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On the equations governing the perturbations of the Schwarzschild black hole

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A coherent self-contained account of the equations governing the perturbations of the Schwarzschild black hole is given. In particular, the relations between the equations of Bardeen & Press, of Zerilli and of Regge & Wheeler are explicitly established.

1. INTRODUCTION

The equations governing the perturbations of the vacuum Schwarzschild metric – the Schwarzschild black hole – have been the subject of many investigations (Regge & Wheeler 1957; Vishveshwara 1970; Edelstein & Vishveshwara 1970; Zerilli 1970*a*, *b*; Fackerell 1971; Bardeen & Press 1972; Friedman 1973). Nevertheless, there continues to be some elements of mystery shrouding the subject. Thus, Zerilli (1970*a*) showed that the equations governing the perturbation, properly analysed into spherical harmonics (belonging to the different l values) and with a time dependence $e^{i\sigma t}$, can be reduced to a one dimensional Schrödinger equation of the form

$$d^2Z/dr_*^2 + (\sigma^2 - V_Z)\,Z = 0 \quad (+\infty > r_* > -\infty), \tag{1}$$

where

$$V_Z = \frac{2n^2(n+1)\,r^3 + 6n^2mr^2 + 18nm^2r + 18m^3}{r^3(nr+3m)^2}\left(1-\frac{2m}{r}\right), \tag{2}$$

$$\frac{d}{dr_*} = \left(1-\frac{2m}{r}\right)\frac{d}{dr} \quad \text{and} \quad n = \tfrac{1}{2}(l-1)\,(l+2). \tag{3}$$

(Here, units are used in which $c = G = 1$.) Accordingly, the reflexion and the transmission coefficients for incident plane waves of various assigned wavenumbers, will clearly suffice to determine the evolution of any initial perturbation of the Schwarzschild black hole. This reduction of the general perturbation problem to the elementary one of determining the reflexion and the transmission coefficients of a one dimensional potential barrier is, of course, a remarkable simplification. But what is the origin of the particular form of the potential V_Z?

Again, considering a particular component of the Riemann tensor – the perturbed Newman–Penrose $\delta\Psi_0$ – Bardeen & Press (1973) showed that *all* the physical results relative to the perturbation of the Schwarzschild black hole can equally be derived from the solutions of the equation

$$\frac{d^2\phi}{dr_*^2} + \left[\sigma^2 - 4i\sigma\frac{r-3m}{r^2} - \frac{l(l+1)\,(r-2m)+2m}{r^3}\right]\phi = 0, \tag{4}$$