Part I

A Geometrical Interpretation of the Electron Jump

A possible geometric interpretation of the electron jump in the hydrogenoid atom

by

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Introduction

We shall limit our discussion to the hydrogenoid atom, with a single electron jumping from a stationary orbit to a more stable position. Under such conditions and according to Bohr's initial suggestion, the electron does not emit while in the stationary orbit. When it jumps from orbit *i* to *k* or from *k* to *i* it will emit or absorb a quantum of energy $h(v_i - v_k)$ corresponding to a decrease or increase of energy (1)

$$hv_{ik} = \pm (E_i - E_k)$$

The presentation of the subject in a pure algebraic form is conform to the intimate mechanism of the phenomenon, in which absorption or emission are perfectly interchangeable phenomena affecting only the signal of the quantum. This 100% efficiency in the reversibility of a physical phenomenon, such as of light absorption or emission, is already an astonishing peculiarity for a natural occurrence. This simple consideration might encourage one to look for a pure geometrical interpretation for this phenomenon, and justifies the attempt to reduce the whole phenomenon to changes in metric, as has been done for the gravitational field. (2,3)

The simplest assumption is to admit that the electron does not emit while in the orbit because it travels along a geodesic of a non-Euclidian space, the metric of which could be defined by the interval :

$$ds^2 = g_{ii}(dx^i)^2 \qquad i = 1,2,3,4$$

where g_{ii} are the components of a fundamental bi-covariant tensor reduced to its diagonal form. (4,5)

For the sake of simplicity, we might admit that the orbit is a two dimensional surface, with a non-Euclidian metric but constant interval, surrounded by an Euclidian four-dimensional space, the Einstein-Minkowski four dimensional continuum of special relativity.[5] Therefore $x^i = X(y^2, y^3)$ if we make y^i and y^4 constants in the orbit. In all the following discussion the coordinates system of the orbital space will be y^i and those of the surrounding Euclidian space x^i.

The dynamics of the particle. As we assume that the particle moves freely in a geodesic of a non-Euclidian surface, its whole energy can be calculated as kinetic energy and expressed in terms of a metric tensor by the equation (4,6)

$$2T = g_{ii}(u^i)^2.$$

With such an assumption it is clear that the four components of "mass" m_{ii} can be identified with the four components of the metric tensor g_{ii} and the whole movement can be described in terms of the Lagrangian equation equivalent to the equations of a geodesic in a non-Euclidian space. The problem can be solved if we find the conditions of a parallel displacement of the vector $u^i = dy^i/dt$, the generalized velocities of the particle. There is no incongruity to introduce the independent variable t when we have considered the coordinate y^4 constant, because the vector u^i refers to the movement in relation to the surrounding Euclidian space and the definition of the metric is independent of the value attributed to u^i. Actually, u^i can have any conventional value and therefore the energy of the particle $E = T$ could be totally identified with the covariant fundamental tensor g_{ii} and confounded with the metric of the space. Since the absolute value of the velocity in each orbit will remain indeterminate, the movement can be described by a system of Langrangian equations (7):

$$\frac{d}{dt}\left(\frac{dT}{du^i}\right) - \frac{dT}{dy^i} = 0$$

which are equivalent to the equations of the geodesic obtained by equalling to zero, the covariant derivatives of the contravariant vector u^i

$$D_t = \frac{du^i}{dt} + \Gamma^i_{rs}\, u^r u^s = 0$$

where

$$\Gamma^i_{rs} = \frac{1}{2} g^{il}\left(\frac{dgrs}{dy^i} + \frac{dgsl}{dy^r} - \frac{dglr}{dy^s}\right) \text{ and } \Gamma^i_{rs} = g^{il}\,\Gamma_{l,rs}$$

The whole movement depends solely upon the metric tensor and of its partial derivatives along the coordinates axis. The derivative du^i/dt is the generalized acceleration of the particle and can be expressed by

$$\frac{du^i}{dt} = \frac{d^2y^i}{dt^2} = - \Gamma^i_{rs} \frac{dy^r}{dt} \frac{dy^s}{dt}$$

These equations can describe the movement in a Riemannian surface of a body moving along a geodesic, if the diagonal components of the metric tensor $g_{ii} = m_{ii}$ $(i = 1,2,3)$[8]. The fourth component $g_{44} = m_{44}$ should be designed as the "time mass" of the particle to be introduced through a different order of considerations. See Annex I.

Energy levels of the atom. The nature of the non-Euclidian space was left indeterminate by the above considerations, the only assumption made is that the interval should be constant. It can be, however, negative or positive. We have to call upon physical data to go further in such a definition.

In order to give any physical significance to the above equations, we have to consider the values of the levels of kinetic energy as given by Bohr's conditions in each stationary orbit. According to data of spectral analysis (9).

$$E_i = - \frac{Rh}{p^2} \quad \text{or} \quad E_i p^2 = - Rh$$

where R is Rydberg's constant, h is Planck's constant and p a simple digit ($p = 3$ or 4 for the visible bands of the emission spectra)[10].

For another orbit (k) the corresponding values for the energy are

$$E_k = -Rh/n^2 \quad \text{or} \quad E_k n^2 = -Rh$$

When the electron jumps from orbit (i) to orbit (k), the expression of the metric tensor g_{kk} in the new orbit can be deduced from that of the previous orbit (g_{ii}) by the equation :

$$g_{ii} p^2 = g_{kk} n^2$$

and the metric tensor in the new orbit can be deduced from that of the previous orbit by the relation

$$g_{kk} = \frac{p^2}{n^2} g_{ii} \quad \text{or} \quad g_{kk} = k^2 g_{ii} \quad k^2 = p^2/n^2$$

If the transformation is homothetic, the same change of metric should occur for all components of g_{ii} and the factor of proportionality (k^2) is always positive and the same for all components of the energy or mass tensor. In other words, while the metric in each orbit is described by an invariant metric tensor (invariance of gauge), when the electron jumps from orbit (i) to (k) the metric tensor will undergo a homothetic transformation, with variation of the "length" of the standard interval (ds^2). This is reminiscent of Weyl's geometry[11] of a metric connection in which there is variation of "gauge" along the pathway of a moving body. It is to be found whether the conditions of homothety applies to the four components of the mass of the metric tensor, i.e. if the three "spatial" components of the mass m_{ii} ($i = 1,2,3$) and the "time" component $m_{44} = g_{44}$ undergo the same homothetic variation.

First of all, we have to consider the signal of the fundamental interval, and therefore of the metric tensor g_{ii}. Since $p^2>n^2$, the absolute values of

$$g_{kk} > g_{ii} \quad \text{or} \quad m_{kk} > m_{ii}$$

This would be against the possibility of an emission from i to k. But if the orbit has a negative curvature, as for instance the internal surface of a sphere, to any increase in absolute value of g_{kk} will correspond a decrease in relative value, therefore $-g_{kk} < -g_{ii}$.

Therefore, we have to postulate a negative interval $-ds^2$ and an imaginary radius for the sphere. The coordinate holding the particle to the center of the atom is therefore an imaginary iR, in such a way that if we take the coordinates y^2 and y^3 of a point of the orbit, the circumference with center in the nucleus will have the equation :

$$(iR)^2 = (y^2)^2 + (y^3)^2$$

and the interval

$$ds^2 = -g_{ii}(dy^i)^2 \qquad (1 = 2,3)$$

We reach the conclusion that the space in the orbit is reminiscent of a surface in Lobatchewski or Bolyai geometry[12,13, and that the particle is being held in the orbit by the imaginary coordinate, therefore along the time axis ($y^4 = ict$). Now, if g_{ii} increases in absolute value, the fundamental tensor ($-g_{ii}$) decreases and there occurs emission of radiation. This change in metric would involve solely a change in the time parameter, as shown in continuation.

The radius of the orbit as a time parameter. If one goes back to the algebraic formulation of Bohr's condition (1) of a stationary state, we have :

$$hv_i p^2 = -Rh \qquad \text{since } (p/n)^2 = k^2$$

$$hv_k n^2 = -Rh$$

we will have

$$k^2 = \frac{v_i}{v_k} = \frac{T_k}{T_i}$$

Therefore, the coefficient of homothety (k^2) is the ratio of two time constants (periods), and the metric of the orbits should obey the fundamental equation :

$$g_{kk} = k^2 g_{ii} \quad \text{or} \quad g_{kk} T_i = g_{ii} T_k$$

As the period shortens, the fundamental tensor dilates in proportion and in absolute value as the inverse of the time parameter, indicating that the particle goes deeper into the atom along the time parameter taken as time coordinate. The only way to compensate for this geometric change, is the emission of radiation of frequency $v_{ik} = v_1 - v_k$. But the obvious consequence is that the metric of the orbit depends solely upon a time constant and from one orbit to the other, only discrete values of the time constant are permissible.

This can be understood intuitively. If we assume that the constant parameter of the orbit is the time parameter and that somehow the electron gravitates at a fixed interval (on time) from the center of the nucleus or of the orbit, the frequency v_i or its equivalent c/T_i is the distance on time from the center of the orbit to the space trajectory. It can only vary by discrete values, when the particle jumps from orbit (i) to (k), with wmission or absorption of a light quantum, which can be expressed by the difference in metric :

$$\pm \Delta E = \pm(g_{ii} - g_{kk}) = h(v_i - v_k)$$

h, being a factor required by the experiment.

While in the orbit, the particle describes a geodesic of a non-Euclidian space with negative but invariant metric tensor ($-g_{ii}$). When it moves from one orbit to the other the values of $-g_{ii}$ are dilated or contracted in the inverse proportion of the discrete values of the periods or in the direct proportion of the frequencies[11], according to the metric of a Weyl's homothetic space. Therefore, the non-Euclidian spaces of the orbits are local cases where the gauge ($k^2 = 1$) is invariant, of a more general geometric space which follows Weyl's metric, with a coefficient of homothety equal to $k^2 \neq 1$.

The time component of mass (m_{44}). We have, of course an intuitive representation of the "space" mass and its components $m_{22} = m_{33} = m_o$ which are all equal (considering only the transversal mass) and measured by the proper mass of the particle (m_o). To have a geometrical representation of the fourth component of mass (m_{44}) we have to consider the variations of it along a single parameter (τ) along which the variations of gauge are manifest according to Weyl's geometry. If F_i is a factor of proportionality changing the metric g_{ii} into g_{kk} by changing the time parameter in the equation of the geodesic and making

$$F_i = d\tau/dt \quad \text{and} \quad d\tau = v_i dt$$

the new interval will be

$$d\tau^2 = F_i g_{ii} (dy^i)^2$$

If we admit that the new parameter depends only on time and that F_i is linearly related to the frequency, or $F_i = v_i$ we have :

$$d\tau^2 = F_i g_{44} (dy^4)^2 \qquad dy^4 = icdt$$

or

$$d\tau^2 = v_i g_{44} (-c^2 dt^2)$$

or, making $d\tau = v_i dt$

$$v_i^2 dt^2 = -v_i g_{44} c^2 dt^2 \qquad \text{since } g_{44} = m_{44}$$

where from

$$v_i = -c^2 m_{44}$$

This relation would indicate that the frequency is related to the time mass as the energy is related to the space mass, i.e., through a proportionality factor that is equal to the square of the light velocity (c^2), in absolute value. Therefore, the "time" component of the mass (m_{44}) has the expression :

$$-m_{44} = \frac{v_i}{c^2}$$

This expression for the component of the time mass $m_{44} = v_i/c^2$ introducing a frequency v_i proper of the electron in the orbit (i) would suggest that the square of the light velocity is a limiting level for the frequencies of the electron and that therefore m_{44} tends to 1, when v_i tends to c^2.

The time mass has the dimensions :

$$M_{44} = \frac{T^{-1}}{L^2\ T^{-2}} = L^{-2}\ T$$

a "time over the square of a length", or a "time over a surface".

We could take any frequency (v_o) as the proper mass of the electron, and actually, the obvious one would be to take the frequency postulated by Broglie ($v_o = 10^{18}$) to be associated with the electron. But we have the limiting condition that it cannot exceed the square of the light velocity (c^2). It is known that the proper frequency of the electron depends upon its velocity, and this of course will arise naturally from the theory, since if space mass changes with velocity, the "time" component will undergo a proportional increase, as shown below. But this increase can go on until the limiting value of the square of light velocity is reached.

If we give to the square of light velocity the dimensions of a frequency (T^{-1}) the time mass will be dimensionless and represent an abstract ratio of two frequencies v_i/v_o. In such a case, the numerical value of the time mass will tend to 1 if the proper frequency of the electron tends to c^2. Actually, if we give to the electron a velocity of 150,000 Km/sec, the value of v_i approaches the square of the light velocity (c^2). This might, for instance, be the velocity of the first electron to be introduced around a nucleus with Z = 72 (Hafnium)[15]. See Annex IV.

If a similar reasoning applies to orbit (k)

$$v_k = -c^2\ \bar{m}_{44}$$

and the difference

$$v_i - v_k = c^2(\bar{m}_{44} - m_{44}) = -c^2(m_{44} - \bar{m}_{44})$$

for the emitted radiation $v_i - v_k = v_{ik}$. Therefore, the emitted radiation v_{ik} will correspond to a reduction of the "time component" of mass amounting to

$$-(m_{44}-\bar{m}_{44}) = (v_i - v_k)/c^2$$

or

$$c^2 \Delta m_{44} = v_i - v_k$$

If the transformation of metric is homothetic (Weyl's geometry), all the other components of the metric tensor $g_{ij} = g_{ji}$ will vary in the same proportion. According to a well known transformation of the quantum theory, we have

$m_{ii} = nhv_i$ and $\bar{m}_{kk} = nhv_k$. We have just shown that for the "time masses"

$$v_i = -c^2 m_{44} \quad \text{and} \quad v_k = -c^2 \bar{m}_{44}$$

or

$$m_{44}/\bar{m}_{44} = v_i/v_k = k^2$$

Since the same coefficient of homothety holds for the space transformation of the components of the metric along the space and time coordinates, the following relations define the homothetic transformation of the metric :

$$\frac{m_{ii}}{m_{kk}} = \frac{m_{44}}{\bar{m}_{44}} = \frac{v_i}{v_k} = k^2$$

it is evident that the spatial components of the mass, inside of the orbit are all equal since $m_{11} = m_{22} = m_{33}$ (the distinction between longitudinal and transversal mass, is of no concern here). When the particle jumps from orbit (i) to (k) there is a reduction in the mass component along the spatial coordinates, expressed by the same ratio k^2 holding for the variation of time mass. Therefore, the postulate of Weyl's geometry is fulfilled, and one can consider the jump as a sudden alteration of gauge (or standard) defining the metric connection along the time coordinate. If the theory is valid one might consider the emission of a quantum as an adjustment of the metric of the orbit (i) to a new standard (gauge) prevailing in orbit (k), and the process of emission would be the consequence of the so-called "segmentar curvature" (Weyl's definition) [11,14].

The mass of the electron expressed in function of Planck's constant. If it is easy to deduce the relative values of the masses in two different orbits, by the laws of emission of radiation, since the same coefficient of homothety k^2 must hold not only for the spatial but also for the "time" component of mass, and establish that the same ratio holds for the components of mass along the four coordinates of space and time, we have to reccur to the experiment again to establish the unknown ratio between space/time mass. We have to propose the simplest assumption to make the theory conform to the experimental data. We are going to see that a great simplification results if we make the coefficient of proportionality equal to $h/2\pi$ and the ratio

$$\frac{m_{ii}}{m_{44}} = \frac{h}{2\pi} = \hbar \quad \text{where} \quad m_{44} = v_o/c^2$$

In that case, the constant $\hbar = h/2\pi$ is the operator transforing "time" mass into space mass

$$m_{ii} = m_{44}\frac{h}{2\pi} = \frac{hv_o}{2\pi c^2}$$

If we substitute in the formula all the known values for

$$h = 6.57x10^{-27} \quad c^2 = 9x10^{20} \text{ and } m_o = 9x10^{-28}$$

we have

$$v_o = \frac{2nc2m_o}{6.27x10^{-27}} = 7.8x10^{20}$$

a value approaching the square of the light velocity. With a correction for the increase of mass, according to the relativity theory, making the velocity of the electron tentatively equal to $1.5x10^{10}$, namely half the light velocity, we have by simple calculation :

$$m_o = 1.054x10^{-27}$$

and the value of the frequency proper (v_o) will approach the square of light velocity $c^2 = 9x10^{20}$.

Assuming that $v_o \rightarrow c^2$ and taking the limiting value c^2 as the fundamental frequency of the electron in its fastest condition, it will likewise measure the "time mass", and the value of the "space mass" of the electron can be calculated by multiplying it by the factor of proportionality $h/2\pi$ divided by c^2. Therefore, at the limit

$$m_o = m_{44}.h/2\pi \rightarrow h/_{2\pi} \quad \text{if} \quad m_{44} \rightarrow c^2$$

or, the time mass tends to 1 as the frequency of the electron tends to c^2, and Planck's constant becomes a measure of the moving mass of the electron. With the above correction, assuming a velocity of $1.5x10^{10}$, half of the light velocity :

$$m_{ii} = 1.054x10^{-27}$$

a value very nearly approaching

$$\frac{h}{2\pi} = 1.10x10^{-27}$$

The small discrepancy might indicate that the limit of the velocity of the electron is a little over the arbitrary 150,000 Km/sec ascribed to it in its fastest condition. Actually there is no means of calculating the actual velocity of the particle in the orbit, since in all geometrical deductions the secular velocity of the particle is cancelled out in the given equations. If the possibility presented above is a valid one, the adjustement of the value of the mass to the known values presented above for h/2π might be one way to determine such a limiting velocity, through relativistic considerations. The calculation given above indicates that such a velocity should be very near half the velocity of light.

To assume that Planck's constant is an operator transforming "time" mass into "space" mass will be conform to what is known of the phenomena of emission and absorption of light. On the other hand, the dimensions usually given to Planck's constant $[h] = [ML^2T^{-1}]$ would be conform with such an assumption. In fact, we have found for the dimensions of the time mass $[M_{44}] = [L^{-2}T]$, therefore :

$$[M] = [M_{44}][h] = [L^{-2}T][ML^2T^{-1}] = [M]$$

the dimension equation is satisfied and we have :

$$[M_{44}] = \frac{[M]}{[h]} = \frac{[M]}{[ML^2T^{-1}]} = [L^{-2}T]$$

The theory advanced above assumes that the square of the light velocity is a fundamental measure of the "time mass" of the electron. However, we have to keep in mind that m_{44} is actually expressed by the relation v_o/c^2 and each time it is introduced into a formula, as the proper frequency of the electron it should be divided by c^2. Therefore $1/c^2$ could be interpreted as a factor to adjust units and be incorporated into the factor of proportionality "space/time" mass which becomes equal to $h/2\pi c^2$ each time we make the time mass = v_o ($=m_{44}$). We have to remark that the value of c^2 is related to the fundamental constants of the electromagnetic field by the relation $c^2 = 1/\delta\mu$, δ, being the dialetric constant in vacuum and μ, the magnetic permeability, so that we can build up a complete picture of the electron with fundamental constants such as δ, μ and h. With this knowledge, the mass of the electron, its frequency and possibly its highest speed can be calculated by the formulas outlined above.

Assuming that the maximal frequency of the electron is $9x10^{20}$ it will not be incompatible with its physical stability when emitting light. According to the theory, the maximum an electron can emit is a radiation with wave length of the order of 10^{-10} to 10^{-11}. The emission of such a quantum of frequency $c/10^{-10}$ could consume the whole frequency of the electron and therefore would mean desintegration. However, the emission of wave lengths such as those of the visible spectrum or even of the U.V. bands emitted by the Hydrogen atom, when excited will not alter but the 7th or 6th case of a number of the order of the square of the light velocity ($9x10^{20}$). In terms of mass emitted it will again be a negligeable fraction of the total mass of the electron.

These values are easy to calçulate by combining Ritz' combination principle[16] and the formulas given by Bohr for the stationary states. The variation of frequency or "time mass" will obviously be :

$$-\Delta g_{44} = v_i - v_k$$

if one takes the two bands in the visible light with wave length $\lambda_k = 6.166x10^{-15}$ and $\lambda_i = 6.56x10^{-15}$, the frequencies are $v_k = 4.861x10^{14}$ and $v_i = 4.57x10^{14}$. When there occurs emission from orbit (k) to (i)

$$-\Delta g_{44} = -(4.861-4.57)x10^{14} = -0.291x10^{14}$$

for the change in time mass, an alteration in the frequency of the electron in the 7th case of a figure of 21 cases !

Likewise, the "space mass" will be altered of a quantity

$$-\Delta g_{ii} = - h(v_i - v_k)/2\pi c^2 = 0.37 x 10^{-34}$$

therefore an alteration in the 7th decimal after the first digit in the measure for the mass of the electron (= 10^{-27}).

Adjusting to wave mechanics. The fundamental idea introduced above is the concept that "time mass" is related to "space mass" through the converting factor (or operator) $h/2\pi$.

Therefore, whenever we have to deal with a relation $2\pi . m_o/h$ it can be cancelled out as the unity (with the dimensions "time mass/square of light velocity"). If we take the relation of de Broglie's wave mechanics $m_o c^2 = hv_o$ it would be written in terms of "time mass"

$$m_o c^2 = \frac{hv_o}{2\pi} \qquad \text{or} \qquad m_o = (m_{44})_o \frac{h}{2\pi}$$

the subscript (o) indicating that we are dealing with resting space mass (m_o) and resting "time mass" $(m_{44})_o$.

In the case of the electron, the frequency $v_o \rightarrow c^2$ if a relativistic correction is applied, and we shall have as indicated before :

$$m = \frac{h}{2\pi} = \hbar$$

If the particle moves with the velocity u, making $\beta = \frac{u}{c}$ its mass will change according to a relativistic effect to

$$m = \frac{m_o}{(1 - \beta^2)^{1/2}}$$

and the above equation becomes in the reference system in motion

$$mc^2 = \frac{hv}{2\pi} = \frac{m_o c^2}{(1 - \beta^2)^{1/2}} = \frac{\hbar\, v_o}{(1 - \beta^2)^{1/2}}$$

according to our definition, the time mass will be expressed by the relation

$$m_{44} = \frac{v}{c^2}$$

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and if $(m_{44})_o$ is the same "time mass" at rest it will undergo the same relativistic transformation as the space mass m_o since

$$v = \frac{v_o}{(1-\beta^2)^{1/2}} \quad \text{and} \quad m_{44} = \frac{v_o}{c^2(1-\beta^2)^{1/2}}$$

or

$$m_{44} = \frac{(m_{44})_o}{(1-\beta^2)^{1/2}}$$

Consequently, in any reference system we should have the same relation $m_{11} = m_{22} \cdots = m_{44}(h/2\pi)$ and the operator $h/2\pi$ is independent of the frame of reference.

By a simple deduction, (as indicated in <u>Annex II</u>) we can draw the conclusion that the vector radius holding the particle to the center $\rho = n^2$, i.e. the square of the quantum number. Since this radius vector is conversely related to the frequency, we have to assume that the frequency proper is inversely proportional to the square of the quantum number, i.e. $v = 1/n^2$, what is conform to the laws of emission and absorption of radiation by the hydrogen atom. With such data we might draw a picture of the hydrogen atom, in which the particle is revolving at distances (<u>on time</u>) of the center, which are proportional to the square of the quantum number. The space coordinates of the orbit remains indeterminate up to now and are irrelevant to the deductions that follow (17).

<u>A geometrical interpretation of Rydberg's constant</u>. Under normal physical conditions of emission and absorption of the three series of bands in the infra-red, the visible and the ultra-violet regions, it is doubtful whether the whole frequency of the electron participates in such changes and it would be reasonable to suppose that the electron has a more stable core and a sort of outer layer that can be lost or rebuilt by the usual emissions and absorptions of light. The limits of this outer layer might be imposed upon it by the value of Rydberg's constant $R = 3.287 \times 10^{15}$. This would constitute the highest frequency to be emitted without a beginning of desintegration, namely under normal physical conditions. In other words, Rydberg's constant (R) indicates the maximum frequency "available" in such circonstances. In fact, according to Bohr's principle when the electron jumps from an

orbit $n = \infty$ to the orbit of lowest level of energy it will emit a quantum of radiation represented by -Rh, as a limit. The light emitted will be one in the U.V. series, with a wave length $\lambda = 0.973 \times 10^{-5}$ and frequency a little less the value of Rydberg's constant $v = 3.29 \times 10^{15}$. A quantum with a frequency identical with Rydberg's constant would be absorbed if the electron would jump from orbit $n = 1$ to one of zero level of energy ($n = \infty$), this should be corrected if the lowest level still should have 1/2(hv). All other frequencies emitted by the hydrogen atom are smaller, unless a beginning of desintegration occurs. The inner core will completely desintegrate by emission of a γ ray of wave length $\lambda = 10^{-11}$ to 10^{-10} (18).

We might assume that the outer layer, which boundaries are defined by Rydberg's constant regulates the geometry of the jump or, in other words, it will define the total "segmentar curvature" of the inner space of the atom model, along the time coordinate. Therefore, in all formulas presented above of the metric tensor, one might assume a sub-metric space corresponding to the time mass $m'_{44} = R/c^2$ and a space mass corresponding to $m'_{11} = m'_{22} = \ldots = m'_{44}(h/2\pi) = Rh/2\pi c^2$. The same laws of emission or absorption of radiation would be applied to this sub-space and measured by formulas which would have the same shape as those given above. So, for the emission of a quantum, the variation of time mass will be, as before :

$$-\Delta m_{44} = (v_i - v_k)/c^2$$

and the variation of space mass :

$$-\Delta g_{ii} = (v_i - v_k) h/2\pi c^2$$

The obvious consequence of this view is to represent the electron as formed of a resistant core enveloped by a changeable halo corresponding to a mass with a value

$$m'_{ii} = \frac{Rh}{2\pi c^2} = 3.6 \times 10^{-33}$$

therefore, the core of the electron would be about 300,000 times the outer layer of the electron. In its more stable position, after emission of the available frequency, the electron is reduced to its core with a space mass :

$$m_{ii} = \frac{h}{2\pi} - \frac{Rh}{2\pi c^2} = \frac{h}{2\pi} \left(1 - \frac{R}{2\pi c^2}\right)$$

Geometric considerations. In this section we are going to show that the law of emission and absorption of radiation can be deduced by pure geometric considerations by postulating that along the time coordinate, the geometry of the atom follows the rules of Weyl's geometry, i.e. the displacement of the particle along the time coordinate (y^4) involves a reduction in length of the bi-tensor g_{ij} in $L = ds^2 = g_{ij}dy^i dy^j$. Although the fundamental interval ds^2 remains constant in each orbit, it will undergo a reduction or expansion when the particle jumps from orbit i to k or k to i (19).

According to the postulates of Weyl's geometry, such a metric connection is characterized by two differential forms :

$$ds^2 = g_{ij}dy^i dy^j$$

and a linear one :

$$d\alpha = \alpha_i dy^i$$

Both forms are invariant for any change of the reference system. By changing the metric, the first is multiplied by a factor v, a positive, continuous and derivable function of the variables; the second is decreased by the differential of log v :

$$-\bar{g}_{ik} = v\, g_{ik} \quad \text{and} \quad \bar{\alpha}_i = \alpha_i - \frac{dv}{v dy^i}$$

or

$$\bar{\alpha}_i = \alpha_i - \frac{d(\log v)}{dy^i}$$

When $\bar{g}_{ik}$ and $\bar{\alpha}_i$ are substituted for the expressions of g_{ik} and α_i in the equations of the geodesics in the metric connection the latter remains invariant. Therefore, as a preliminary step we have to deduce the equations of the displacement of a vector along the geodesic in a space of Weyl.(As shown in Annex III). We can deduce a set of fundamental equations relating α to the g_{ii} when a vector of length $L = ds^2$ is displaced by congruence along the y^s coordinates, reducing by a quantity

$$\Delta L = L\, \alpha_s \Delta y^s \quad \text{where} \quad \alpha_s \Delta y^s = \Delta\alpha$$

In the particular case we are considering (that) the y^s coordinate is supposed to be the time coordinate y^4, since all other components of ΔL are null, according to our assumption that the ds^2 remains constant in each orbit.

The fundamental equation relating α to the g_{ii} are (see Annex III):

$$- \frac{\Delta g_{ii}}{\Delta y^s} = - g_{ii} \frac{\Delta \alpha}{\nabla y^s}$$

This equation can be transformed in the following way : since $g^{ii} = 1/g_{ii}$ therefore $g^{44} = c^2/v_o$ and $g^{22} = g^{33} = 1/m_o = 2\pi/h$

$$g^{ii} \frac{\Delta g_{ii}}{\Delta y^s} - \frac{\Delta \alpha}{\Delta y^s} = 0$$

and, by permutation of all indexes of g and y, we have the following set of equations :

$$g^{44} \frac{\Delta g_{44}}{\Delta y^4} - \frac{\Delta \alpha}{\Delta y^4} = 0$$

and

$$g^{22} \frac{\Delta g_{22}}{\Delta y^4} - \frac{\Delta \alpha}{\Delta y^4} = 0$$

and a similar one for g^{33}.

With these fundamental relations we should be able to describe the movements of the particle along the time coordinate to be conform with the laws of emission of light according to quantum theory. For reasons that will become clear in the next paragraphs, the function α is related to the frequency v and we can assume that $\Delta\alpha = \Delta v/v$, in such a way that the parameter v introduced previously is made equal to the frequency v.

Since $g^{44} = c^2/v_o$

$$\frac{c^2}{v_o} \frac{\Delta g_{44}}{\Delta y^4} = - \frac{\Delta \alpha}{\Delta y^4} = - \frac{\Delta \nu}{\nu \Delta y^4}$$

and

$$c^2 \Delta g_{44} = - \Delta v = - (v_i - v_k)$$

expression that is identical with the one given in page 7, if we make $g_{44} = m_{44}$. The decrease in "time mass" will appear as emitted frequency divided by the factor c^2.

Now, let us calculate the variation of the "space mass" of the metric tensor $g_{22} = g_{33} = m_o$. The fundamental equation takes the form :

$$g^{22}\frac{\Delta g_{22}}{\Delta y^4} = -\frac{\Delta\alpha}{\Delta y^4} \quad \text{where} \quad g^{22} = \frac{1}{m_o} = \frac{2\pi}{h}$$

therefore :

$$\frac{2\pi}{h}\frac{\Delta g_{22}}{\Delta y^4} = -\frac{c^2}{v_o}\frac{\Delta g_{44}}{\Delta y^4} = -\frac{v_i - v_k}{v_o \Delta y^4} = -\frac{\Delta v}{v_o \Delta y^4}$$

with the simplification, making $v_o = c^2$ we have the relation :

$$2\pi\, c^2\, \Delta g_{22} = - h(v_i - v_k)$$

between the decrease of space mass ($m_{22} = m_{33} = \ldots$) and the frequency of the radiation emitted. As the only permissible variation of angular moment is that embodied in the decrease of "space mass", we can take $c^2 \Delta g_{22}$ as the expression of the variation of the angular moment $-\Delta p_i$ and have :

$$2\,\Delta p_i = h(v_i - v_k)$$

and since $v_i - v_k$ according to Ritz principle, is the frequency of the emitted radiation, its wave length will be

$$\lambda = \frac{1}{v_i - v_k}$$

and

$$\Delta p_i = \frac{h}{2\pi\lambda}$$

for the moment attached to the emitted quantum, a well known deduction from the postulates of wave mechanics. It is interesting that this relation has been obtained by geometric considerations, indicating that the fundamental equations above might describe the phenomenon of light emission as a "congruent displacement" along the time coordinate, according to the postulates of Weyl's geometry (20).

Let us go now to the physical interpretation of the equations obtained in the preceding section. If a vector is transported along a geodesic, the values of g_{ik} and of α vary according to the following conditions :

$$\bar{g}_{ik} = v\, g_{ik} \quad \text{and} \quad \bar{\alpha} = \alpha - \frac{dv}{v}$$

if we substitute in one of our fundamental equations, we obtain:

$$\frac{1}{g_{22}}\frac{\Delta g_{22}}{\Delta y^4} + \frac{\Delta v}{\Delta y^4} = -\frac{\Delta\alpha}{\Delta y^4} + \frac{\Delta v}{v\Delta y^4}$$

Now, we can define the transport along the geodesic of a Weyl's space, or a transport by congruence of a vector or tensor, in conditions in which the variation of length is completely accounted for by changes in metric, i.e. by equalling to zero each side of the above equation :

$$\frac{1}{g_{22}} \quad \frac{\Delta g_{22}}{\Delta y^4} = - \frac{\Delta v}{v \Delta y^4} \qquad \text{or} \qquad \Delta g^{22} = - g_{22} \frac{d\nu}{\nu}$$

and

$$\frac{\Delta\alpha}{\Delta y^4} = \frac{\Delta v}{v \Delta y^4} \qquad \text{or} \qquad \Delta\alpha = \frac{\Delta v}{v} = d(\log v)$$

It is easy to see that our new parameter v is identical with the factor F_i introduced in the deduction of the "time component of mass" and supposed to be linearly related to the frequency. Now, we can make it identical with the frequency and have :

$$\Delta\alpha = \frac{\Delta v}{v} = \frac{v_i - v_k}{v_i}$$

and have likewise :

$$\Delta g_{22} = - \frac{\Delta v}{v} g_{22}$$

If these assumptions are correct, we have to give a great significance to the increment $\Delta v/v$ or $(v_o - v_i)/v_o$ i.e. the ratio of decrement of the frequency to the highest frequency should determine the decrement or total variation of the energy of the orbit g_{22} or, in other words, the real law of emission will depend upon the successive powers of the ratio $\Delta v/v$, and this can be compared with experimental data derived from spectral analysis.

<u>The law of emission of radiation</u>. We have to go back to the mechanism of radiation to find out if any quotient of that sort might be of significance to trace back the paths followed by the electron along the time coordinate. We assume that along such time coordinate, the particle should follow a geodesic the equations of which are given by the equations :

$$\Delta g_{22} = - g_{22} \frac{\Delta v}{v} \qquad \text{and} \qquad \Delta\alpha = \frac{\Delta v}{v}$$

To find the physical significance of such equations we have two alternatives :

a) to consider Δv as the variation of the whole frequency associated with the electron; this of course is not required by the theory and difficult to fit with the experimental data available;

b) the variation of Δv refers to that part of the electron frequency represented by Rydberg's constant (R).

In that case, the whole geometry of the electron jumps will be regulated by variations of the "outer layer", representing 1:300,000 of the mass of the electron. It is this variable "plasma" or halo which contributes with the needed frequency for changes in position of the electron inside the atom, along the time coordinate (y^4). Therefore, we can make v indicate the levels of frequency imposed by Ritz' combination principle and consider the maximum value for $v_m = R$ (Rydberg's constant). In such a case, the interval

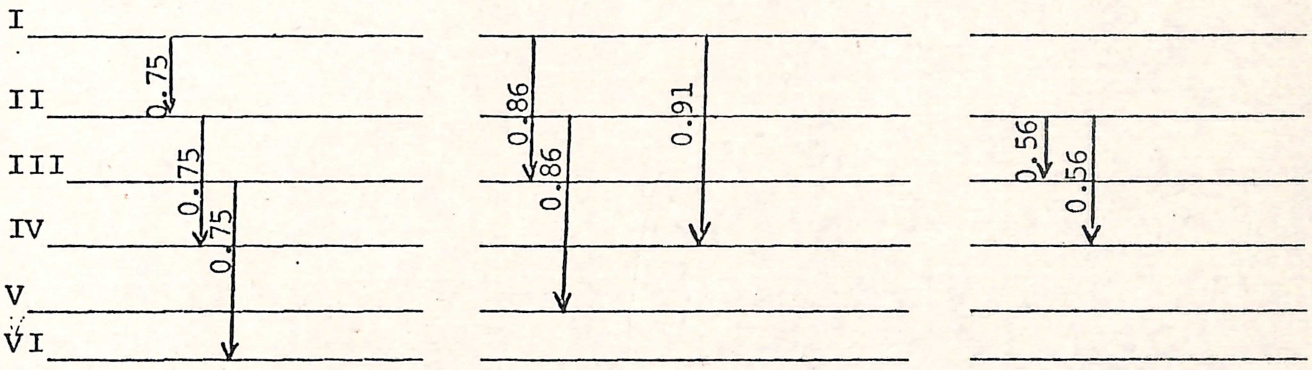
$$\Delta\alpha = \frac{\Delta v}{v} = \frac{v_i - v_k}{v_i} \quad \text{and} \quad \Delta g_{ii} = -g_{ii}\left(\frac{v_i - v_k}{v_i}\right)$$

would repeat itself for each jump, from one orbit (i) to k_1, k_2 k_3... In other words, if the numerical expression of this ratio for a first jump is $(v_1-v_2)/v_1$ the next jumps will be regulated by the square, the cube and the fourth powers of this ratio. Therefore, for each interval $\Delta\alpha$ we will have the powers $(\Delta\alpha)^2$ $(\Delta\alpha)^3$ $(\Delta\alpha)^4$ and so forth, as possible pathways along the geodesic corresponding to the time coordinate. This simple and attractive possibility allows a direct recalculation of the levels of frequencies of the emitted radiation by the hydrogen atom, in terms of the postulates of Weyl's geometry. Following this line, we have found, as indicated below, three of such intervals 0.91 0.86 and 0.75. From these three values we can deduce the frequencies of the known bands of the hydrogen spectrum. Obviously, these series of frequency are not arranged as the classical series of Balmer, Paschen and Lyman, in the visible, the infra-red and the ultraviolet, but they are serially distributed along the whole spectrum in the natural sequence U.V. → Vis. → I.R. that is more natural from a geometrical point of view than the artificial distribution in the I.R., visible and U.V.

To distinguish from the classical series, we might call the new grouping of bands, as "classes", each one characterized by a geometrical interval, as indicated above. Furthermore, the three intervals indicated above corresponds to the number of orbits (bonds) overlapped by the jump, in such a way that the interval 0.75 corresponds to a single bond, the interval 0.86 to a double bond and the interval 0.9 to a triple bond, overlapped by the jump when the electron starts from the highest level of energy. It is remarkable that some of these intervals

$( )^n$ repeat themselves in the different classes. For instance 0.75 repeat in all four classes with remarkable regularity; also the interval 0.86, the fundamental interval for a double jump, repeats itself in the second triple jump, and possibly in the second quadruple jump. One of the sub-intervals $(0.75)^2$ = 0.56 appeared twice in our calculations in the second single jump and the fourth double jump. The following table I, gives detaila of the calculations of the intervals $\Delta\alpha=\Delta v/v$ using seven or eight levels of frequencies which are utilized to deduce the emitted frequencies $(v_i - v_k)$ according to Ritz' combination principle.

We might represent in a condensed scheme the repetitions of intervals and sub-intervals, limiting our calculation to the six levels of energy utilized in Table I. We are going to develop further this idea, after some considerations in order to define better the law of emission according to the geometrical postulates assumed above.



The law of emission. What we need now is a general principle that might take the form of a conversative law wherefrom the law of emission and absorption of radiation by the particle could derive according to the experiment. We have found in the preceding paragraph that there is a constant interval regulating the emission of radiation in each jump. We might assume, according to that, the constancy of the relation dv/v and introduce this as a law of nature :

$$d(\log v) = \text{const.}$$

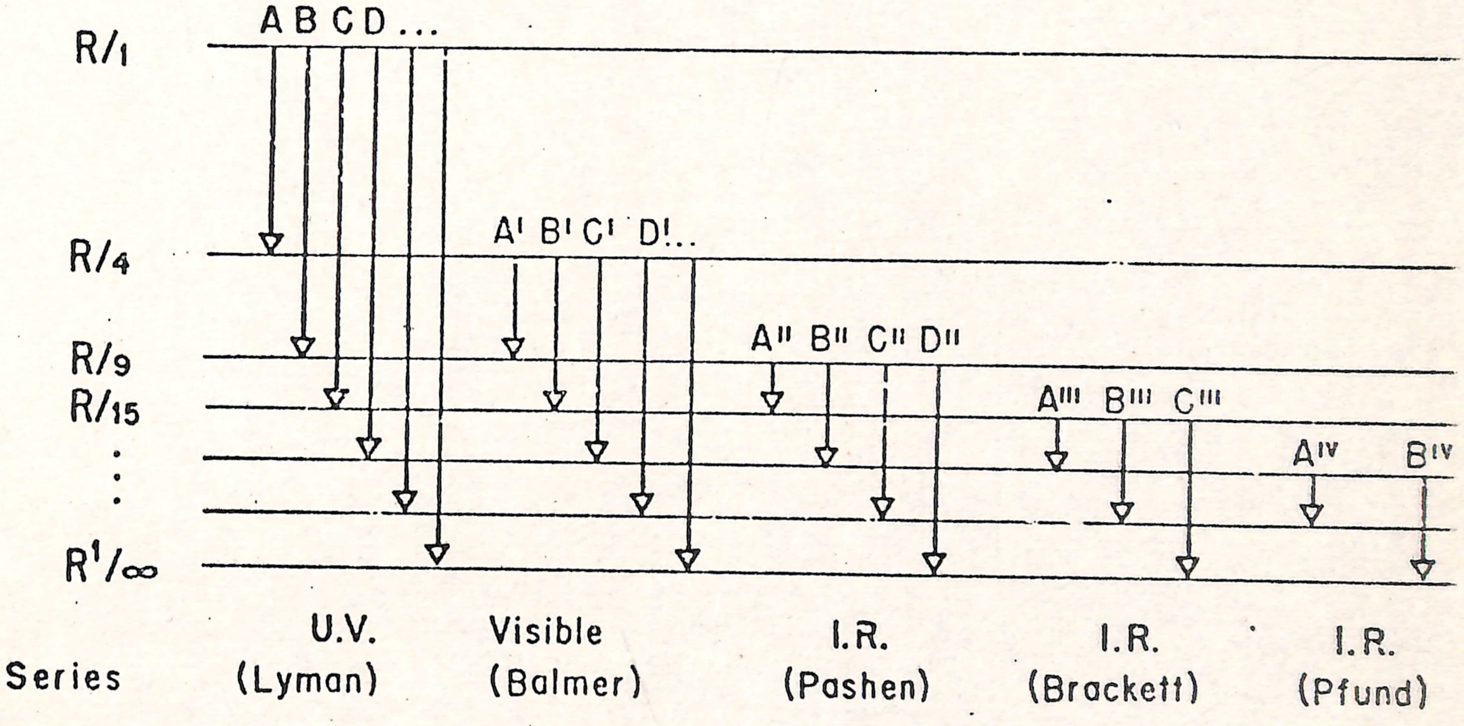
Therefore, the conservative law regulating the geometry of the jump might be the above equation with its immediate consequence :

$$d^2(\log v) = 0$$

With such a simple formulation we might be able to formulate on some new basis the electro-magnetic field and deduce an electro-magnetic definition for the "time mass". For the moment, it is enough that we take that relation as a basic postulate to present the phenomenon of emission of radiation in its simplest geometrical way.

Let us consider, first, the six levels (n = 6) considered before. It is known that the value of $n \to \infty$ can be very great, but the phenomenon will be fully understood if we limit our considerations to a reasonable value of n (=6). If the electron jumps from 1 to 6 it will emit 97% of the whole frequency stored in the outer orbit, but since the bands will be more and more packed together, a small difference in residual frequency will mean a large increase in the value of the number of quanta emitted, till the maximum frequency is approached, coinciding with the value of Rydberg's constant $R = 3.29 \times 10^{15}$ (22).

The limiting frequencies for the classical series in the U.V., Vis. and I.R., are respectively R = 3.287 R/4=0.821 R/9 = 0.365 R/16 = 0.205 and so forth. The phenomenon can be represented by the well known diagram :



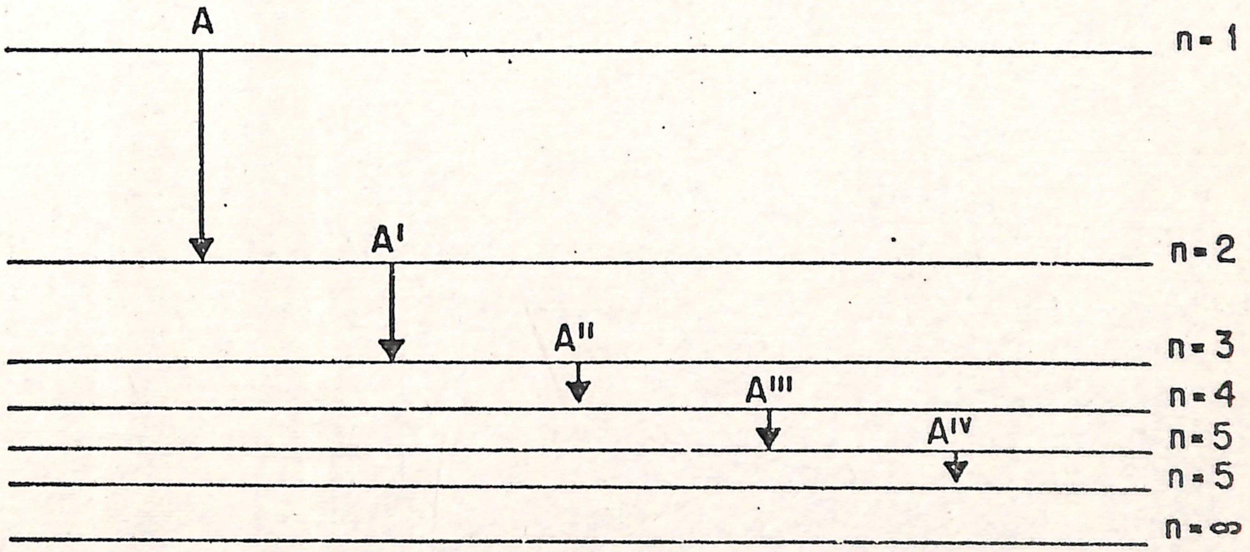
It is known that there are simple relationship between all bands which depend upon a single jump A A' A'' A''' A^{iv}... from one orbit to the following one. They usually have the highest intensities in each series. Therefore it is assumed that they represent the bands of highest probabilities inside the series, since the largest number of particles are doing such a single jump for each time interval. There is another reason to make the single jump the commonest one. They are all geometrically equal. In fact, if we accept the idea that the law of emission depends upon the constancy of an interval that can be expressed by d(log v) constant, we can now classify the bands under a different criterium and have a "class" of bands which obey such a law. This "class" will contain the bands with the highest intensity in the series, and therefore they will represent emission of single jumps. In such a case, if we have for the first jump $(v_1-v_2)/v_1$ we will have for the second jump a similar interval and therefore :

$$\frac{v_2-v_3}{v_2} = \left(\frac{v_1-v_2}{v_1}\right)^2$$

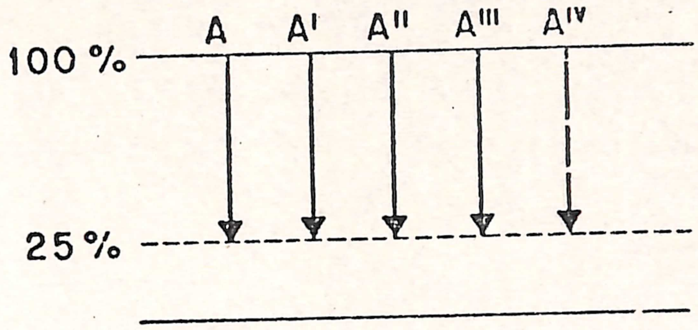
and for the third single jump, in the infra-red, the cube of the fundamental interval :

$$\frac{v_3-v_4}{v_3} = \left(\frac{v_1-v_2}{v_1}\right)^3$$

This succession of single jumps will give a "class" of bands distributed all along the spectrum from the highest Lyman series to the softest I.R. series. This class will contain the bands with the highest energy of emission in each series. The frequencies will be obtained by multiplying Rydberg's constant by the successive powers of the fundamental interval 0.75 as indicated in the following diagram :



The size of the arrows A, A', etc are proportional to the frequencies of the emitted radiations and therefore localize them in each one of the usual series in the U.V., the Vis. and the I.R., corresponding to the classical series of Lyman, Balmer, Pashen, Brackett and Pfund. However, this new class constitutes a more rational series of bands, since they correspond to single jumpers. All of them are emitted by single jumps, though the initial levels are lower and lower starting with level 1, following the steps 2,3,4, etc. If we give their values in terms of percentage of the highest frequency level from where they start, they can be represented by the same interval 0.75 of the limiting frequency of the series :



It is in that sense that we can talk about the "same interval" characteristic of all bands A A' A'' A''' and so forth.

Though in terms of frequency they are very much different, ranging from the U.V. to the I.R., in terms of "interval" they are all alike and each one can be obtained from the previous one by multiplying it by the constant factor

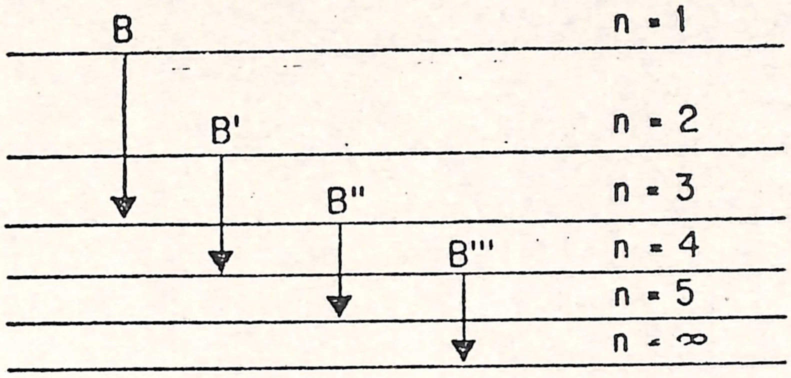
$$d(\log v) = \text{constant} = 0.75$$

The possible significance of this parameter as a geometric interval regulating the metric of the jump, has been already stressed. The constancy of such an interval could be taken as a law of nature, regulating the movements of the particle along the time coordinate. In each single jump a constant parcel of that "plasma" will be consumed. If we start with the whole cake represented by Rydberg's constant, the partition must be in perfect squares 1:4:9:16:25... as if we were dealing with an instrument of measure that would stretch proportionally to the squares of the single digits 1,2,3,4,5... to cut a cake that must be divided to serve an increasing number of customers (guests). The other equitative division would be the following : for the first coming customer and each one following it, a certain proportion of what is left will be given, let us say 75%. This is a wise law of division adopted by nature to divide Rydberg's constant among the successively increasing wave lengths, since the end of the series is unknown, or indefinite.

The same is true for the "double jumpers" representing a "class" of the following intensitites. In that case, the interval will be

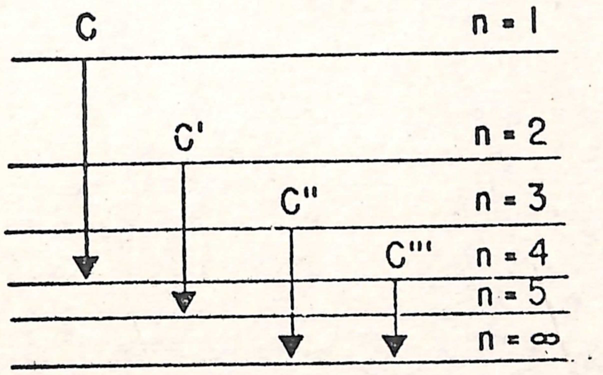
$$\frac{v_1 - v_3}{v_1} = 0.86$$

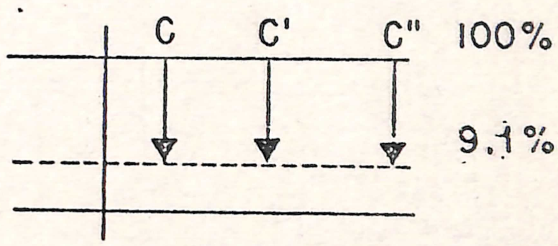
It will repeat itself in all double bonds (jumps) in the different series forming a new class B B' B'' B'''.... according to the following diagram :



And again if we translate frequencies in terms of percentages we will have a diagram similar to the previous one, indicating that for each double jump 86% of the frequency will be emitted and 14% will be left, and all the intervals for a double jump are equal.

As shown in Table I, a similar law will hold for the triple jumpers with an interval $(v_1-v_4)/v_1 = 0.91$. And similar diagrams can be draw to show that all triple jumps correspond to the same "geometric interval" :





In a certain sense, the corrections presented above, transforming absolute frequencies in terms of percentages of the initial values, would constitute a way of correcting for the so-called "segmentar curvature" of the atomic universe, according to Weyl's postulates.

Conclusion

We have tried to give a coherent view of the phenomenon of emission and absorption of radiation by the electron in the hydrogen atom, reducing it to a geometrical pattern.

It has always been tempting to reduce physical phenomena to changes in metric and remarks by Gauss[21] Helmholtz[22] Clifford[23] and many others, in the last Century, pointed in that direction. However, up to now, it has only been successfully applied to the gravitational field in the general theory of relativity. Attempts to reduce the electromagnetic field to variations in metric of space, presented by Weyl and others met with failure because of the impossibility of the proposed scheme to account for the discontinuity which is peculiar to the phenomenon of emission and absorption of light. We thought to overcome this difficulty by introducing the idea that Planck's constant is a mere proportionality factor operating the transformation of "time mass" into "space mass". In that sense, along the time coordinate, Planck's constant constitutes an interval "thickness of the universe" inside of which the electron can move or change velocity and position, without changing "gauge" or metric, as indicated by the constant interval $-ds^2 = g_{ik}dy^i dy^k$. This metric is regulated by the frequency proper of the orbit, and therefore to each level of the metric potential (g_{ik}) there corresponds a definite value of the time coordinate. The particle is supposed to move at fixed distance "on time" of the center of the orbit, and the metric of the orbit is regulated by a certain value of the time coordinate.

This attitude is defensible if one assumes a perfect fusion of the four parameters of Minkowski-Einstein universe. If the particle moves with any velocity approaching that of light, a time parameter fixing it to the center of the atom, is not only a possibility but an unavoidable consequence of the relativistic equations. In consequence it is understandable that the particle brings with itself a clock-like device to fix time as a parameter of the orbit. Since time is continuously flowing, the only way to make it a constant of the orbit is to transform it into a frequency, or to make it to flow alternately in opposite directions in a stationary wave. By so doing, the time parameter of each orbit becomes "gauged" or fixed and the geometry of the jump along the time coordinate can be regulated by changes of frequency alone.

To go once more to the law of radiation, if the electron moves from orbit j to orbit i, along the time axis to the center of the orbit, it will emit radiation and for each frequency emitted it becomes fixed to a more internal orbit with a new level of frequency v_i, in such a way that the emitted frequency $v_{ij} = v_j - v_i$ according to Ritz' combination principle. Since this parameter is the only one regulating the levels of energy in each orbit we are bound to assume that v_{ij} is the factor which change the metric of the orbit and therefore :

$$\bar{g}_{ii} = v_{ij} g_{jj}$$

and the fundamental interval $-ds^2$ will be affected proportionaly when the electron jumps from orbit j to orbit i. As the electron changes of "affine connection", there is a change of "gauge" and the variation of gauge then occurring can be called "segmentar curvature" and depends solely upon the frequency proper of the orbit. The postulates of Weyl's geometry are fulfilled, if we can define a parameter v_{ij} affecting the "gauge" by multiplying the fundamental tensor (g_{ij}), and another parameter varying linearly along the pathway, and that we have denoted by α, in such a way that both parameters are connected through the differential equation :

$$\frac{d\alpha}{dy^4} = \frac{dv}{v.dy^4}$$

or

$$d\alpha = d(\log v) = \text{constant}$$

according to the principles of Weyl's geometry. From this relation, we can deduce a law of conservation

$$d^2(\log v) = 0$$

This new law of conservation regulates the mechanism of emission and absorption of light (radiation) by the particle in the hydrogen atom. We have seen that this law introduces a new way of calculating the frequencies of the emitted radiations, leading to new "classes" of single, double and triple jumpers, distributed along the whole spectrum of the hydrogen atom. From these considerations a new law of distribution of the emitted frequencies has been proposed, as one giving always a certain percentage (say 75%, 86%, 91%) of what is left to be emitted. This appears to be a more rational approach to the distribution of frequencies among a number of unknown possibilities, than any equitative distribution. If we have to cut a cake to satisfy equally any newcomer, if we dont know how many will be coming, the obvious procedure would be to give to the next the same proportion of what was left of the cake, instead of giving equal parts that might be in excess or default according to the number of newcomers. Multiplying each slice of the "time mass" cake by Planck's constant permits the transformation of slices of "time mass" into "space mass" or energy.

The proposed theory constitutes only a new view of well established facts. The agreement between theory and experimental data as shown in Table I is not quite satisfactory, it might be necessary to introduce a correction term difficult to define at the moment. Unfortunately we have no theory of the jump that might be matched with that presented above, since in "old" and in "new" quantum theory what happens to the particle while jumping from orbit i to j is completely left unaccounted for. In the theory presented above emission and absorption of radiation are understood as adjustements of the "gauge" of the particle to the geometry of the new orbit and that the alterations of "time" and "space" mass are homothetic with a coefficient $k^2 = v_i/v_k$ or $k^2 = T_k/T_i$. Therefore, all transformations of "gauge" are deterministic and follow the postulates of Weyl's geometry.

Summary

The present paper represents the results of reflexions on the intimate mechanism of emission and absorption of radiation by such a simple system as the hydrogen atom. We have departed from simple and straightforward premisses :

1. The inability of the electron to emit while in a stable orbit derives from the fact that the orbit is a geodesic of a non-euclidian surface the whole energy of it being accounted for as kinetic energy.

2. The coordinate along which the electron is bound to the nucleus is the time coordinate of a Minkowski-Einstein four dimensional continuum and therefore the sphere containing the orbit of the electron could be understood as one having an imaginary radius (iR) and could be expressed in terms of a Bolyai-Lobatchewski concave surface, having as its simplest geometrical representation the so-called hypersphere of negative curvature.

3. The changes in kinetic energy when the electron jumps

from orbit i to k follows the rules of a parallel displacement in a Weyl's geometric continuum, along the time coordinate.

4. Planck's constant $(h/2\pi)$ can be envisaged as measuring the ratio between the components of mass, along the space coordinates and the component of mass along the "time axis", i.e. between "space mass" (m_{ii}) and "time mass" (m_{44}) in such a way that $m_{44} = (h/2\pi)m_{11}$.

5. Time mass has the dimensions of a "time over the square of a length", i.e. $m_{44} = L^{-2}T$ and will be numerically calculated as the ratio of a frequency over the square of the light velocity : $m_{44} = v^2/c^2$ or, in a pure dimensional form :

$$T^{-1}L^{-2}T^2 = L^{-2}T$$

In such conditions, Planck's constant is a dimensionless quantity invariant in any frame of reference.

6. If Planck's constant is a simple factor of proportionality between "space mass" (m_{ii}) and "time mass" (m_{44}), the uncertainty principle $\Delta E.\Delta t = h/2\pi$ springs necessarily from that premisse. If we develop the expression

$$\frac{\Delta m_{11}}{\Delta m_{44}} = \frac{\Delta c^2 m_{11}}{\Delta c^2 m_{44}} = \frac{\Delta E}{\Delta v} = \frac{h}{2\pi}$$

since Δv can always be made equal to Δt^{-1}, it follows

$$\Delta E.\Delta t = h/2\pi$$

7. Rydberg's constant can introduce a sort of "outer layer" of the electron, as a sub-metric space corresponding to a "time mass" $m'_{44} = R/c^2$ that can be changed each moment the electron emits or absorbs radiation. This changing "halo" or "plasma" of the electron corresponds to a space mass of $m_{11} = Rh/2\pi c^2 = 3.6\times10^{-37}$ g, therefore only 3.3×10^{-7} of the "inner core" of the electron. This outer layer would represent the "available frequency" that can be emitted at once if the electron travels from infinite to its most stable orbit, or in "parcels" when it jumps from orbit i to k, absorbed when the movement of the electron along the time coordinate is in the opposite direction.

8. If, as assumed above, the emission or absorption of radiation results of a parallel displacement according to the rules of Weyl's geometry, along the time coordinate, such an

emission involves a reduction of the length of the metric tensor g_{ij} in

$$ds^2 = g_{ij}dy^i dy^j$$

According to the above considerations, ds^2 is constant in each orbit (invariance of gauge), but negative ($= -g_{ij}dy^i dy^j$), and bound to suffer a increment or decrement when the electron undergoes a parallel displacement (according to Weyl's postulates) along the time coordinate. By applying the derivation peculiar to this kind of displacement in a Weyl's space, we can deduce a new law of radiation giving primary importance to a logarithmic ratio $\Delta v/v$ the increasing potencies of which (from 1 to n) will regulate the frequencies of the emitted radiation. Therefore, the new law of radiation would conform to the constancy of that ratio or to $d(\log v) = \text{const.}$, or $d^2(\log v) = 0$, that might be the expression of a new law of permanence in the universe.

9. If this law of radiation is applied to the numerical values of the frequencies emitted by the electron when jumping a single, double or triple bond, an interesting coincidence between expected and observed could be obtained, as indicated in Table 1. Accordingly, the spectral lines could be classified in a more natural way, in classes containing, the first one, all emissions of single jumps, for which the fundamental interval was found to be 0.75; the second one, all double jumps, with an interval of 0.86 (experimental 0.89) and the third one all triple bonds or jumps, with an interval of 0.95 (experimental 0.937) and so on. Such intervals correct all emissions for the "segmentar curvature" as expressed by the postulates of Weyl's geometry, and will be conform to the general law $d(\log v) = \text{const.}$ or $d^2(\log v) = 0$.

10. To understand in a crude way the significance of this new radiation law, we might think of Rydberg's constant (R) as a cake or frequencies (time mass) to be divided equitatively in each jump with emission of radiation. If we thought of a way to divide a cake when we dont know the number of coming customers, nothing more equitative than to give to each one a certain proportion of what was left. This is a most wise law of division invented by nature to distribute the available frequency represented by Rydberg's constant, among increasing quanta of radiation, since the end of the series is unknown or indefinite.

11. The above theory is entirely speculative and constitutes a different way to interpret events that have been fully explained by other theories.

TABLE I

Values of the differential $=\Delta v/v$ for single, double, triple and quadruple bonds, taking $v_m = R$ or Rydberg's constant as the highest value of the frequency commanding the jumps.

Frequencies: $\times 10^{15}$	Differences $(v_i - v_k)$ $\times 10^{15}$	Ratios $\frac{(v_i - v_k)}{v_i}$		Expected valu of $\frac{(v_m - v_{m-1})^n}{(v_m)^n}$
Single jumps			Ratios (corrected)	
v_1 = 3.287,870				
v_2 = 0.821,975	2.465,895	0.7499	0.7472	0.7500
v_3 = 0.365,318	0.456,657	0.5555	0.5444	0.5625
v_4 = 0.205,491	0.159,828	0.4375	0.4125	0.4218
v_5 = 0.131,352	0.074,139	0.3601	0.3164	0.3164
v_6 = 0.091,330	0.040,019	0.3059	0.2353	0.2373
v_7 = 0.067,099	0.024,223	0.2553	0.1655	0.1779
Double jumps				
v_1 = 3.287,870				
v_3 = 0.365,318	2.922,552	0.8889		0.8660
v_2 = 0.821,975				
v_4 = 0.202,491	0.616,485	0.7500		0.7499
v_3 = 0.365,318				
v_5 = 0.131,352	0.233,966	0.6404		0.6494
v_4 = 0.205,491				
v_6 = 0.091,330	0.114,161	0.5556		0.5614
$v_5 \rightarrow v_7$ = 0.067,099	0.064,253	0.4892		0.4871

Table I (continuation)

Triple jumps

v_1 = 3.287,870			
v_4 = 0.205,491	3.082,238	0.9375	0.9130
v_2 = 0.821,875			
v_5 = 0.131,551	0.690,361	0.8400	0.8333
v_3 = 0.365,318			
v_6 = 0.091,330	0.273,988	0.7500	0.7610
$v_4 \to v_7$ = 0.067,099	0.138,342	0,6734	0.6951

Quadruple jumps

v_1 = 3.287,870			
v_5 = 0.131,352	3.156,518	0.9600	0.9350
v_2 = 0.821,875			
v_6 = 0.091,330	0.730,643	0.8890	0.8742
v_3 = 0.365,318			
v_7 = 0.067,099	0.298,219	0.8163	0.8174
v_4 = 0.205,491			
v_8 = 0.051,373	0.154,116	0.7500	0.7642

Note : The discrepancy between found and expected ratios in the lowest levels of single jumpers is difficult to understand according to the theory, and might depend on some correction term still undisclosed. If we subtract from all differences a constant value say Σ = 0.00911, we may correct the values of the ratios as indicated in the first section (correction) of Table I.

Falta Bibliograf da Parte I

Part I

Notes and References

1) The laws of emission and absorption of energy (E) by the hydrogenoid atom can be found in any advanced Text-Book of Physics, since N. Bohr (1913) Phil. Mag. 26, 1 and 476, emitted his theory of stationnary orbits. See also:

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2) Einstein, A. 1905. Zur Elektrodynamie bewegter Körper. Ann. der Physik 17; Idem. 1916. Die Grundlage der allgemeine Relativitätstheorie. Ann. der Physik 49; 1916

3) The geometrical foundations of the gravitational field became the subject of many monographs and Text-Books, ever since Einstein (1916) proposed his theory of the gravitational field as a Riemannian space-time continuum known as the General Theory of Relativity. See:

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Denis-Papin, M., Faurex, R. et Kaufmann, A. 1960. Exercices de Calcul tensoriel avec leurs solutions. Edit. Eyrolles, Paris.

Denis-Papin, M. et Kaufmann, A. Cours de Calcul Tensoriel Appliquē. Edit. Albin-Michel, Paris.

5) The concept of Minkowxki-Einstein four dimensional continuum can be found in:

Becquerel, J. (1922). Loc. cit. (3).

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Minkowski, H. 1908. Raum und Zeit, cit. in Donder (1925). Loc. cit.

Einstein, A. 1922. The meaning of Relativity. Methuen, London.

Einstein, A. 1922. Loc. cit. (3).

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6,7) For all problems of setting dynamic equations in the form of absolute differential geometry. See:

Appell, P. (1955). Loc. cit. (4).

Denis-Papin et Kaufmann (1961). Loc. cit. (4).

8) Idem, Ibidem.

9) For the composition of the hydrogen spectrum, any Text-Book of advanced Physics can be consulted. See:

Richtmeyer, F.K., Kennard, E.H. and Cooper, J.N. 1969. Introduction to Modern Physics, 6^{th}, Edit. McGraw Hill Book Co., New York, Tokyo.

Bockhoff, P.J. 1969. Elements of Quantum Theory. Addison-Wesley Publ. Co., Reading, Mass.

Sherwin, C.W. 19 . Introduction to Quantum Mechanics. Holt, Rinehart, N. York.

Slater, J.C. 1955. Modern Physics. Mac Graw-Hill Book Co. Inc., N. York, Tokyo.

10) Balmer (1885) showed that the frequencies of the hydrogen spectrum in the visible obeyed an extremely simple formula (Balmer series) $C(\frac{1}{4} - \frac{1}{n^2})$ = frequency, for each value of the integer n. Later on it was found that such a series would extend to the whole spectrum, if instead of 4, another digit (m^2) was introduced in the formula, and therefore what is known as Balmer formula is the empirical frequency = $R\ (\frac{1}{m^2} - \frac{1}{n^2})$, R being the constant of Rydberg $R = 3.287,870 \times 10^{15}$ cycles/sec. The use of R, in cycles per sec, instead of wave numbers/cm^{-1} as the constant appears in most text-books of Physics (see references at the end of this section) $R_H = 109.677.459 \pm 0.05\ cm^{-1}$) would be justified in view of the great historical importance of the coincidence found by Bohr (1913) between the theoretical value of $R = 2\pi^2 me^4/h^3$ and the experimental value $R = 3.290 \times 10^{15}$ as found in the literature derived from direct spectroscopic measurements. In a seprate paper (Rocha e Silva, 1978). What is in a name? Rydberg constant: $\underline{R}$, $\underline{R}_H$, $\underline{R}_{oo}$ or $\underline{R}_{BR}$. To be published.

11) The technique of calculation of a parallel displacement in a Weyl's space can be found in Weyl H. (1922) Op. cit. (14).

Also in: Apell, P. (1955). Loc. cit. (4).

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12) What we called Hypersphere has been more frequently called Pseudosphere or hyperbolic non-Euclidian geometry. According to the relative value of Gaussian curvature (K), the three species of surfaces are usually considered: $K = 0$, Euclidian space or surface; $K > 0$, Riemannian space or the external surface of a sphere, as appears in general relativity; $K < 0$, a Boyayan or Lobatchewskian surface, as the internal surface of the sphere, the pseudo-sphere. For a sphere of radius a, the curvature is meaured by $K = 1/a^2$, herefore positive ($K > 0$). A surface with constant and negative Gaussian curvature ($K < 0$) is called a pseudosphere, and can be generated of the revolution of a curve called a *Traktrix*. The geometry of the pseudosphere was studied by Beltrami (1868-69, Loc. cit. (13), and introduced as a possibility to represent intuitive space, by Helmholtz (18). For a recent discussion see J.L. Richards, 1977, *Br. J. of Philosophy* 28, 235-253. In the present paper it was suggested as a model for the intraatomic surface, or space, identifying K with the frequency (v_i) of the orbit. See Annexes II and III.

13) Beltrami, E. 1868-69. *Teoria fondamentali degli spazii di curvatura constante*. Annali di Matem. 2, 232-255. Cit. in Cartan (1928). Loc. cot. (4).

14) Weyl, H. 1922. *Temps, Espace et Matière. Léçons sur la Théorie de la Rélativité Générale*. Libr. Sc. Albert Blanchard, Paris.

15) Einstein, A. 1921. *La géometrie et l'expérience*. Gauthier-Villars, Paris. See also

Einstein, A. 1921. *L'éther et la Théorie de la Rélativité*. Gauthier-Villars et Cie., Edit., Paris.

16) For the formulation of Ritz' combination principle. See...

17) See Annexes I, II, III and IV.

18) See any Textbook of advanced Physics, as mentioned in the end of this Section (Refs.).

19) See Loc. cit. (4, 7, 11, 14).

20) See Refs. (4, 11, 14).

21) Gauss,

22) Helmholtz,

23) Clifford,

24) For the relations between the theory advanced above and old and new. Quantum theory, Wave Mechanics and Relativity, the following Text-books may be consulted: