ON THE "HIDDEN VARIABLES" THEORY OF A. BACH

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Received 16 December 1980

We study the hidden variables theory proposed recently by A. Bach. We show that the author's mathematical results do not establish a hidden variables representation of Quantum Mechanics but, instead, express the possibility of writing the density operators as continuous mixtures of pure quantal states.

Half a century after von Neumann's "impossibility proof", the status of hidden variables theories still remains unclear. In particular, the conditions that a theory should satisfy in order to be accepted as establishing a hidden variables representation of Quantum Mechanics have not been elucidated. The only undoubtedly necessary – but not sufficient – condition is that such a representation should provide a richer description of microsystems than that of Quantum Mechanics.

Another condition which is sometimes considered as necessary and sufficient is the following: we say to have established a hidden variables theory iff we find a certain set Σ such that:

(a) every state W defines a probability measure on Σ,

(b) every observable A defines a random variable (i.e. a measurable function) f_A on Σ, and

(c) the quantum-mechanical expectation value of A in the state W is the mean value of the random variable f_A for the probability measure μ_W, i.e.:

$$E_W(A) = \int_{x\in\Sigma} d\mu_W(x) f_A(x) . \qquad (1)$$

Now one could interpret the set Σ as the set of all hidden variables; the measure μ_W as the probability distribution of these hidden variables if the quantum-mechanical state is W; and $f_A(x)$, $x \in \Sigma$ as the value of the quantity A for an individual system in the hidden state x.

However, we wish to make clear that, contrary to a widely held opinion, the above mentioned condition is neither necessary nor sufficient. It is not necessary since the observed value of the quantity A may depend not only on the hidden variable of the system measured, but also on the apparatus. That it is not sufficient can be seen from two examples in which this condition is trivially satisfied and yet we always stay inside quantum theory.

(1) Take Σ to be the set of all statistical operators, μ_W to be the point measure concentrated on W, i.e. $\mu_W(\{W\}) = 1$, $\mu_W(\Sigma\backslash\{W\}) = 0$ and $f_A(W') = \mathrm{Tr}\,(W'A)$ $\forall\ W' \in \Sigma$. Then (a), (b) and (c) hold, and (1) expresses a trivial tautology: $E_W(A) = \mathrm{Tr}(WA)$.

(2) Take Σ to be the Hilbert space H representing the system and let $W = \Sigma_i \lambda_i P_{[\varphi_i]}$ be the spectral decomposition of the density operator W ($[\varphi_i]$ is the ray containing the normalized vector φ_i and $P_{[\varphi_i]}$ the projector on $[\varphi_i]$). Define μ_W as a measure concentrated on the vectors φ_i such that $\mu_W(\{\varphi_i\}) = \lambda_i$. Set furthermore $f_A(\varphi) = (\varphi, A\varphi)$. Then (a), (b) and (c) do hold, and (1) simply means

$$E_W(A) = \sum_i \lambda_i (\varphi_i, A\varphi_i) ,$$

i.e. it is a statement of quantum theory (it is a definition of the mixture of pure states) and has nothing to do with hidden variables.

Very recently, Bach [1,2] proposed a hidden variables theory based on a mathematical formula (formula (35) of ref. [2]). The mathematical results of Bach's work are correct; however, it is our opinion, that

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