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Distribution of Recoil Nucleus in Pair Production by Photons

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The angular and momentum distribution of the recoil nucleus in pair production by a photon is calculated covariantly by a method which utilizes the unitarity of the S matrix. The results are in disagreement with a recent experiment, particularly for small angles and high momentum transfers. The exact total cross section for pair creation is also given.

I. INTRODUCTION

RECENT attempts to measure the momentum and angular distributions of the recoil nucleus in pair production by energetic photons[1] have made a theoretical investigation of these effects desirable. There exists in the literature only an estimate of the momentum distribution.[2] An opportunity is also afforded of illustrating a method of calculation of somewhat more general applicability, which is based on the unitary character of the S matrix. A simplified and covariant calculation of the total pair production cross section and its asymptotic form (Bethe-Heitler formula[3]), is also carried through.

II. METHOD OF CALCULATION

In this discussion, the effect of the nucleus is represented by its static field

$$A_\mu(\mathbf{x}) = \int \exp(-i\mathbf{q}\cdot\mathbf{x})A_\mu(\mathbf{q})d^3\mathbf{q}, \tag{1}$$

which is viewed as an external field. Higher radiative corrections are disregarded (see Appendix II), as is excitation of atomic states by the incident photon, and only the lowest term in the external field is computed (first Born approximation). The validity of some of these assumptions has been questioned,[4] but these effects are in any case not large.

The total cross section for pair production by a photon of energy k_0 (units in which $\hbar = c = 1$ are used throughout) is then obtained in the form:

$$\sigma(k_0) = \int d^3\mathbf{q}A_\mu(\mathbf{q})A_\nu(-\mathbf{q})T_{\mu\nu}(\mathbf{q}; k), \tag{2}$$

where $\mathbf{q}$ is the momentum transferred to the field and k is the energy momentum 4-vector of the photon, so that the integrand of (2) gives directly the differential cross section for obtaining a recoil nucleus of momentum $\mathbf{q}$.

To obtain $\sigma(k_0)$ we consider the scattering matrix, S, developed in a power series in the charge e:

$$S = 1 + eS_1 + e^2S_2 + \cdots. \tag{3}$$

If S^* is the Hermitian conjugate of S, the condition for the unitarity of S becomes

$$\begin{aligned} S^*S = 1 &+ e(S_1^* + S_1) + e^2(S_1^*S_1 + S_2^* + S_2) \\ &+ e^3(S_1^*S_2 + S_2^*S_1 + S_3^* + S_3) \\ &+ e^4(S_1^*S_3 + S_3^*S_1 + S_2^*S_2 + S_4^* + S_4) + \cdots = 1, \end{aligned} \tag{4}$$

so that each of the expressions in parentheses must vanish. Here, apart from mass terms,

$$\begin{aligned} e^nS_n = \frac{(-i)^n}{n!} \int_{1,\cdots,n} & P[j_{\mu 1}(1), \cdots, j_{\mu n}(n)] \\ & \times P[A_{\mu 1}(1) + \phi_{\mu 1}(1), \cdots, A_{\mu n}(n) + \phi_{\mu n}(n)], \end{aligned} \tag{5}$$

where $j_{\mu 1}(1) = ie\bar{\psi}(x_1)\gamma_{\mu 1}\psi(x_1)$, $\phi_{\mu 1}(1)$ is the electromagnetic field operator at x_1, etc., and the integral is taken over all the indicated 4-spaces.[5]

Consider next the expectation value for a state with a single photon of momentum k_0 of the e^4 term of (4), limiting ourselves to terms quadratic in the external field. If the external field cannot create pairs, which is true of a static field, the S_1 terms give zero. Furthermore, if the initial state in S_2 is a one-photon state, the final state can only have an electron-positron pair. The $e^4S_2^*S_2$ term is then the sum of the squares of all matrix elements between the initial state and all possible final states containing a pair; that is, the total probability for the creation of any pair by the photon. The cross section $\sigma(k)$ is thus given by $\sigma(k_0) = (k_0|e^4S_2^*S_2|k_0)$, which by virtue of (4) becomes

$$\sigma(k_0) = e^4(k_0|-S_4^* - S_4|k_0) = -2Re(k_0|e^4S_4|k_0). \tag{6}$$

This form is much more convenient for computational purposes than is the conventional non-covariant summation over final states.[6] It is to be noted that the absence of internal photon lines in the matrix elements of interest here makes unnecessary any explicit mass subtraction.

Employing the usual prescription of Feynman and Dyson[5] to evaluate the S_4 matrix element and averaging over the polarizations of the photon, one obtains

$$\begin{aligned} T_{\mu\nu}(q, k) = +(e^4/8\pi k_0)Re[&A_{\mu\nu}(q, k) + B_{\mu\nu}(q, k) \\ &+ B_{\mu\nu}(-q, k) + A_{\mu\nu}(q, k) \\ &+ B_{\mu\nu}(q-k) + B_{\mu\nu}(-q, -k)]. \end{aligned} \tag{7}$$

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[6] Some other applications of Eqs. (4)–(6) are discussed briefly in Appendix I.