21-80
127

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Conformal Invariance of Zero-Mass Klein-Gordon–Type Equation.

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BRACKEN (1) in a recent paper has shown that $D^{m,n}_{M=0}$ representations of the Lorentz group that satisfy Klein-Gordon–type equations are not covariant under transformations of the conformal group if $mn \neq 0$. We wish to point out that the proof is alright as far as it goes but it would not hold if one takes into account additional symmetries and constraints usually associated with these fields.

Using Bracken's notation (we take $P_\mu = -i\partial_\mu$, and our l is negative of that used by BRACKEN), the commutator identities one requires are ($k = l + 1 - \frac{1}{2}n$, where n = space-time dimension)

$$a_\mu \equiv [k_\mu, P^2] = 4i(x_\mu P^2 - 2ikP_\mu + s_{\mu\nu}P^\nu)\,, \tag{1}$$

$$g^{\lambda\mu}[k_\lambda, a_\mu] = 16\{9(s_{\nu\lambda}x^\lambda P^\nu + ikD) - 3x^2P^2 - 2s_{\mu\alpha}s^{\mu\alpha} - kl\}\,. \tag{2}$$

The principal assumptions involved in the proof of noninvariance are ($\varphi \equiv \varphi^{m,n}_{m=0}$)

$$P^2\varphi = 0\,, \qquad k_\mu P^2 \varphi = P^2 k_\mu \varphi\,. \tag{3}$$

The second of those equations implies that the field φ and its transform $\mathscr{C}\varphi$, under the special conformal transformation $\mathscr{C}$ both satisfy the original field equation (namely, the first of eqs. (3)). On applying (3) to (1), (2), we obtain Bracken's result: $m(m+1) + n(n+1) = l(l-1)$. If we now make use of the fact that action of $S_{\mu\nu}$ on the vector potential ($D^{\frac{1}{2},\frac{1}{2}}_{M=0}$ representation) is given by $(S_{\mu\nu}A)_\varrho = i(g_{\mu\varrho}A_\nu - g_{\nu\varrho}A_\mu)$ we find from (1)-(3) that these imply the Lorentz condition

$$\partial_\mu A^\mu = 0\,, \qquad l \neq 4\,. \tag{4}$$

On the other hand, this condition is known not to be covariant under special conformal transformations. The proof thus breaks down in this case. The point, of course, is that according to Bracken's proof when $mn \neq 0$ the fields φ and $\mathscr{C}\varphi$ do not satisfy the same wave equation; that is to say that the transformed field does not satisfy eqs. (3).

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(1) A. J. BRACKEN: *Lett. Nuovo Cimeno*, **2**, 574 (1971).

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