments of measure, to be situated on the Z-axis of K, at the virtual distance h from each other, so that the gravitational potential in S_2 is greater than that in S_1,

(*) According to the latest determinations, Rydberg's constant Rc = 3.281,147 (Hänsch et al. 1979). The use of this new value for f_m will give a "found ratio" $\Delta f/f = 0.75001$, coincident with the one adopted for "single jumps" as shown in Table I.

hγ (hγ)

by hγ, equivalent to the gravitational potential difference between S_2 and S_1 ($G = h\gamma$). By a simple algebraic deduction, a photon with a frequency v_1 as measured in the top of the distance $\left\{\begin{matrix}S_2\\S_1\end{matrix}\right.$ will have at its arrival in S_1 the frequency v_2, according to the equation:

$$\frac{\Delta v}{v} = \frac{v_1 - v_2}{v_1} = \frac{G}{c^2} = 0.7404 \times 10^{-28}$$

as a definition of the unitary relativistic "red-shift" in which $G = 6.664 \times 10^{-8}$ and $c^2 = 9 \times 10^{20}$

Such a coincidence per se would suggest the possibility of a "red-shift" of 0.7500 as that calculated in Table I to be produced by a field of force of the order of 10^{28} times the gravitational field in the surface of the earth. This was actually confirmed by inserting all constants in formula (11), to calculate the ratio between the electrostatic (F_e) and the gravitational field (F_g) inside the hydrogen atom. Such a ratio (about 10^{28} times the gravitational field) giving a red-shift of the order of 0.75 coincides exactly with that calculated in Table I for "single jumps" (0.7500^n where n can take all values of the digits 1, 2, 3... n). It is obvious that n coincides with the quantum numbers of the orbits.

The main consequence of the theory proposed above is that to attribute to the "local time" or frequency of the orbit, the driving factor to retain the electron in each orbit, as proposed in previous papers (Rocha e Silva, 1978, 79). Accordingly, the jumps are regulated by changes in "time mass" ($v/c^2 = m_{44}$) according to the rules of Weyl's geometry. The definition of each orbit (stationary orbits) depends solely on the definition of the coeficient of "red-shift". For single jumps, this coefficient is almost exactly 0.75 to the nth power, n taking the values of the quantum numbers (1, 2, 3...n), as a first approximation.

An important question may be argued. What is the role to be played by the Schrödinger equation to define the energetic levels in Bohr's atom? We may suggest a sort of compromise between the Theory of Relativity and the Quantum mechanics points of view. In its intra-atomic or secular behaviour, the electron has to accupy space, and therefore when dealing with energy (E_1) and "space mass" ($m_{ii} = E_1/c^2$) the orbit frequency (v_i) must be multiplied by Planck's constant ($E_i = \frac{h}{2\pi} v_i$) as indicated in the beginning of this paper (2 and 2a). Therefore, in its spatial existence, the electron obeys the rules of Heisenberg (1930) uncertainty principle: $\Delta q.\Delta p \geq \frac{h}{2\pi}$.

In its double existence, along the time coordinate (y^4 = ict) and in its spatial orbitals (E_i), the electron obeys, from one side, the deterministic "red-shift law" going across from one orbit to the other through the "spatial uncertainty" expressed by Planck's constant ($h/2\pi$). In other words, it is through a kind of "tunnel effect" that the electron can find its pathways from orbit k to i, with emission of a monochromatic photon, according to the equation

$$v_{ik} = (v_i - v_k) = -(v_k - v_i)$$

This phase of the phenomenon is highly deterministic which agrees with the experimental evidence that time intervals, as measured by periods of emitted radiation display the unbelievable precision of one second in many centuries, as in the case of the band $9.192,6318 \times 10^6$ Hertz, of Coesium, stable to $1:10^{11}$ adopted as a standard of time as an universal clock; or the first hydrogen Maser developed by Ramsey et al. (1960), with a radio-wave output whose frequency is of the order of 1.4×10^9 Hertz, reproducible with an accuracy of one part in thirty millions-million ($1:10^{12}$). According to Encyclopedia Britannica (1979 Edition), a clock controlled by such a maser would get out of step "no more than one second in 100,000 years.

Incidently, if we talk of lasers and masers, the above theory may be submitted to an obvious test. If a blue laser be submitted to a strong enough electrostatic field, of the order of 10^{28} e.s.u or even less (above 10^{20} electron-volts) it may change its frequency to a green or red color, with emission of a green-yellow photon depending upon the strength of the applied electrostatic field.

In conclusion, we can admit that the unsurmountable gap between Relativity and Quantum Theory can be bridged up by knowledge afforded by the Theory of Relativity itself. In commemoration of the 100th birthday of Albert Einstein we may quote one of his latest phrases: "If the statistical quantum theory does not pretend to describe the individual system (and its development in time) completely, it appears unavoidable to look elsewhere for a complete description of the individual system; in doing so it would be clear from the very beginning that the elements of such a description are not contained with the conceptual scheme of the statistical quantum theory. With this one would admit that, in principle, this scheme could not serve as the basis of theoretical physics. Assuming the success of efforts to accomplish a complete description, the statistical quantum theory would, within the framework of future physics, take an approximately analogous position to the statistical mechanics within the framework of classical mechanics. I am rather firmly convinced that the development of theoretical physics will be of this type; but the path will be lengthy and difficult." Einstein, February 1, 1949 (in P.A. Schilpp, 1951).

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