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The solution of Dirac's equation in Kerr geometry

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Dirac's equation for the electron in Kerr geometry is separated; and the general solution is expressed as a superposition of solutions derived from a purely radial and a purely angular equation.

1. INTRODUCTION

Teukolsky's (1972) separation of the variables of the equations governing the electromagnetic, the gravitational, and the two component neutrino-field perturbations of a Kerr black hole has been central to much of the later developments. But the lack of a similar separation of the variables of Dirac's equation for the electron has been an obstacle to progress along many desired directions (particularly, for the treatment of massive fields in the context of Hawking's (1975) quantal process of the evaporation of black holes). In this short paper, we shall show that Dirac's equation can also be separated and the solution expressed in terms of certain radial and angular functions satisfying decoupled equations; in consequence problems associated with an electron in the vicinity of Kerr black holes become amenable to treatment.

2. DIRAC'S EQUATION IN THE NEWMAN–PENROSE FORMALISM

In Penrose's (1972) spinor notation, the four components of the wave function which satisfy Dirac's equation are represented by two spinors P^A and Q^A (say); and Dirac's equation (in the units in which $c = \hbar = 1$) is written in the form

$$\nabla_{AA'} P^A + i\mu_e \overline{Q}_{A'} = 0 \tag{1}$$

and

$$\nabla_{AA'} Q^A + i\mu_e \overline{P}_{A'} = 0, \tag{2}$$

where $2^{\frac{1}{2}}\mu_e$ is the mass of the electron (in the chosen units) and $\nabla_{AA'}$ is the symbol for covariant differentiation.

Following Newman & Penrose (1962), we introduce a basis ζ_a^A for the spinor space and a corresponding basis $\overline{\zeta}_{a'}^{A'}$ for the conjugate space. To the spinor basis is associated, at each point of the space-time, a null tetrad, $(\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \overline{\boldsymbol{m}})$ satisfying the orthogonality relations, $\boldsymbol{l}\cdot\boldsymbol{n} = 1$, $\boldsymbol{m}\cdot\overline{\boldsymbol{m}} = -1$, and $\boldsymbol{l}\cdot\boldsymbol{m} = \boldsymbol{l}\cdot\overline{\boldsymbol{m}} = \boldsymbol{n}\cdot\boldsymbol{m} = \boldsymbol{n}\cdot\overline{\boldsymbol{m}} = 0$.

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