ANNEX III

The so-called "segmentar curvature" according to Weyl's geometry imposes a new condition to the displacement of a vector along one or more of its coordinates. This segmentar curvature is identically null in any Riemannian or, a fortiori, Euclidian space. If it is not null, it will make impossible the free movement of a particle without changes in its fundamental structure, and in special, the constancy of the angular momentum should not be expected to hold in a space obeying Weyl's postulates. But, this is precisely what characterizes the geometry of the jump of an electron from one orbit to the next. In pictorial words, we might say that the particle does not "fit" into the geometry of the new orbit, without losing part of its substance, and if we assume that such a change takes place along the time coordinate holding the electron to the center of its orbit, the law of emission and absorption of radiation will appears as a condition imposed to the particle to adjust itself to the geometry of the new orbit. It is easy to show that this change in geometry involves a reduction or expansion of the metric tensor, and the path of the particle along the time coordinate is a geodesic in a space of Weyl. The equations of the geodesic in such a space can be deduced by the following considerations.

A vector of length $L = ds^2$ will reduce of a quantity

$$\Delta L = - L\alpha_s \Delta y^s \qquad \alpha_s \Delta y^s = \Delta\alpha$$

when displaced by congruence along the y^s coordinate. In the particular case we are considering, the y^s coordinate is the time coordinate y^4, since all other components of ΔL are null, according to our assumption that the space in the orbit is Riemannian, and therefore the ds^2 remains constant :

$$\Delta L = (-g_{ii}u^i u^i)\Delta\alpha = \Delta(g_{ii}u^i u^i)$$

$$\Delta(g_{ii}u^i u^i) = 2\, g_{ii}u^i \Delta\, u^i + u^i u^i\, \Delta\, g_{ii}$$

$$u^i \doteq - \Gamma^{i}_{is}\, \Delta y^s (u^i)$$

$$-2\, g_{ii}u^i u^i\, \Gamma^{i}_{is} \Delta y^s + u^i u^i \Delta\, g_{ii} = - (g_{ii}u^i u^i)\Delta\alpha$$

or

$$-2\, g_{ii}\Gamma^{i}_{is} = -2\, \Gamma_{i,is} = - \Gamma_{i,is} - \Gamma_{i,si}$$

$$\Gamma_{i,is} = \frac{1}{2}\left(\frac{dg_{ii}}{dy^s} + \frac{dg_{is}}{dy^i} - \frac{dg_{si}}{dy^i}\right)$$

$$\Gamma_{i,is} = \frac{1}{2}\left(\frac{dg_{ii}}{dy^i} + \frac{dg_{is}}{dy^i} - \frac{dg_{si}}{dy^s}\right)$$

since all $dg_{is}/dy^i = 0$ if $i \neq s$

the fundamental equations relating α to the g_{ii} are

$$-\frac{dg_{ii}}{dy^s} = -g^{ii}\frac{d\alpha}{dy^s}$$

or

$$\frac{dg_{ii}}{dy^s} - g_{ii}\frac{d\alpha}{dy^s} = 0$$

or

$$a^2 = \frac{c^4}{2m_{11}^2 v_o^2}\left(\frac{dg_{11}}{dx_4}\right)^2$$

and

$$a.v_o = \frac{c^2}{\sqrt{2}.m_{11}}\left(\frac{dg_{11}}{dx_4}\right)$$

<u>a</u> should be considered a differential operator over v_o to give the rate of variation of the frequency along the time coordinate and therefore

$$a.v_o = \frac{\Delta v}{\Delta x_4} \qquad a = \frac{\Delta}{\Delta x_4} \qquad a^2 = \frac{\Delta^2}{\Delta x_4^2}$$

in such a way that we have the following relation :

$$\frac{c^2}{\sqrt{2}.m_{11}}\frac{\Delta g_{11}}{\Delta x_4} = \frac{\Delta v}{\Delta x_4}$$

or

$$\frac{c^2}{\sqrt{2}.m_{11}}\Delta g_{11} = \Delta v$$

and considering that $m_{11} = \frac{h}{2\pi}$ and suppressing the radical $\sqrt{2}$ for reasons of symmetry (for instance, taking twice the non-null values of the geodesic coefficients in the expression of R_{1414}), the above expression can be put in the final form :

$$c^2 \Delta g_{11} = \frac{h}{2\pi}\Delta v$$

an expression identical with the law of radiation obtained before.

In the above deduction, the curvature itself (a^2) is inversely proportional to v_o^2 what seems contrary to the intuitive idea of a continuum that becomes more and more curve as v_o increases and the particle approaches the nucleus along the time coordinate. However, we have to take into account that a^2 is negative and therefore, if its absolute value increases there is actually a decrease of the negative curvature of the inside space-time continuum. Actually we should take $a.\sqrt{-1}$ as the expression of the operator over v_o and the law of emission becomes :

$$c^2 \Delta g_{11} = \frac{hi}{2\pi} \Delta v = - \frac{h}{2\pi i} \Delta v$$

A space with a negative curvature $-a^2$ is a hypersphere, or a Bolyai-Lobatchewski variety, into which we can introduce some of the deductions of Riemannian geometry, as the constancy of the value of $-a^2$ in each orbit, but from which it deviates in a very striking property as having an infinity of parallels from one point to any geodesic drawn on its surface.

The non-null values are the following :

$$\Gamma_{4,11} = \frac{1}{2} \left(\frac{\partial g_{41}}{\partial x_1} + \frac{\partial g_{14}}{\partial x_1} - \frac{\partial g_{11}}{\partial x_4}\right) = - \frac{1}{2} \frac{dg_{11}}{dx_4}$$

$$\Gamma_{4,44} = \frac{1}{2} \left(\frac{\partial g_{44}}{\partial x_4} + \frac{\partial g_{44}}{\partial x_4} - \frac{\partial g_{44}}{\partial x_4}\right) = \frac{1}{2} \frac{dg_{44}}{dx_4}$$

$$\Gamma_{1,14} = \frac{1}{2} \left(\frac{\partial g_{41}}{\partial x_4} + \frac{\partial g_{11}}{\partial x_4} - \frac{\partial g_{14}}{\partial x_1}\right) = \frac{1}{2} \frac{dg_{11}}{dx_4}$$

$$\Gamma_{1,41} = \frac{1}{2} \left(\frac{\partial g_{11}}{\partial x_4} + \frac{\partial g_{14}}{\partial x_1} - \frac{\partial g_{41}}{\partial x_1}\right) = \frac{1}{2} \frac{dg_{11}}{dx_4}$$

where form :

$$\Gamma^{1}_{41} = g^{11} \Gamma_{1,41} = \frac{g^{11}}{2} \frac{\partial g_{11}}{\partial x_4} \qquad \Gamma^{4}_{41} = g^{44} \Gamma_{4,41} = 0$$

$$\Gamma^{1}_{44} = g^{44} \Gamma_{1,44} = 0 \qquad \Gamma^{4}_{44} = g^{44} \Gamma_{4,44} = \frac{g^{44}}{2} \frac{dg_{44}}{dx_4} .$$

and the curvature tensor is reduced to

$$R_{1414} = \Gamma^{1}_{41} \Gamma_{1,41} - \Gamma^{4}_{44} \Gamma_{4,11}$$

or

$$R_{1414} = \frac{g^{11}}{4} \left(\frac{dg_{11}}{dx_4}\right)^2 + \frac{g^{44}}{4} \left(\frac{dg_{11}}{dx_4}\right) \left(\frac{dg_{44}}{dx_4}\right)$$

If we introduce the values of g^{11} and g^{44} in terms of "space" and "time" masses :

$$R_{1414} = \frac{1}{4m_{11}} \left(\frac{dg_{11}}{dx_4} \right)^2 + \frac{c^2}{4v_o} \left(\frac{dg_{44}}{dx_4} \right) \left(\frac{dg_{11}}{dx_4} \right)$$

Since the expression of the curvature a^2 is given by the quotient R_{1414}/g we will have :

$$a^2 = \frac{R_{1414}}{g} = \frac{c^2}{4m_{11}^2 v_o} \left(\frac{dg_{11}}{dx_4} \right)^2 + \left(\frac{c^4}{4m_{11} v_o^2} \right) \left(\frac{dg_{44}}{dx_1} \right) \left(\frac{dg_{11}}{dx_4} \right)$$

According to previous assumptions, we can make the following simplifying assumptions :

$$\frac{dg_{44}}{dx_4} = \frac{dg_{11}}{m_{11} dx_4} \qquad m_{11} = m_o \qquad v_o = c^2 \qquad g_{44} = \frac{g_{11}}{m_o} \qquad m_{11} = \frac{h}{2\pi}$$

and the curvature :

$$a^2 = \frac{c^4}{4m_{11}^2 v_o^2} \left(\frac{dg_{11}}{dx_4} \right)^2 + \frac{c^4}{4m_{11}^2 v_o^2} \left(\frac{dg_{11}}{dx_4} \right)^2$$

and

$$4a^2 = \frac{1}{\gamma o^2}$$

or

$$2a = \frac{1}{\gamma o}$$

ANNEX IV

Space and time coordinates of the orbit

If we consider the particle moving around the nucleus, we have to consider the two possible parameters along the space and time coordinates. We can reduce the three space coordinates to one, along the x axis, considering only the projections of the o thers along that axis. The time coordinate is supposed to be nor mal to the x axis. Therefore, the movement of the electron redu- ces itself to a simple harmonic movement along the x axis, at a fixed "time distance" to the nucleus.

The particle might be supposed to turn around the center 0 in the plane of the paper, but also at a fixed distance on ti- me given by the vector OP → r.sen Θ, as shown in Fig. 1. The ra- dius of its "space orbit" would be the distance OX which might be considered as a space interval $ds^2 = dx^2 + dy^2 + dz^2$, or with the simplification introduced $ds^2 = dx^2$. To begin with, let us assume that the particle moves at a very low speed in a circle situated in the plane (x,y) or along a line (x) as indicated in Fig. 1. If the speed increases to values approaching the veloci ty of light (c), the interval ds^2 will remain invariant but it will have a different expression, since the time coordinate comes now into play, as if the vector ds(= r) had turned of a certain angle Θ. We shall have

$$ds^2 = dx^2 + d\tau^2$$

As long as the velocity of the particle remains stationary, the value of ds^2 remains constant. Actually

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

and the geometry will be one of a quadridimentional hyperboloid of revolution around the time axis, as indicated in Fig. 2. If we reduce it to the simpler expression

$$ds^2 = c^2dt^2 - dx^2$$

the picture of the movement of the particle in its spatial orbit will be simplified as shown in Fig. 2. In the plane (xy) the or- bit will be an elliptical or circular section of the hyperboloid. If we

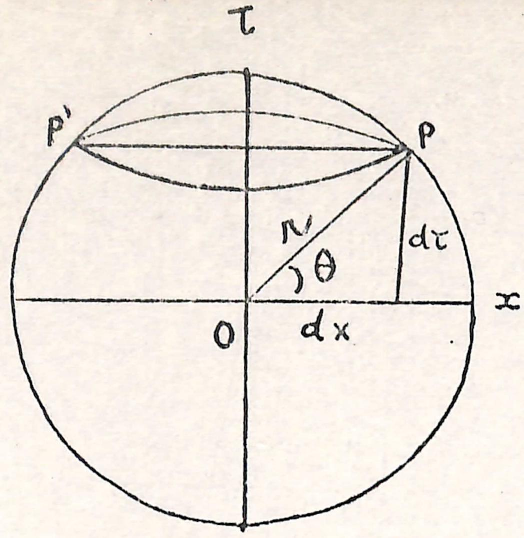

Fig. 1. Trajectory of the particle (PP') in reference to the time (τ) and space (x) coordinates. The interval $ds^2 = dx^2 + dx^2$ is maintained invariant for any velocity of the particle. If the velocity tends to 0, the interval tends to dx (along the x coordinate). If the velocity tends to c (light velocity), the interval ds tends to dx.

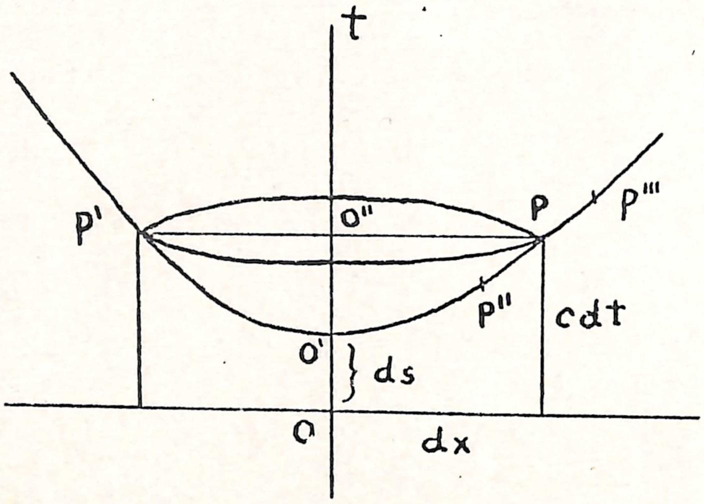

Fig. 2. The hyperboloid P'O'P represents the trajectory of the particle, if instead of τ, the ordinate is taken as dt (the real time parameter), and the interval ds = 00' assumes the relativistic form $ds^2 = c^2 dt^2 - dx^2$. In the plane (xy) the orbit will be an elliptical or circular section of the hyperboloid. If we assume that the point 0 is the center of the atom and P any point in the spatial orbit of the particle, the equation of the hyperbola P'O'P will be $\pm ds^2 = dx^2 - c^2dt^2$.

assume that the point 0 is the center of the nucleus, and P any point in the spatial orbit of the particle, the equation of the hyperbola PO'P' will be

$$\pm\, ds^2 = dx^2 - c^2dt^2$$

Initially $ds^2 = dx^2$. When the velocity increases, the values of c^2dt^2 become more and more important and the radius of the new orbit dx will vary according to a simple law, easily deduced from the principles of relativity. But to change the position of the center of the atom from 0" (center of the spatial orbit) to 0, origin of the time coordinates, is a conceptual change introduced by the theory advanced in this paper. In fact, if we assume that for each stationary orbit, the time parameter is the one fixing the position of the spatial orbit, we have to deduce the radius of the orbit from the time interval $d\tau = cidt$, and assume the constancy of the interval $ds^2 = dx^2 + d\tau^2$, or $-ds^2 = c^2dt^2 - dx^2$, and the geometry of the orbit is one obeying the postulates of Bolyai or Lobatchewski geometry, and the meaning of "increasing" or "decreasing" dx, depends on the signal given to the interval ($\pm ds^2$). Let us take + signal and have as the expression of the interval

$$ds^2 = c^2dt^2 - dx^2$$

and divide both sides of the equation by c^2dt^2

$$\frac{ds^2}{c^2dt^2} = 1 - \frac{dx^2}{c^2dt^2} = 1 - \beta^2$$

where $\beta^2 = v^2/c^2$

or

$$ds^2 = c^2dt^2\,(1 - \beta^2) = c^2dt^2 - \beta^2c^2dt^2$$

if we compare this equation with that of the interval $ds^2 = c^2dt^2 - dx^2$ it is clear that one can make

$$dx^2 = \beta^2c^2dt^2$$

and

$$d\tau^2 = c^2dt^2 \quad \text{as before (disregarding signals)}$$

We come to the simple and attractive conclusion that the square of the radius of the orbit is a fraction (β^2) of the "time coordinate". Actually this result would be expected, since for each value of the velocity ($v = dx/dt$), the time coordinate ($d\tau$) will be constant and also the radius (a) a constant fraction (β) of the time coordinate. It is in that sense that we might assume that the particle "turns around the nucleus at a fixed distance on time" for each orbit, as much as we can say that the radius of its orbit projection upon the (xy) plane is constant, calculated by Bohr as $a = n^2.5x10^{-9}$. However, the interval ds^2 can assume a minus-plus value, corresponding to the two symmetrical shells of the hyperboloid of revolution. That means that the signal should be adjusted to real situations, as those derived from experiment.

For the sake of simplicity, and assuming a spherical symmetry to the orbit, in such a way that its projection upon the (xy) plane will be a circonference of radius $(\beta^2c^2dt^2)^{1/2}$, the interval of universe will be

$$ds^2 = c^2dt^2 - \beta^2c^2dt^2$$

this interval will be the square of the distance 00', the main parameter of the equilateral hyperbola represented in Fig. 2. If the velocity of the particle changes, the point P will migrate in the direction P" or P"' with corresponding changes in the diameter PP' (2a) of the spatial orbit, the interval remaining constant. Actually, the constancy of this interval might be a fundamental property of the hydrogen atom, and all movements of the particle are bound to be contained in the characteristic hyperbola of parameter ds = 00'. However, the relation between the diameter (2a) and the velocity given by the above formulation is far from being simple. Since β is also a function of the velocity, and since there is an inverse relationship between dt^2 and $(1-\beta^2)$ or between dt and $(1-\beta^2)^{1/2}$ an increase in dt will bring about a decrease in $(1-\beta^2)^{1/2}$ that is a tendency of the velocity ($v \rightarrow c$) to approach the velocity of light. We might expect that for the smaller values of the radius, when the particle approaches the nucleus, the velocity of the particle would tend to the velocity of light, since $\beta \rightarrow 1$. This occurs when Z increases to a maximumn above 125. For the Laurentium (Z = 103) the velocity of the inner electron is near c. This of course will give a reason for the upper limit of the Mendeleiev Table. See Annex IV (cont.).

Now, we have to attempt to calculate the interval ds^2 in terms of physical constants. We know according to the theory advanced above, that the time coordinate dt should be connected to the reciprocal of Rydberg constant ($R = 3.29 x 10^{15}$). In the lowest level of energy ($n = 1$) we may have accordingly

$$dt = \frac{1}{4\pi\nu R}$$

and for orbits with increasing values of $n(>1)$, the value of dt will change with the cube of the quantum number, and we should have

$$dt = \frac{n^3}{4\pi\nu R}$$

The radius $dx = \beta c dt = v dt$ or

$$dx = \frac{\beta c n^3}{4\pi R}$$

in which $\beta c = v$ is the velocity of the electron in the orbit of quantum number n. Note that these values agree with the theory advanced above

$$dx^2 = \beta^2 c^2 . \left[\frac{(n^3)^2}{(4\pi R)^2} \right]$$

or

$$dx = v.dt = v. \left[\frac{n^3}{4\pi R} \right]$$

For $n = 1$

$$\beta = 0.0073 \text{ and } v = \beta c = 2{,}2 \times 10^8$$

$$dx = \frac{2{,}2 \times 10^8}{12.56 \times 3.29 x 10^{15}} = 5.49 \times 10^{-9}$$

very near the radius of Bohr $a_o = 5.3 \times 10^{-9}$
For $n = 100$

$$\beta = 0.000073 \quad \text{and} \quad v = 2.2 \times 10^6$$

and the radius

$$dx = \frac{2.2 \times 10^6 \times 10^6}{12.56 x 3.29 x 10^{15}} = 5.32 \times 10^{-5}$$

that agrees with the radius calculated by Stebbings (1976) (1) for the radius $a_{100} = 5.3 \times 10^{-5}$.

This calculation would obviously agree with that given by Bohr, in which the radius increases with the square of

---

(1) Stebbing R.F. High Rydberg atoms : Newcomers to the Atomic Physics Science. Science 193, 537 (1976).

$n(=10^4)$ for the orbit with n = 100.

But, let us start again by giving details of Bohr's calculation.

According to the simplified formula proposed by Bohr for the frequency (w) of the orbit of quantum number (n)

$$w = \frac{2R}{n^3}$$

where R is Rydberg's constant. If we make $dt = 2\pi w$ as a measure of the time parameter to this orbit, we can have :

$$d\tau = cidt = \frac{cn^3}{4\pi R} \quad (i)$$

and since $dx = a = \beta dt$ and $\beta = v/c$ we will have

$$dx = \frac{n^3 . v}{4\pi R}$$

We know, by the quantification rule of the momentum that $v = \frac{2\pi e^2}{\pi h}$ for the expression of the velocity of the particle in the orbit of quantum number (n). On the other hand, the value of R can be expressed in terms of e,m and h, by the expression formulated by Bohr :

$$R = \frac{2\pi^2 me^4}{h^3} \quad (=3.29\times 10^{15})$$

combining all formulas given above we will have for the radius

$$dx = a = \frac{2ne^2 n^3}{\pi h} \times \frac{h^3}{8\pi me^4} = v. \frac{n^3}{4\pi R} = 5.53\times 10^{-9}$$

(for n=1)

an expression identical with the one given above for the value of $dx = v.dt = \beta c.dt$, and so forth.

Therefore, we can have a clear cut measure of the radius of the orbit, by assuming that it is a $\beta = v/c$ fraction of the time coordinate (or parameter) and therefore according to the simple model of the hyperbolic space-time continuum of special relativity. The conceptual advantage of this kind of deduction is that we have all indications that the time parameter of the orbit is a constant that can be determined with the highest accuracy (from spectroscopic measurements) and that for each time parameter the actual position of the spatial orbit is left indeterminate, though we can theoretically determine a value for the radius as a β fraction of the time parameter.

The coincidence indicated above between the radius of the orbit calculated from the model outlined above, and the radius calculated by Bohr, obviously indicate that the model can fit the older quantum theory. The advantage of considering the time parameter as the one fixing the distance of the particle to the nucleus still compatible with a calculation of a definite value for the radius of the orbit, would permit to have an intuitive representation of the indeterminacy of the orbit since for each time value one can have an infinity of planes for the spatial orbit. If we consider the projections over the experimental plane of the infinity of orbital planes with a single "time parameter", one might have an intuitive representation of the astonishing "electronic cloud", the density of which is distributed in a Gaussian way around the Bohr's radius. Any one who has seen a malabarist turning a disk at the top of the nose will understand how a constant value of a parameter, the distance of the tip of the nose to his chin, for instance, can generate an infinity of possibly space orbits, the projection of which upon a horizontal plane (xy) will generate a sinusoid along one of the dimensions of the plane, with a maximum frequency corresponding to the distance a of the radius of Bohr.

Data from R.F. Stebbings. 1976. High Rydberg Atoms; newcomers of the atomic physics. Science 193, 537-542 (1976).

Annex VI

Adjusting to wave mechanics

It remains now the important task to adjust the theory advanced above to the postulates of wave mechanics. The fundamental idea introduced in this report is the concept of "time mass" related to "space mass" through the converting factor $h/2\pi$. In this case whenever we have to deal with a relation $2\pi m_o/h$ it can be cancelled out as unit, with the dimensions of a "time mass" = time over a square length or $L^{-2}T$. This of course empties Planck's constant of any physical meaning, becoming a "pure number" without dimensions.

Let's take, for instance, the fundamental relation of wave mechanics $m_o c^2 = h\nu_o$. In terms of time mass it will be written:

$$m_o c^2 = \frac{h\gamma o}{2\pi} \qquad \text{or} \qquad m_o = (m_{44}) \frac{h}{2\pi}$$

with the fundamental definition $m_{44} = \dot{\nu}_o/c^2$. In the case of the electron, the frequency $\nu_o \to c^2$ if a relativistic correction is applied and we will have $m = h/2\pi$.

If the particle moves with the velocity v, making $\beta = \frac{v}{c}$, its mass will change according to a relativistic effect to

$$m = \frac{m_o}{(1 - \beta^2)^{1/2}}$$

and the above equation becomes in the reference system at motion:

$$mc^2 = h\nu/2\pi = m_o c^2/(1 - \beta^2)^{1/2} = h\nu_o/(1 - \beta^2)^{1/2} \cdot 2\pi$$

According to our definition, the "time mass" will be expressed by the relation $m_{44} = \nu/c^2$ and therefore if $(m_{44})_o$ is the time mass "at rest", it will undergo the same relativistic transformation as the "space mass" m_o (or m_{11} m_{22} m_{33}), since $\nu/\nu_o(1 - \beta^2)^{1/2}$ and $m_{44} = \nu_o/(1 - \beta^2)^{1/2} c$. Therefore, it is clear that any reference system will permit to write the general relations:

$$m_{11} = m_{22} = \ldots = m_{44} \frac{h}{2\pi}$$

and $h/2\pi$ is a factor independent of the state of motion of the system.

In terms of relativity, $h/2\pi$ is a pure "scalar" factor, invariant in relation to any reference system. Actually it is the only "scalar" with a physical significance appearing in the equations of movement.

Since the mass of the particle is supposed to be proportional to its kinetic energy in the orbit, any variation of momentum will be proportional to variations in mass and, therefore, by a judicious choice of unitage, we can make

$$c^2\Delta g_{22} = 2\pi c\Delta p_i = - hc(v_1 - v_2)$$

for the change of momentum (angular) when the particle jumps from orbit 1 to 2 and since $v_1 - v_2 = v_{12}$, the frequency of the emitted radiation with wave length λ, we will have for the variation of momentum $p_i = -ch/2\pi\lambda$ and since the radius of the orbit can be related to the wave length by the well known quantic relation $2\pi R = n.\lambda$ we should be able to deduce directly a value for the frequency associated with the most stable orbit of the electron ($n = 1$).

<u>The probability function</u>. But the important point to relate these findings to quantum mechanics is to arrive at a wave funciton Ψ the square of which would indicate the "probability" of finding the electron in any one of its orbit. We can antecipate the result by assuming that the probability would be maximum (= 1) in the most stable orbit of the electron, and it will be 0, when the particle has been pulled out from its attraction by the nucleus. The probability of finding the particle in any of the excited orbits will be less and less, as <u>n</u> (the quantum number) increases. For each orbit, however, the probability will assume a definite value that can be deduced from spectral analysis, or by calculation of the eigen values of the function Ψ.

If we assume that in any of its orbits the electron can be expressed by a monochromatic wave, with the frequency <u>v</u> proper of the orbit, it seems reasonable to assume that the value of Ψ will be a characteristic of the orbit. To arrive at a reasonable formulation of the function Ψ, we can take the element of the Hamiltonian action

$$\Delta A = \Delta p_x dx - \Delta E dt$$

identical with

$$\Delta A = \left[\Delta p_x \frac{dx}{dt} - \Delta E\right] dt = \Delta\left[\frac{ds}{dt}\right] dt$$

where S is the expression of a potential function, giving the moment $p_x = \frac{dS}{dx}$ as its gradient over the space coordinates and the energy $E = \frac{dS}{dt}$ its derivative along the time coordinate.

It is obvious that one can take ΔA as the expression of ΔS and assume that the element of Hamiltonian action is the integral between two time values of a function $\frac{dS}{dt} = \dot{S}$ of the moment and the energy. We know, for instance, that

$$S = \int L dt \quad \text{where} \quad L = T - V$$

where L is the Lagrangian of the system.

Actually $dS/dt = \dot{S} = -H$, the Hamiltonian and therefore:

$$\Delta \left[\frac{dS}{dt}\right] dt = \Delta A$$

We can assume that the movement of the electron when confined to any of its orbit is conservative and that in the most stable orbit $\Delta A = 0$** This variation only occur along the time coordinate which fixes the electron to the nucleus, when the electron jumps from one orbit to another. Since the value of ΔA will be negative for any spontaneous jump, it can be taken with values varying from 0 to $-\infty$, when the electron will be free at an infinite distance of the nucleus. We can

---

** Note: This assumption is warranted by the consideration of all orbits and jumps as possible paths of the particle, and the minimum or extremum will be that in the stablest orbit (for convenience we make it n = 1). The t appearing in the derivatives of S, should be taken along the vector axis fixing the particle to the nucleus and could be described by :

$$\frac{dS}{d\tau} = H$$

and the integral

$$A = \int_{\tau_i}^{\tau_j} H . d\tau$$

would ultimately indicate the variation of action due to absorption or emission of radiation.

postulate that the function could be of the form:

$$\Psi = e^{i\Delta A}$$

since Ψ will be then = 1 in the most stable orbit, when $\Delta A = 0$ and zero, when ΔA approaches $-\infty$. Therefore Ψ can have the meaning of a probability function and also its square will be always smaller or equal to 1.

If we wish to have Ψ in function of the momenta and the energy, we have to consider the fact that the experiment does not give us any indication about the spatial components of either parameter along the space coordinates, but give a very accurate expression of the "time components" of both when they vary along the time coordinate (jump with emission or absorption of light.

Let us call, as before, Δp_{44} the time component of the momentum and ΔE_{44} the "time component" of the energy. The two following expressions will spring from the previous discussion:

$$\Delta p_{44} = \frac{2\pi i}{h} \Delta p_x$$

and

$$\Delta E_{44} = \frac{2\pi i}{h} \Delta E$$

Therefore, the "time component" of the Hamiltonian action will be:

$$\Delta A_{44} = \frac{2\pi i}{h} (\Delta p_x dx - \Delta E\, dt)$$

and the function becomes:

$$\Psi_{44} = e^{\frac{2\pi i}{h} (\Delta p_x dx - \Delta E\, dt)}$$

identical with Schrödinger's equation:

$$\Psi = Ae^{\frac{2\pi i}{h}(p_x x - Et)}$$

Since ΔA_{44} has the same domain (along the time coordinate) as ΔA, i.e. can only have values from 0 to $-\infty$, the values of Ψ_{44} will also vary from 1 to 0 and will indicate a probability of finding the electros in any of its orbits.

If Ψ_{44} is twice derived in relation to x and once to t, we have the expressions:

$$\frac{h}{2\pi i}\frac{d\Psi_{44}}{dx} = p_x\Psi_{44} \qquad -\frac{h^2}{4\pi^2}\frac{d^2\Psi_{44}}{dx^2} = p_x^2\Psi_{44}$$

and

$$-\frac{h}{2\pi i}\frac{d\Psi_{44}}{dt} = E\Psi_{44}$$

where

$$p_x = \frac{h}{2\pi i}\frac{d}{dx} \quad \text{and} \quad E = -\frac{h}{2\pi i}\frac{d}{dt}$$

appear as operators acting upon the wave function Ψ_{44}. From these values and considering that

$$H = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + V = E$$

and assuming that V = 0, we have one of the forms of Schrödinger equation:

$$(H - E)\Psi_{44} = 0$$

or

$$\nabla^2\Psi_{44} - \frac{8\pi^2 m}{h^2}E\Psi_{44} = 0$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]\Psi + \frac{8\pi^2 m}{h}E\Psi_{44} = 0$$

If we impose now the condition that

$$\int\Psi^2\, dxdydz = 1$$

the values of E can only assume discrete values which will be the same as those imposed by the quantification of the orbit. Therefore, without assuming any physical structure for Ψ_{44} it assumes really the nature of a function of "pure probability" this indicating that its value is maximum, i.e. $\Psi_{44}^2 = 1$ when the electron is in its most stable orbit. In other words, the most probable configuration of the hydrogen atom is that in which $\Psi_{44} = 1$. The emission of radiation will indicate a change toward a more probable situation. On the contrary, when the electron receives a quantum -dA of action it will change its position toward one of less probability and when it is rejected from the atom it will rise to levels of less and less probability until it becomes infinitely removed from the nucleus and the probability of finding it at any point becomes zero.

To attach to Ψ a physical significance, a real wave propagating in space, leads to the usual contradictory result of "ghost waves", waves with a velocity superior to that of light and so forth. To give it the abstract meaning of finding a particle in any of its orbits, seems to me the most logical attitude, since probabilities cannot be "propagated" but are only abstract mathematical entities.

The quantification rule. Another way to arrive at quantification conditions is to assume that the frequency proper (ν_o) of the electron can be derived from a potential function Φ whose gradient along the time coordinate (τ) is the exact expression of γ_o. Therefore:

$$d\phi = \nu_o d\tau$$

The only limiting assumption to make is that along the trajectory, the integral

$$\phi = \int \nu_o d\tau = n.2\pi$$

is a multiple of 2π, the factor n being the principal quantum number n. This can be done, since ν_o having the dimensions of an "inverse time", its multiplication by "time" can be a dimensionless number $2\pi.n$.

Now, we can assume that the value of ν_o is a constant of the particle, since it means the basic frequency of the electron and the value of the integral $\int d\phi = n.2\pi$ will depend solely upon the values attributed to $d\tau$ in each orbit. For relativistic reasons, the value of $d\tau$ will depend upon the velocity v of the particle in each orbit, according to

$$d\tau_o = \frac{d\tau - v.dx/c^2}{(1 - \beta^2)^{1/2}} \qquad \beta^2 = \frac{v^2}{c^2}$$

or

$$d\phi = \frac{\nu_o \left(d\tau - \frac{vdx}{c^2}\right)}{(1 - \beta^2)^{1/2}} \qquad d\tau_o = d\tau \left(1 - \frac{v^2}{C^2}\right)^{1/2}$$

It is easy to calculate the value of ϕ in function of the energy $E = m_o c^2/(1 - \beta^2)^{1/2}$ and the linear momentum $p_x = m_o v/(1 - \beta^2)^{1/2}$ where m_o is the mass (space mass) of the particle.

By simple transformations:

$$\frac{E}{m_o c^2} = \frac{1}{(1 - \beta^2)^{1/2}} \quad \text{and} \quad \frac{\nu}{c^2} = \frac{p_x (1 - \beta^2)^{1/2}}{m_o c^2} = \frac{p_x}{E}$$

The expression of

$$d\phi = \frac{E\nu_o}{m_o c^2} (d\tau - p_x dx/E)$$

or

$$d\phi = \frac{\nu_o}{m_o c^2} (Ed\tau - p_x dx)$$

To the approximation of a constant $(\frac{\nu_o}{m_o c^2})$, the expression of $d\Phi$ is the same as the classical expression of the element of Hamiltonian action $-dA$

$$-dA = (Edt - p_x dx) \quad \text{as given above.}$$

But, what seems to be quite remarkable is that the constant $\nu_o/m_o c^2$ is nothing else than the inverse ratio of time mass (ν_o/c^2) to space mass (m_o). According to the theory it would be equal to $2\pi/h$

$$\frac{\nu_o}{m_o c^2} = \frac{2\pi}{h} \quad \text{since} \quad \frac{m_o}{m_{44}} = \frac{h}{2\pi}$$

Therefore

$$d\phi = \frac{2\pi}{h} (Edt - p_x dx)$$

is the "time component" of the Hamiltonian action:

$$\oint d\phi = - \frac{2\pi}{h} \Delta A = - dA_{44}$$

If we introduce the limiting condition that

$$\oint d\phi = 2\pi . n$$

we arrive at the Bohr-Sommerfeld condition

$$\oint (Edt - p_x dx) = nh.$$