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A generalized quantum field theory

John R. Fanchi*
1629 South 108 East Avenue, Tulsa, Oklahoma 74128
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An expression in quantum-field-theoretic language of the four-space formulation (FSF), especially the FSF group properties, is derived by generalizing Schwinger's formulation of Lagrangian quantum field theory (LQFT). The resulting theoretical framework includes a mass operator in addition to the energy-momentum and angular momentum operators. It also contains LQFT as a special case. Broad conclusions regarding conservation laws (of rest mass, energy-momentum, and angular momentum) are obtained from the general formalism. Many mathematical details concerning the FSF group and FSF transformations are presented.

INTRODUCTION

The most widely used formulation of Lagrangian quantum field theory (LQFT) is that by Schwinger.[1] His formulation began with the definition of an action operator W_S in terms of a function $\mathcal{L}_S$ of field operators $\phi_\alpha(x)$ and their first derivatives such that

$$W_S = \int_{R_S} \mathcal{L}_S(\phi_\alpha(x), \partial_\mu \phi_\alpha(x))dx\,. \tag{1}$$

The quantity R_S denotes an infinite four-volume in space-time that is bounded by the spacelike surfaces σ_1, σ_2 as in Fig. 1, and the invariant measure dx is defined as

$$dx \equiv dx^1 dx^2 dx^3 dx^4\,. \tag{2}$$

The notation used here is that of Roman,[2] namely

$$\partial_\mu = \partial/\partial x^\mu \tag{3a}$$

and

$$\partial^\mu = g^{\mu\lambda}\partial_\lambda = \partial/\partial x_\mu\,, \tag{3b}$$

where the nonzero elements of the fundamental metric tensor $g_{\mu\nu}$ are

$$g_{00} = 1 = -g_{11} = -g_{22} = -g_{33}\,. \tag{4}$$

By performing the variation δW_S and then postulating that δW_S is equal to the difference between the generators of canonical transformations F_S at σ_1 and σ_2, i.e.,

$$\delta W_S = F_S[\sigma_2] - F_S[\sigma_1]\,, \tag{5}$$

Schwinger developed LQFT. A correspondence between LQFT and classical field theory (CFT) can be drawn as follows.

Classically[3] the invariant parameter used to trace the evolution of a system point in configuration space is the proper time τ_{cl}. Furthermore, the Lagrangian L_{cl} used in a covariant formulation of Hamilton's principle must satisfy certain specified transformation properties, e.g., Lorentz invariance. The subsequent classical action integral has the form

$$I_{cl} = \int_{\tau_{cl1}}^{\tau_{cl2}} L_{cl}\, d\tau\,, \tag{6a}$$

and Hamilton's principle requires that

$$\delta I_{cl} = 0 \tag{6b}$$

where both I_{cl} and L_{cl} are invariant scalars, and the proper times τ_{cl1}, τ_{cl2} are kept fixed. In a relativistic CFT the quantity L_{cl} becomes an integral of a classical Lagrangian density $\mathcal{L}_{cl}$ such that

$$L_{cl} = \int_{R_{cl}} \mathcal{L}_{cl}\, dx\,, \tag{7}$$

where R_{cl} is the appropriate four-volume.

The point of interest here is the similarity of Eqs. (1) and (7). Schwinger's procedure is classically analogous to varying L_{cl} rather than I_{cl}. This raises the question: What would QFT look like if the action operator was redefined to more closely parallel CFT? The answer to this question will be obtained by defining an action operator A analogous to I_{cl} and then evaluating δA. The resulting formalism is aesthetically appealing for two reasons: It closely parallels CFT; and it retains all of LQFT as a special case. Beyond aesthetics, however, is a physically significant motivation for performing the ensuing derivation.

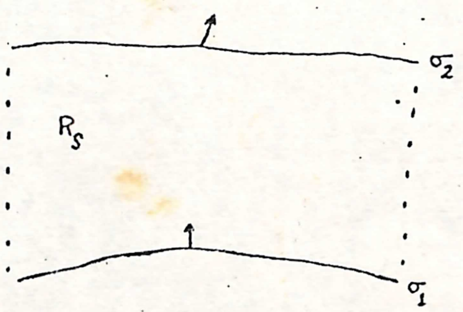

FIG. 1. Schwinger's infinite four-volume R_S. The arrows denote unit four-vectors normal to the spacelike surfaces σ_1, σ_2.