

Quantum Mechanics of Relativistic Spinless Particles

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Received September 15, 1977

A relativistic one-particle, quantum theory for spin-zero particles is constructed upon $L^2(x, ct)$, resulting in a positive definite spacetime probability density. A generalized Schrödinger equation having a Hermitian Hamiltonian H on $L^2(x, ct)$ for an arbitrary four-vector potential is derived. In this formalism the rest mass is an observable and a scalar particle is described by a wave packet that is a superposition of mass states. The requirements of macroscopic causality are shown to be satisfied by the most probable trajectory of a free tardyon and a nontrivial framework for charged and neutral particles is provided. The Klein paradox is resolved and a link to the free particle field operators of quantum field theory is established. A charged particle interacting with a static magnetic field is discussed as an example of the formalism.

1. INTRODUCTION

At present a consistent relativistic one-particle quantum theory for spin-zero particles has been developed only for free particles² and it is generally believed that interactions, such as the electromagnetic interaction, are properly interpreted only within the context of many-body theories.³ It is shown in this paper, however, that a consistent one-particle theory of relativistic spinless particles (RSP) in the presence of an arbitrary four-vector potential can be constructed using a formalism developed from ideas first introduced by Fock⁽³⁾ and Stückelberg⁽⁴⁾ over thirty years ago. More recently these ideas have been discussed by Nambu,⁽⁵⁾ Schwinger,⁽⁶⁾ Feynman,⁽⁷⁾ and Cooke.⁽⁸⁾ A logical framework for the ideas presented in Refs. 3-8 is developed in this paper. Furthermore, a number of physical

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² A statement to this effect is made explicitly in Ref. 1.

³ This contention is prevalent in, for example, the widely used text by Bjorken and Drell.⁽²⁾