On the equations governing the axisymmetric perturbations of the Kerr black hole

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On the equations governing the axisymmetric perturbations of the Kerr black hole

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It is shown how Teukolsky's equation, governing the perturbations of the Kerr black hole, can be reduced, in the axisymmetric case, to a one-dimensional wave equation with four possible potentials. The potentials are implicitly, dependent on the frequency; and besides, depending on circumstances, they can be complex. In all cases (i.e. whether or not the potentials are real or complex), the problem of the reflexion and the transmission of gravitational waves by the potential barriers can be formulated, consistently, with the known conservation laws. It is, further, shown that all four potentials lead to the same reflexion and transmission coefficients.

1. INTRODUCTION

It is now generally believed that black holes occurring in nature must belong to the Kerr family. The Kerr metric (Kerr 1963) as written in the coordinate system, first introduced by Boyer & Lindquist (1967), is

$$ds^2 = \frac{1}{r^2+a^2\cos^2\theta}\{-\Delta(\mathrm{d}t - a\sin^2\theta\,\mathrm{d}\varphi)^2 + [(r^2+a^2)\,\mathrm{d}\varphi - a\,\mathrm{d}t]^2\sin^2\theta\} + (r^2+a^2\cos^2\theta)\left[\frac{(\mathrm{d}r)^2}{\Delta} + (\mathrm{d}\theta)^2\right], \tag{1}$$

where

$$\Delta = r^2 - 2Mr + a^2, \tag{2}$$

M is the inertial mass, and a is the specific angular momentum of the black hole. The event horizon occurs at the larger of the two roots of the equation $\Delta = 0$ (so long as $a < M$).

The equations governing perturbations of the Kerr metric are, of course, basic for many problems in relativistic astrophysics concerned with what may happen in the neighbourhood of Kerr black holes (cf. Rees 1974). By considering a particular combination of the components of the Riemann tensor – the Newman–Penrose Ψ_0 – and restricting himself to perturbations with a dependence on t and φ of the form $e^{i(\sigma t + m\varphi)}$ (where m is an integer positive, negative, or zero), Teukolsky (1972, 1973) was able to separate the variables r and θ and obtain a pair of decoupled equations. The separation of the variables is a remarkable achievement in view of the