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Further Unnoticed Symmetries in Special Relativity

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Bias is beautiful.

In an earlier paper [1] the author drew attention to some unnoticed symmetries in the special theory of relativity revealed through a quantity called bias[1] B,

$$B = \sqrt{(c - v)/(c + v)}, \tag{1}$$

where c is the velocity of light and v, the velocity of a "particle." Here we point out another symmetry which we shall call *towards and away* (TA) *symmetry* which follows naturally from the theory.

Considering only one-dimensional space, let O be an observer and $\mathscr{A}$ and $\mathscr{B}$ two events separated in space and time, respectively, by x and t in the frame of reference in which O is at rest. If we assume that the positive direction of the x axis is towards the right and the two events are to the right of O, then $(t + (x/c))$ can be interpreted as the time interval, as observed by O between the signals sent by the events $\mathscr{A}$ and $\mathscr{B}$. If O' is an observer moving with a velocity v with respect to O, the time interval between the signals reaching O' is $(t + (x/c))(c/(c + v))$ in the frame in which O is at rest since $(c + v)$ is the *exterior relative velocity* (see Appendix) between O' and the light signal as observed by O. The time interval between the signals in the frame in which O' is at rest is $(t' + (x'/c))$ *assuming that the velocity of light in the rest frame of O' is the same as in the rest frame of O.*

For reasons which will be immediately apparent we shall write[2]

$$t' + \frac{x'}{c} = k\left\{\left(t + \frac{x}{c}\right)\Big/\left(1 + \frac{v}{c}\right)\right\}. \tag{2}$$

[1] Herman Bondi has called this the k factor and developed a k calculus around it.

[2] The right-hand side of (2) can also be derived as follows: A signal starting from $\mathscr{B}$ at time t later than the signal at $\mathscr{A}$ is equivalent to a signal starting from a point at a distance $x + ct$ from $\mathscr{A}$ at the same time as the signal at $\mathscr{A}$. The time interval between these two signals reaching the moving observer O' is $(x + ct)/(c + v)$ as observed by O.

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