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A Universal Duality Principle For Hadrodynamics and Electrodynamics+

(preliminary copy)

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Abstract

We extend to hadrodynamics Born's dimension selecting duality principle for generalized electrodynamics. In two dimensions, this principle is obeyed by the parameter invariant Nambu theory of hadronic strings, in four dimensions, by the Nielsen-Olesen theory of the dual strings. A geometrodynamical interpretation of Duality is presented.

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One intriguing puzzle in recent strong interaction physics has been the central importance of 2-dimensional field theories in the description of processes in the 4-dimensional physical world.[1)] In this paper, we propose a possible solution.

We view the occurrence of 2-dimensional theories as evidence of the workings of a fundamental principle in the arena of strong interactions. This dimension selecting Duality Principle, proposed by Max Born,[2)] is obeyed by Maxwell and Born-Infeld electrodynamics in 4 dimensions.[3)] We show that it is also satisfied by the Nambu hadrodynamics of strings[4)] in 2 dimensions and the Nielsen-Olesen local field theory of the dual strings in 4 dimensions.[5)]

We consider a classical continuum system S in an n-dimensional differentiable world manifold W with or without affine, conformal, or metric structure. Associated with S is a Lagrangian $\mathcal{L}(\psi^\alpha, \psi^\alpha_i)$ restricted to be a function of ν fields $\psi^\alpha(x^i)$ and their first derivatives ($\partial_i = \partial/\partial x^i$)

$$\psi^\alpha_i = \partial_i \psi^\alpha , \qquad \begin{matrix}\alpha = 1,2\ldots\nu, \\ i = 1,2\ldots n.\end{matrix} \quad (1)$$

$x^1, x^2, \ldots x^n$ label a local coordinate system. ψ^α and ψ^α_i describe some dynamical properties of S such as elastic potential, fluid velocity, etc. The two spaces of the ψ^α and the x^i are assumed to transform independently with their respective metrics left arbitrary.

For the sake of concision and physical clarity, we use some k-vector calculus.[6)] A k-vector is an antisymmetric tensor of rank k. We define the operations of the Div and Curl and Dual of a k-vector. When $\mathbb{T}$ is a contravariant k-vector density of weight 1 ($K>0$) in an n-dimensional manifold W, $\operatorname{div}\mathbb{T}$ is a contravariant (k-1) vector density

$$(\operatorname{div} \mathbb{T})^{ij\ldots l} = \partial_h T^{ij\ldots lh} \quad (2)$$

with $\partial_h = x^h$ ordinary derivative. Moreover,

$\mathrm{div}\,\mathrm{div}\,\mathbf{T} = 0$.

When $\mathcal{U}$ is a covariant k-vector $(K < n)$, curl $\mathcal{U}$ is a covariant (k + 1)-vector

$$(\mathrm{curl}\,\mathcal{U})_{hij\ldots\ell} = \frac{1}{k!}\delta^{abc\ldots d}_{hij\ldots\ell}\,\partial_a \mathcal{U}_{bc\ldots d} \qquad (3)$$

where $\delta^{ab\ldots d}_{hi\ldots j}$ is the mixed Kronecker symbol.
Moreover, curl curl $\mathcal{U}$ =0.

If P and Q are k-vectors of weight M , their duals P^* and Q^* are (n-k)-vectors

$$P^{*ab\ldots c} = \frac{1}{k!}\epsilon^{ab\ldots c\, ij\ldots\ell} P_{ij\ldots\ell}, \qquad (4)$$

$$Q^*_{ab\ldots c} = \frac{1}{k!} Q^{ij\ldots\ell}\epsilon_{ij\ldots\ell\, ab\ldots c} \qquad (5)$$

of weight $M+1$ and $M-1$ respectively. $\epsilon^{ab\ldots\ell}$ is the Levi-Civita pseudo-tensor density. For any type of k-vector Y , $(Y^*)^* = Y$. We also note the useful relation between Div and Curl

$$(\mathrm{curl}\,\mathcal{U})^* = \mathrm{div}\,\mathcal{U}^* \qquad (6)$$

provided $\mathcal{U}^*$ is taken covariant when $\mathcal{U}$ is a scalar. End of prelude on k-vectors.

We consider ψ^α_i as the i-th component of a 1-vector Ψ^α in n-space. If ν 1-vectors Ψ^α are to be derived from ν potentials ψ^α , as in (1), they must satisfy the $\nu\frac{n(n-1)}{2}$ "integrability conditions," (I.C.).

$$\mathrm{curl}\,\Psi^\alpha = 0 \qquad (7)$$

asserting the equality of mixed derivatives $\partial_i\partial_j\psi^\alpha$ and $\partial_j\partial_i\psi^\alpha$ (Poincaré Theorem). (7) is a non-dynamical, geometric element of the theory. It is a topological statement of smoothness about the fields ψ^α_i i.e. they are at least continuously differentiable functions.[7] (7) is an implementation of a differential structure on the manifold W. By Hamilton Principle, the dynamical elements of our theory, the Euler-Lagrange equations of motion (E-L equns), are the necessary

conditions for the extremal of the action integral

$$I = \int_D \mathcal{L}(\psi^\alpha, \psi^\alpha_i)\, d^n x \ . \qquad (8)$$

D is a simply connected domain in W. We obtain

$$q^\alpha - \partial_k \pi^k_\alpha = q^\alpha - \mathrm{div}(\Pi_\alpha) = 0 \ . \qquad (9)$$

In continuum mechanics, $q^\alpha = \frac{\partial\mathcal{L}}{\partial\psi^\alpha}$ corresponds to the "external forces" on S and $\pi^k_\alpha = \frac{\partial\mathcal{L}}{\partial\psi^\alpha_k}$, the conjugate field, to the "internal stresses." Π_α is a contravariant 1-vector.

For the theory to be dual, the Duality Principle requires that the roles of its I.C. (7) and E-L equns (9) be interchangeable. The necessary conditions are: 1) $q^\alpha = 0$,the system is "force-free", 2) the I.C. and E-L equns must be equal in number

$$\nu\frac{n(n-1)}{2} = \nu \qquad (10)$$

with the only positive soltution

$$n = 2 \quad \text{for any } \nu \ . \qquad (11)$$

Hence Duality is a dimension selecting principle: Dual systems must be force free and their dynamical fields ψ^α_i are functions of only two internal variables $x^1 = \sigma$ and $x^2 = \tau$.

The necessary conditions cited above are also sufficient. Indeed, using (6), equations (7) and (9), with $q^\alpha = 0$, become

$$\mathrm{div}\,\Psi^*_\alpha = 0 \ , \qquad (12)$$

$$\mathrm{curl}\,\Pi^{*\alpha} = 0 \qquad (13)$$

respectively.
Consequently, (13) are now the I.C. for the existence of ν potentials $\pi^{*\alpha}$ such that $\pi^{*\alpha}_i = \partial_i \pi^{*\alpha}$. $\pi^{*\alpha}$, like ψ^α , is determined up to the group of potential transformations $\pi^{*\alpha\prime} = \pi^{*\alpha} + \mathrm{const.}$. (12) are the E-L equns which extremize the dual action

$$I^* = \int_D \mathcal{H} \, d^2\alpha \quad , \quad d^2\alpha = d\alpha d\tau \, . \qquad (14)$$

is the covariant Hamiltonian obtained through a Legendre transformation

$$\mathcal{H}(\pi^{*\alpha}_i) = \mathcal{L} - \pi_\beta \cdot \Psi^\beta = \mathcal{L} + \pi^{*\beta} \cdot \Psi^*_\beta \qquad (15)$$

since by solving for Ψ^α_i in $\pi^i_\alpha = \frac{\partial \mathcal{L}}{\partial \Psi^\alpha_i}$ as a function of we can express $\mathcal{H}$ as a function of $\pi^{*\alpha}_i$. In the special case where $\mathcal{L} = \pi_\rho \cdot \Psi^\rho = -\pi^{*\beta} \cdot \Psi^*_\beta$, $\mathcal{L}$ is then self-dual and $I = I^*$ in (8).

Making use of (6) and defining the complex fields $\Phi^{\alpha\pm} = \Psi^\alpha \pm i\,\pi^{*\alpha}$ which reflect the symmetry between the field Ψ and its dual canonical conjugate $\pi^{*\alpha}$, we can replace the systems (7,9) and (12,13) by the single equation

$$\mathrm{curl}\, \Phi^{\alpha +} = 0 \quad \text{or} \quad \mathrm{Curl}\, \Phi^{\alpha -} = 0 \qquad (16)$$

Alternatively, we have

$$\mathrm{div}\, \Phi^{*+}_\alpha = 0 \quad \text{or} \quad \mathrm{div}\, \Phi^{*-}_\alpha = 0 \, . \qquad (17)$$

Formally, we then have a dual theory.

2-dimensional dual field theories are the most general dual systems, ones with the maximum possible number of E-L equns and independent I.C.. We call them maximal dual systems (MDS). Those dual systems which do not meet this criterion we classify under the general heading of minimal dual systems (mds).

The structure (i.e. equations) of a field theory is determined by the set of principles which the theory obeys. It can and does happen that one or more of these principles dictates a specific choice of I.C., different in form and number from the maximal case (7). A case in point is Maxwell and Born-Infeld electrodynamics in n-dimensions. The principle involved is gauge invariance of the second kind.

Let us consider in n-space the electromagnetic system $\mathcal{L}(\partial_\mu A_\nu)$

where A_ν $(\nu = 1, 2, \ldots n)$ is the usual vector potential. In accordance with conditions for Duality, we have set $\partial\mathcal{L}/\partial A_\nu = 0$. By gauge invariance, it follows that $\partial_\mu A_\nu$ can enter the Lagrangian only through the combination

$$F_{\mu\nu} = (\mathrm{curl} A)_{\mu\nu} \qquad (18)$$

invariant under $A'_\mu = A_\mu + \partial_\mu \Lambda(x)$. $F_{\mu\nu}$ are the components of a covariant 2-vector F. (18) are the $\binom{n}{3}$ I.C. for the existence of the potential A_ν, and have the well known form

$$\mathrm{curl}\, F = 0 \, . \qquad (19)$$

Not all the equations (19) are independent. In the instance n=4, only three of the four are independent. Defining the contravariant 2-vector density P $\left(P^{\mu\nu} = 2 \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}} \right)$

$$\mathrm{div}\, P = 0 \qquad (20)$$

are then the n E-L equns. Duality selects for this system the dimension of W

$$n = 4 \, . \qquad (21)$$

This result is remarkable as the physical world is 4-dimensional and our best known gauge field theory, Maxwell's electrodynamics ($P = F$), is thereby dual in our world.

Born-Infeld electrodynamics is the class of systems with $\mathcal{L}(f, h)$ for n=4. ($f = \frac{1}{2} F_{\mu\nu} F^{\mu\nu}$ and $h = \frac{1}{2} F_{\mu\nu} (F^*)^{\mu\nu}$ are the field invariants.) It provides examples of minimal dual systems. (19) and (20) have the alternative representation

$$\mathrm{div}\, F^* = 0 \quad , \qquad (22)$$

$$\mathrm{curl}\, P^* = 0 \qquad (23)$$

and their roles are interchanged in the process. Just as in (12) and (13), (23) are now the I.C. for the existence of the conjugate potentials B^*_μ such that $P^* = \mathrm{curl}\, B^*$ and (22) are the E-L equns from the action

$$I^* = \int \mathcal{H}(P^*_{\mu\nu})\, d^4x \tag{24}$$

with $\mathcal{H} = \mathcal{L} - \frac{1}{2} P\cdot F = \mathcal{L} + \frac{1}{2} P^* F^*$.

By defining the complex field $\Omega^\pm = F \pm i P^*$ we can replace (19,20) or (22,23) by the single equation

$$\mathrm{curl}\, \Omega^+ = 0 \quad \text{or} \quad \mathrm{Curl}\, \Omega^- = 0 \,. \tag{25}$$

Alternatively, we have

$$\mathrm{div}\, \Omega^{*+} = 0 \quad \text{or} \quad \mathrm{div}\, \Omega^{*-} = 0 \tag{26}$$

By inspection of (16) and (25), their formal structural identity is manifest. Thus we establish that maximal dual theories are the 2-dimensional counterparts of Born-Infeld electrodynamics. The field Ψ^α is a 1-vector in 2-space while F is a 2-vector in 4-space. Since in n-space Ψ^*_α is a (n-1)-vector, F^* a (n-2)-vector, we see the simple origin of the 2 and 4-dimensionality of the world-manifolds in the two classes of dual theories. The Duality Principle demands complete symmetry between $\Psi^\alpha(F)$ and $\Psi^*_\alpha(F^*)$, these vectors and their duals must be of the same rank. Hence n=2 (4).

Without specializing the metric or the Lagrangian, the dynamics of dual systems are completely specified by

- a 2-component irrotational covariant k-vector $V^\pm$
- a 2 component solenoidal contravariant k-vector $V^{*\pm}$

where $k = 1, 2$, $V^\pm = \Phi^\pm, \Omega^\pm$.

Through analogies with electrodynamics or fluid dynamics, the reader can readily make other completely equivalent characterizations of dual systems. Regardless of the characterization, only one point is important;[6] the single defining property of dual systems is that their dynamics are metric independent topological laws of conservation of attributes, be they charges, currents, energy-momentum

etc.

The restrictive effects of the Duality Principle on a field theory are two-fold. On the one hand, the I.C. whose number depends on the tensorial nature of the geometrical objects (the fields), and the global dimension of the underlying world-manifold W, are geometrical continuity equations. The requirement that they can play the role of dynamical E-L equns of motion for the same system S sets a severe constraint on the geometrical structure of W by picking a definite dimension of W. On the other hand, as the E-L equns are required to be able to play the role of I.C., the dynamics are restricted to take the topological form of a conservation law.

What then is Duality? One interpretation emerges rather clearly from our formulation. The Duality Principle asserts the most complete symmetry between geometry and dynamics. By allowing for the interchangeability of intrinsically different roles between the I.C., which are the "trivial" (differential) geometric elements of a field theory[8] and the E-L equns, which are the dynamical elements. Duality puts geometry on the same, equal and interchangeable footing with dynamics and vice-versa. Thus Duality has all the earmarks of a geometrodynamical principle

DYNAMICS OF DUALITY = GEOMETRY

That gravitation and electrodynamics are geometric, has long been an accepted working framework for classical physics.[9] In light of our interpretation of the Duality Principle and the fact that the hadrodynamics of strings in Dual Resonance Models also obeys this principle (demonstrated below), there arises the exciting prospect that hadron physics in its prequantized form can be cast in geometrical terms. That this is indeed the case can be seen from the following examples of dual systems.

Since Duality does not specify the nature of the metric in the 2-space(σ,τ), the latter can be Euclidean or Minkowskian. Therefore we have two classes of maximal dual systems, one being a Wick rotated ($\tau \to i\tau$) image of the other. Some examples of 2-dimensional dual field theories are harmonic systems:

a) The Derichlet Problem with $\Psi^\alpha = (\psi^\alpha_\sigma, \psi^\alpha_\tau)$

$$\mathcal{L} = -\frac{1}{2}\Psi^\alpha \cdot \Psi^\alpha . \tag{27}$$

As $\Pi^\alpha = \Psi^\alpha$, Ψ^α is both irrotational and solenoidal, it is analytic. Hence ψ^α is harmonic. The I.C. and E-L equns are the Cauchy-Riemann equations of the theory of analytic functions. (27) is also the free Lagrangian of Nielsen electrostatic analog for the n-particle Veneziano model.[10] The Minkowskian version of (27) yields the 2-dimensional Klein-Gordon equations of the Nielsen-Susskind string theory.[10-11]

b) The Problem of Plateau[12] or the theory of a minimal 2-surface S embedded in a $(\nu+2)$ dimensional space.

$$\mathcal{L} = -\left[\left(1+\sum_{\alpha=1}^{\nu}\psi^{\alpha 2}_\sigma\right)\left(1+\sum_{\alpha=1}^{\nu}\psi^{\alpha 2}_\tau\right) - \left(\sum_{\alpha=1}^{\nu}\psi^\alpha_\sigma \psi^\alpha_\tau\right)^2\right]^{1/2} . \tag{28}$$

For $\nu = 1$ and $\psi(\sigma,\tau) = z$ we have the case of a minimal 2-surface S in 3-space (σ, τ, z). The I.C. and E-L equns are

$$\mathrm{curl}\ \Psi = 0 , \tag{29}$$

$$\mathrm{div}\ (\Psi \rho) = 0 \tag{30}$$

where $\rho^{-1} = (1 + \Psi\cdot\Psi)$ is a measure of the curvature of S. (27) is just the weak field $|\Psi|^2 << 1$ or flat space limit of (28). (30) expresses simply the defining property of a minimal surface, the vanishing of the mean curvature everywhere on S. Being elliptic, the system (29,30) can only describe static phenomena such as the shape of a soap film bounded by a wire or stationary motions of a liquid or gas. One such application made by Pryce[13] concerns 2-dimensional Born nonlinear electrostatics with

$$\mathcal{L} = -(1 - \Psi\cdot\Psi)^{1/2} . \tag{31}$$

Complete solutions to a systems of charges were found. Wheeler[14] has emphasized the importance of the analogy between (28) and the Einstein gravitational equations. The hyperbolic counterpart of (28) is

$$\mathcal{L} = -(1 + \psi^2_\sigma - \psi^2_\tau)^{1/2} \tag{32}$$

the Lagrangian for a 2-dimensional scalar analog of the Born-Infeld nonlinear electrodynamics. Similar systems have been considered by Blokintsev[15] and Heisenberg.[16] These essentially nonlinear theories appear to be particularly convenient for description of processes in which many particles are created in a single elementary act.

(32) can be generalized to the case of a 2-extremal surface in a $(\nu+2)$ dimensional pseudo-Euclidean space. Physically, it could correspond to the case of ν nonlinear Born-Infeld fields interacting in a specific way. Such a problem was discussed by Barbashov and Chernikov[17] who solved both the classical Cauchy problem and the quantized theory. For quantization to work, the time and space coordinates had to be operators along with the field functions. The result was therefore a theory in quantized space-time. These authors chose isometric coordinates for σ and τ. The nonlinear E-L equns of the type (30) then reduced to linear two dimensional Klein-Gordon equations with the nonlinear constraint $T_{ij} = 0\ (i,j=1,2)$, T_{ij} being the energy momentum tensor of the system. In analogy to electrodynamics, $T_{ij} = 0$ was applied as a weak constraint to eliminate timelike modes and the Fourier moments of components of T were found to obey a closed gauge algebra now known as the Virasoro algebra. We refer the reader to the original paper for greater details. It is noteworthy that this work was done in 1966, thus antidating by several years the nearly identical, independent techniques and results in DRM.[4]

c) The most general representation of a minimal 2-surface S is achieved by taking the extremal of the parameter invariant action resulting not from (28) but from

$$\mathcal{L} = \sqrt{g} \quad , \quad g = \det(g_{ij}) \tag{33}$$

where setting $\psi^\alpha = x^\alpha$ the position vector in space, the metric tensor intrinsic to S is $(\partial_{i,j} = \partial_\sigma, \partial_\tau)$

$$g_{ij} = \partial_i x^\alpha \partial_j x^\alpha , \quad \alpha = 1, 2, \ldots \nu , \tag{34}$$

and $g_{ij} g^{kj} = \delta_i^k$, g^{kj} being the reciprocal matrix. S is thus a Riemannian 2-manifold embedded in ν -space.

The I.C. and E-L equns are $(P^\alpha = (x^\alpha_\sigma, x^\alpha_\tau))$

$$\mathrm{curl}\, P^\alpha = \partial_i x_j - \partial_j x_i^\alpha = 0 \tag{35}$$

which is also the covariant curl as the affine connection[18] of the manifold, Γ^i_{jk} is symmetric, $\Gamma^i_{jk} = \Gamma^i_{kj}$.

$$\mathrm{div}\, P^\alpha = \frac{1}{\sqrt{g}} \partial_i (\sqrt{g}\, g^{ij} \partial_j x^\alpha) \tag{36}$$

$$= \frac{1}{\sqrt{g}} \partial_i (\sqrt{g}\, x^{\alpha i}) = 0$$

respectively. (36) is recognized as the covariant divergence of the contravariant vector P^α in the <u>curved</u> Riemannian space S[18]. It is Beltrami's continuity equation of flow for a solenoidal and irrotational (35) fluid over S. x^α is harmonic. The system (35,36) is just the curved space counterpart of (a). The Duality property can be checked using the properly defined dual quantities in the presence of the metric g_{ij} :

$$x^{*\alpha i} = \frac{1}{\sqrt{g}} \epsilon^{ij} x^\alpha_j \quad , \quad x^{*\alpha}_i = -\sqrt{g}\, \epsilon_{ij} x^{\alpha j} . \tag{37}$$

A 2-dimensional Riemann surface is locally conformally flat (Euclidean). This means that a neighborhood of each of its points can be mapped onto a domain of the plane. This simple feature is not generally true for higher dimensional manifolds. In consequence of this conformal flatness,

the nonlinear equation (36) or

$$\partial_\sigma \left(\frac{g_{\tau\tau} \partial_\sigma x^\alpha - g_{\sigma\tau} \partial_\tau x^\alpha}{\sqrt{g}} \right) + \partial_\tau \left(\frac{g_{\sigma\sigma} \partial_\tau x^\alpha - g_{\sigma\tau} \partial_\sigma x^\alpha}{\sqrt{g}} \right) = 0 \tag{38}$$

can be made explicitly harmonic if isometric coordinates are chosen for which the nonlinear constraints are satisfied

$$g_{\sigma\tau} = 0 \quad , \quad g_{\tau\tau} = g_{\sigma\sigma} \tag{39}$$

i.e. parameters corresponding to a conformal mapping of the surface S onto the plane τ, σ. (39) is equivalent to the energy momentum tensor of the system $T_{ij} = 0$.[4c)]

Alternatively, (39) are recognized as the 2-dimensional counterparts of the gauge or harmonic coordinate conditions often used in general relativity[19)] $\left(\Gamma^j = \frac{1}{\sqrt{g}} \partial_i (\sqrt{g}\, g^{ij})\right)$

$$\Gamma^j = g^{ik} \Gamma^j_{ik} = 0 \quad , \quad j = 1,2 . \tag{40}$$

As (36) is also

$$\Lambda^2 x^\alpha - \Gamma^j \partial_j x^\alpha = 0 \tag{41}$$

where $\Lambda^2 = g^{ij} \partial_i \partial_j$ is the invariant 2-D'Alembertian.
With $g^{\sigma\sigma} = \frac{g_{\tau\tau}}{g}$, $g^{\tau\tau} = \frac{g_{\sigma\sigma}}{g}$ and $g^{\sigma\tau} = \frac{-g_{\sigma\tau}}{g}$,
It can be readily verified that (40) is equivalent to (39) and $g^{-1/2} \Lambda^2$ reduces to $\partial^2_\sigma + \partial^2_\tau$. Since the dynamical invariance of the system (33) is general coordinate invariance in the σ, τ space, we are doing general relativity in 2-space.

This maximal dual system is remarkable in that for $\alpha = 4$ (or 26), it is the Euclidean free field Lagrangian for Nambu's hadrodynamics of strings.[4a)] In the quantized theory, (39) becomes the Virasoro gauge conditions. We shall not elaborate further on the classical and quantum Minkowskian theory of the massless purely transverse relativistic string as it has been fully analyzed by GGRT.[4d)]

Examples of minimal dual systems are provided by Born-Infeld electrodynamics[3)] in a review article of Born.[2)]

For our purpose, only the specific Born Lagrangian $\mathcal{L}(f) = -\frac{1}{a^2}\left(\sqrt{1+a^2 f} - 1\right)$ is of interest. In the weak coupling limit $a^2 \ll 1$, it reduces to the standard $\mathcal{L}(f) = -\frac{1}{2} f$ of Maxwell theory. In the strong coupling limit $a^2 \gg 1$, we have the essentially nonlinear form of

$$\mathcal{L}(f) = -\frac{1}{a}\sqrt{f} \quad . \tag{42}$$

(40) is identical to the Lagrangian of Nielsen-Olesen local field theory of the dual strings[5,20)] which is thereby dual in 4 dimensions. (40) is remarkable in that its field equations admit the Nambu string (36) as a special solution if the string field $F_{\mu\nu}$ is chosen to be

$$F_{\mu\nu}(x) = \int d^2\sigma \, \frac{\partial(z_\mu, z_\nu)}{\partial(\sigma, \tau)} \, \delta^4(z(\sigma,\tau) - x) \quad . \tag{43}$$

This form of a world sheet singularity first appeared in Dirac monopole theory[21)] based on the linear Lagrangian $\mathcal{L} = -\frac{1}{2} f$. The nonlinearity of (42) is a crucial element. In Dirac's case, no equations of motion for the field z in the σ, τ space are obtainable so the string is unphysical. However, in the Nielsen-Olesen theory, the string carries energy and momentum; its E-L eqns (36) simply state the local conservation of the 4-energy momentum flow on the curved world sheet S. Born-Infeld electrodynamics is not unique in having this feature. As a class of unitary field theories, it shares with Einstein's general relativity a nonlinearity through which the equations of motion of the singularities in the fields need not be postulated independently but are already contained in the field equations themselves. The local field theory of dual strings will be more fully discussed in a forthcoming work.[22)]

In conclusion, we have shown the dimension selecting Duality Principle to be universal as it is obeyed by electrodynamics and hadrodynamics of strings. Furthermore, Duality is interpreted as a geometrodynamical principle which asserts the equivalence of dual dynamics and geometry. We hope that this equivalence and the structural identity between free electrodynamics and hadrodynamics will show the way to the inclusion of spinor fields in hadrodynamics and to interacting field theories of strings.

Another paper discusses extensively those topics treated here as well as many related topics.[23)]

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