Critique of conventional relativistic quantum mechanics

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Relativistic quantum-mechanical theories presently taught in upper-division undergraduate and graduate programs are critically examined. Following an historical sketch of the development of relativistic quantum mechanics, a discussion of the still unresolved difficulties of the presently accepted theories is presented. This review is designed to complement and update the discussion of relativistic quantum mechanics presented in many widely used texts.

I. INTRODUCTION

The goal of this paper is the presentation of a critical review of conventional relativistic quantum mechanics. The expression "conventional relativistic quantum mechanics" refers to those theories that are presently taught in upper-division undergraduate and graduate programs.[1] These theories will be discussed in detail below.

A critique of conventional theories is necessary for a number of reasons. From an academic point of view the most important reason arises from the presence of discussions of conventional theories in widely used texts.[1] A critique of conventional theories that is suitable for consumption by embryonic physicists will complement these texts by broadening the educational experience of students encountering relativistic quantum mechanics for the first time.

Another reason for providing a critique of conventional theories arises from the appearance of a new relativistic quantum-mechanical theory known as the four-space formulation (FSF).[2-4] Acceptance or rejection of the FSF, which is compatible with "relativistic dynamics,"[5-8] depends on how well the FSF can resolve the difficulties beseiging conventional theories. Evaluation of the FSF presupposes an awareness of these difficulties on the part of the physics community. The presentation given below can enhance this awareness.

First the question of what has been done is addressed. The historical sketch of relativistic quantum mechanics up through the FSF sets the stage for a discussion of the still unresolved difficulties associated with the conventional theories. These difficulties are discussed in detail.

The ensuing presentation will not consider field theories. This simplifies the discussion and makes the critique more suitable for nonspecialists. Field theoretic considerations are also not as important since the mathematical apparatus of the widely used Lagrangian quantum field theory (LQFT)[9] is a special case of the generalized quantum field theory (GQFT) associated with the FSF.[10-12] A pedagogical review of the FSF, including discussion of conceptual differences between LQFT and GQFT, will be presented elsewhere.

II. HISTORICAL SKETCH

Three of the most important relations of nonrelativistic quantum mechanics (NRQM) were established in the 1920s. They are Schrödinger's equation[13]

$$i\hbar \frac{\partial \psi_s}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi_s + V\psi_s, \tag{1}$$

Born's[14] probability density

$$\rho_s = \psi_s^* \psi_s \geq 0, \tag{2}$$

and the associated normalization condition

$$\int \rho_s \, d^3x = 1. \tag{3}$$

Shortly after Schrödinger's first paper on NRQM the following equation appeared[15-17]:

$$\left(\frac{\hbar}{i}\frac{\partial}{\partial x_\mu} - \frac{e}{c}A^\mu\right)\left(\frac{\hbar}{i}\frac{\partial}{\partial x^\mu} - \frac{e}{c}A_\mu\right)\psi = m_0^2 c^2 \psi, \tag{4}$$

where m_0 is the rest mass of the particle and the nonzero elements of the metric are

$$g_{00} = 1 = -g_{11} = -g_{22} = -g_{33}. \tag{5}$$

Originally, Eq. (4), now known as the Klein–Gordon (KG) equation, was thought to be the wave equation for relativistic particles. This belief was short lived.

Combining Eq. (4) and its complex conjugate yields the continuity equation

$$\partial j^\mu / \partial x^\mu = 0, \tag{6}$$

where

$$j^\mu \equiv \frac{i\hbar}{2m_0}\left(\psi^* \frac{\partial \psi}{\partial x_\mu} - \psi \frac{\partial \psi^*}{\partial x_\mu}\right) - \frac{eA^\mu}{m_0 c}\psi^*\psi. \tag{7}$$

A continuity equation having the same form as Eq. (6) can also be derived from Schrödinger's equation. In an effort to mimic the successful NRQM, it was assumed that j^0/c should be interpreted as the relativistic probability density. Immediately problems of interpretation arose because j^0 could have negative values. These problems cast a shadow of doubt on the adequacy of the KG equation as the wave equation for relativistic particles. An alternative was sought.

Dirac's[18] relativistic theory was published a couple of years after the KG equation. Negative probability densities occurred in this theory also, but the ability of the Dirac equation to accurately describe the electron, including its magnetic moment, quickly gained Dirac's theory acceptance as the proper description of relativistic particles. The KG equation was resurrected only after Pauli and Weisskoph[19] reinterpreted Eq. (4) as the field equation for spinless particles in the second quantization formalism. They did not, however, resolve the negative probability density problem.

Wave equations for relativistic particles (RP) received attention in later years, but usually as background or pre-

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