

**Relativistic Quantum Mechanics:
a Space-Time Formalism for Spin-Zero Particles.**

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Summary. — A single-particle quantum formalism is constructed for spin-zero particles based upon state functions and operators defined on a four-dimensional space-time manifold. This yields a generalized Schrödinger equation having a Hermitian Hamiltonian and a derivative with respect to proper time. This formulation requires that particle mass be treated as an observable, not as a specified constant. A consistent probability interpretation results and a proper classical limit is exhibited. It is proposed that this formalism should properly replace the conventional Klein-Gordon formalism.

1. — The four-space formalism.

The fact that a satisfactory probability interpretation can be attached to the Klein-Gordon (KG) equation *with fields* (*),

$$\left(i\hbar \frac{\partial}{\partial x_\mu} + \frac{e}{c} A_\mu \right) \left(i\hbar \frac{\partial}{\partial x^\mu} + \frac{e}{c} A_\mu \right) \psi = m_0^2 c^2 \psi,$$

only by invoking the Foldy-Wouthuysen approximation technique in the two-component formalism introduced by FESHBACH and VILLARS (1) and elaborated

(*) Here the Einstein notation is used with the metric of signature $g_{00} = 1, g_{\mu\mu} = -1, \mu = 1, 2, 3$, and x_0 being ct .

(1) H. FESHBACH and F. VILLARS: *Rev. Mod. Phys.*, **30**, 24 (1958).

by BJORKEN and SCHWENGER (2) as a consistent relativistic formalism for particles. The root of the problem is that the probability distribution should treat a distribution of particles of zero mass as a single particle, thus a temporal distribution of the particle.

This hypothesis is based on a particle in space-time. The *expectation values* of the space-time co-ordinates, that is, the observed space-time coordinates.

(1)

with ρ a distribution function.

(2)

and

(3)

with integration over the space-time manifold.

One may object that this formalism is restricted to a single particle, only how a particle is distributed in space-time points which may be seen to emerge from a distribution.

Now, if an observable is defined by $\langle x^\mu \rangle, \mu = 0, 1, 2, 3$, of such points on a space-time submanifold of the space-time manifold parameter τ . This formalism is based on a distribution of particles.

(2) J. D. BJORKEN and S. D. SCHWENGER: *Phys. Rev.*, **147**, 959 (1964).

(3) S. S. SCHWEBER: *Phys. Rev.*, **147**, 959 (1962).

(4) PARTICLE DATA GROUP: *Phys. Rev. D*, **9**, 1 (1974).