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On a transformation of Teukolsky's equation and the electromagnetic perturbations of the Kerr black hole

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Teukolsky's equation, governing the perturbations (scalar, electromagnetic, and gravitational) of the Kerr black hole, is transformed, by a simple change of variables, in a manner such that there is formally no difference in the treatments of the axisymmetric and the non-axisymmetric modes: the rôle of 'm' is effectively eliminated. By considering in detail the case of electromagnetic perturbations we show how, in all cases, the problems can be reduced to problems in the theory of penetration of one dimensional potential barriers with, however, certain novel features. The phenomenon of super-radiance, peculiar to the Kerr metric, emerges, for example, in an unexpected guise. The case of scalar perturbations is considered briefly in an appendix.

1. Introduction

In a recent paper (Chandrasekhar & Detweiler 1975; this paper will be referred to hereafter as (I)) Teukolsky's equation governing the axisymmetric (gravitational) perturbations of the Kerr black hole was transformed to a one dimensional equation as a natural generalization of the Zerilli and the Regge–Wheeler equations governing the perturbations of the Schwarzschild black hole. While this transformation was perhaps satisfying from a formal point of view, it must be conceded that only the non-axisymmetric perturbations – the m-modes with an $e^{im\varphi}$-dependence on the azimuthal angle φ (where m is a positive or a negative integer) – include the physically interesting aspects, such as the phenomenon of super-radiance, which are peculiar to the Kerr black hole. In this paper, we shall accordingly turn our attention to Teukolsky's equation governing the different m-modes and show how, by a single change of the independent variable, we can reduce it to a form in which m no longer appears explicitly and all the physically interesting features of the Kerr black hole emerge as novel aspects of the one dimensional barrier-penetration problem familiar in elementary quantum theory. We shall illustrate the particular appropriateness of the transformation by considering in detail the electromagnetic perturbations of the Kerr black hole. We shall return to the consideration of the gravitational perturbations in a later communication.

In an accompanying paper Detweiler (1976) has considered a transformation of Teukolsky's equation for the case $s = 1$ from a different point of view.