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Ilmo. Sr.

Professor Mario Schenberg

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Remetente ... Odylio Denys de Aguiar
Endereço ... INPE (Instituto de Pesquisas Espaciais)- Caixa Postal 515
CEP 12200 - São José dos Campos - SP

São José dos Campos, 13 de dezembro de 1982

Caro Professor Schenberg

Não consegui xerocar ainda o artigo que estou para lhe mandar sobre uma contestação da Relatividade Especial. O artigo é de autoria de A.H. Winterflood e tem 55 páginas (o setor gráfico está sobrecarregado). Dei uma relida no 1º capítulo deste e visualizei um erro, mas não estou certo.

Sobre a demonstração de Maxwell que o senhor se referiu, tive oportunidade de conversar com dois colegas. Após a discussão concluímos melhor o que o senhor havia falado. Deve se tratar do fato que Maxwell utilizou as equações de Newton (que correlacionam velocidades e posições entre as partículas) no tratamento do movimento de partículas em um gás, e que mais adiante desprezou a correlação entre partículas, assumindo irreversibilidade (coisa que as equações de Newton não conduzem, pelo contrário conduzem à reversibilidade) ou seja assumindo "caos molecular" como resultante. Realmente sua demonstração é a rigor errada, porém conclui corretamente, tendo a intuição de que os processos naturais pressupõem interações não correlacionadas entre partículas, ou seja, que a componente aleatória nas interações prevaleça sobre a determinística na presença de um número muito grande de partículas. Se estiver errado, por favor me escreva, dizendo o que o senhor se referia.

Estão seguindo os históricos da escola, ITA e INPE. Não são recomendativas as notas do ITA, pois não me dedíquei muito aos cursos (dedicava mais tempo ao Centro Acadêmico, juntamente com Clóvis Goldemberg e os outros). Lhe envio depois o artigo do Winterflood e também uma cópia daquele trabalho de graduação do ITA. Segue aparte uma modesta retribuição pelo favor a que o senhor se ofereceu.

Gostaria de eventualmente discutir com senhor alguns temas de relatividade e gravitação, a começar pelos seus dois artigos da Revista Brasileira de Física (encontrei o segundo "Causality and Relativity" publicado em 77).

Sinceramente

Adylio

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June 21, 1982

Mr. Odylio Denys de Aguiar
Rua Massaguaçu, 130, Cidade Jardim
12200-São José dos Campos - S.P.
Brazil

Dear Mr. de Aguiar:

This replies to your letter of April 15. I do not know of any experimental physics group anywhere in the world which is trying to do serious experiments on the production of gravitational waves by interactions between electromagnetic waves and electrostatic or magnetostatic fields. Many theoretical estimates have been made of the strength of such produced gravitational waves and of the prospects to detect them. All correct estimates of which I am aware conclude that it is hopeless to generate detectable waves in this manner. The best estimates I know of are those made by Leonid Grishchuk and his colleagues in Moscow. I do not have precise references, but you might check the names Leonid Grishchuk, Ya. B. Zel'dovich, and Vladimir B. Braginsky in bibliographies of physics publications; the relevant papers were published in the early or mid-1970's.

On the other hand, there are several experimental groups working on the detection of cosmic gravitational waves by their interaction with electromagnetic fields. These groups include our own here at Caltech, the group of Professor Rainer Weiss at MIT, the group of Professor Billing at the Max Planck Institute for Quantum Optics in Munich, Germany, the group of Ronald Drever at Glasgow University in Glasgow, Scotland, and a group in Novosibirsk, Russia. All of these groups are using laser interferometry to attempt the detection of gravitational waves. There are also ideas floating around for detection of gravitational waves by their interaction with microwave radiation in microwave cavities. I enclose one reference, by a colleague of mine, on this subject.

So far as work for the Ph.D. is concerned, I would suggest that you consider the following experimental gravity research groups: Professor Vladimir B. Braginsky, Physics Department, Moscow State University, Moscow 117234, U.S.S.R. (he is using microwave-cavity transducers to monitor the motion of a sapphire-crystal bar gravitational-wave detector); Professor Rainer Weiss, Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139 (he is using laser interferometry to monitor the motions of freely swinging, widely separated masses, for gravitational-wave detection); Professor Ronald W. P. Drever, Gravity Physics 130-33, California Institute of Technology, Pasadena, California 91125 (laser interferometry as above); Professor Ronald W. P. Drever, Department of Natural Philosophy, Glasgow University, Glasgow, G12 8QQ, Scotland (laser interferometry as above); Professor H. Billing, Max-Planck-Institut fürAstrophysik, D-8046 Garching, F. R. Germany (laser interferometry as above); Professor David Douglass, Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627 (bar detector for gravitational waves, monitored by SQUID transducer); Professor Joseph Weber, Physics Program, University of Maryland, College Park, Maryland 20742 (bar detector); Professor William Hamilton, Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803 (bar detector).

I hope that this information is of some help to you.

Sincerely,

Kip S. Thorne (by jba)

Kip S. Thorne
Professor of Theoretical Physics

KST:jba
Enclosure

Dictated by Prof. Thorne just before a one-month trip to Europe; typed and signed in his absence by JoAnn Boyd Anderson.

MICROWAVE CAVITY GRAVITATIONAL RADIATION DETECTORS ☆

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Received 26 September 1978

The coupled electro–mechanical system consisting of a microwave cavity and its walls can serve as a gravitational radiation detector. A gravitational wave interacts with the walls, and the resulting motion induces transitions from a highly excited cavity mode to a nearly unexcited mode.

Microwave cavities with superconducting walls may have a variety of applications as detectors of non-newtonian gravitational fields. The response of a microwave cavity to a time-changing gravitational field is quite complicated. Both the electromagnetic field in the cavity and the cavity walls interact directly with the gravitational field; in addition, the electromagnetic field and walls interact with one another at the boundary between the two.

I have developed a formalism for analyzing this complicated electro–mechanical system in the presence of a weak gravitational field. A previous paper [1] sketched an application of the formalism to a proposed experiment to measure dragging of inertial frames. This letter presents results of applying the formalism to microwave cavities designed to detect gravitational radiation. Subsequent papers [2] will give details of the formalism and of its various applications.

In 1971 Braginsky and Menskii [3] suggested using microwave cavities to detect high-frequency gravitational waves ($\nu \sim$ (cavity's fundamental mode frequency)). I have analyzed their high-frequency detectors and have also found new designs for, and developed the theory of, detectors designed to operate at much lower frequencies. After the first formal (but unpublished) write-up of my analysis [4], I became aware that Pegoraro et al. [5] had arrived at some similar designs for low-frequency detectors.

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Both high- and low-frequency microwave cavity detectors operate in essentially the same way. A gravitational wave incident on the cavity couples its electromagnetic modes and thereby induces transitions between modes. The coupling is due to the *direct interaction* of the electromagnetic field with the wave and to an *indirect interaction* in which the wave interacts directly with the cavity walls, whose resulting motion couples the electromagnetic modes. In the simplest detectors, the cavity is designed so that two of its modes are strongly coupled by the gravitational wave. One of these two modes (mode 1) is driven into steady-state oscillation at its eigenfrequency; initially the other mode (mode 2) is nearly unexcited. A passing gravitational wave with Fourier components near the splitting frequency between the two modes "pumps" quanta from mode 1 to mode 2, and the wave is detected by monitoring the resulting excitation of mode 2.

Focus attention now and for the remainder of this letter on low-frequency detectors – those designed to operate at frequencies much lower than the cavity's fundamental mode frequency. Since the wave's characteristic wavelength is much larger than the cavity's dimensions, it is convenient to describe the wave in Fermi-normal ("physical") coordinates [6]. In these coordinates the motion of the cavity walls is described by the local displacement vector ξ, which is governed by the usual equations for an elastic medium subject to a tidal force produced by the gravitational wave and to stresses at its boundary produced by the electromagnetic field.

However, analysis of the electromagnetic field in these coordinates is complicated because the boundary of the cavity is moving. To handle this difficulty [2], one transforms to new coordinates in which the boundary is at rest, and one chooses the new coordinates so that they differ from the old coordinates only in a small region near the boundary. In the new coordinates one uses the artifice of writing the curved-space, generally-covariant Maxwell equations in a form which is identical to the flat-space Maxwell equations for a moving, anisotropic medium [7]. With the Maxwell equations recast in this form and with the boundary at rest, the boundary conditions are the familiar ones. All information about the interaction of the electromagnetic field with the gravitational wave is contained in the "dielectric tensor" $\boldsymbol{\varepsilon}$ and the "velocity" g of the (fictitious) moving medium. At linear order in the gravitational wave amplitude, $\boldsymbol{\varepsilon}$ and g split cleanly into terms representing the direct and indirect interactions. The indirect interaction terms are proportional to the physical displacement of the cavity boundary; the direct interaction terms are smaller by a factor ~ (cavity dimension/gravitational wave wavelength)^2 and can be neglected for low-frequency detectors.

The recast Maxwell equations and the mechanical equations can be decomposed into normal-mode equations. The Coulomb-gauge vector potential is expanded in terms of the cavity's normalized electromagnetic eigenmodes A_n: $A = \Sigma_n c_n A_n$ ($\int A_n \cdot A_m \, dV = \delta_{nm}$); and the local displacement vector is expanded in terms of the walls' normalized mechanical eigenmodes ξ_α: $\xi = \Sigma_\alpha d_\alpha \xi_\alpha$ ($M^{-1} \int \rho \xi_\alpha \cdot \xi_\beta \, dV = \delta_{\alpha\beta}$, where ρ and M are the density and mass of the walls). The result is a set of coupled equations for the normal-mode coordinates c_n and d_α in the absence of dissipation [2].

In the case of interest, mode 1 is highly excited at its eigenfrequency by an external source ($c_1 = \mathrm{Re}(Ae^{i\omega_1 t})$; (total energy in mode 1) $= U_1 = \omega_1^2 |A|^2/8\pi$), and mode 2 is strongly coupled to mode 1 by wall motion. Typically, only one mechanical mode ($\alpha = \mathrm{m}$) couples strongly to the gravitational wave and, at the same time, produces displacements of the cavity boundary which strongly couple the two electromagnetic modes. Neglecting all other electromagnetic and mechanical modes, one obtains equations for c_2 and d_m in the presence of a highly excited mode 1. With addition of empirical damping terms and neglect of high-frequency stresses on the walls, these equations become

$$\ddot{c}_2 + 2\beta\dot{c}_2 + \omega_2^2 c_2 = \omega_1^2 c_1 d_\mathrm{m} \mathcal{L}^{-1}, \tag{1a}$$

$$\ddot{d}_\mathrm{m} + 2\beta_\mathrm{m}\dot{d}_\mathrm{m} + \omega_\mathrm{m}^2 d_\mathrm{m} = (\omega_1^2/4\pi M) c_1 c_2 \mathcal{L}^{-1} + f_\mathrm{m} \,. \tag{1b}$$

Here ω_1 and ω_2 are the angular eigenfrequencies of modes 1 and 2 when the cavity is *fixed* in the shape it has after it is distorted by the time-averaged stresses produced by the field in mode 1: ω_m is the angular eigenfrequency of the mechanical mode; $\mathcal{L}^{-1}$ is a field-wall "matrix element" given by

$$\mathcal{L}^{-1} \equiv \int_S (T_2 \cdot T_1 - A_2 \cdot A_1)(\xi_\mathrm{m} \cdot \mathrm{d}a),$$

where $T_n \equiv \omega_n^{-1} \nabla \times A_n$ and $\mathrm{d}a$ is the outward-directed surface element of the cavity boundary; and

$$f_\mathrm{m} \equiv -M^{-1} \int \rho (\xi_\mathrm{m})^j R_{0j0k} x^k \mathrm{d}V,$$

where the R_{0j0k} are the "electric" components of the wave's Riemann tensor. The terms in eqs. (1) involving $\mathcal{L}^{-1}$ represent the coupling of modes 1 and 2 by wall motion (eq. (1a)) and the force exerted on the wall by the electromagnetic field (eq. (1b)); f_m represents the coupling of the mechanical mode to the gravitational wave.

It is convenient to introduce a dimensionless complex quantity μ defined by $c_2 = \mathrm{Re}(\mu A e^{i\omega_1 t})$. The Green function solution for μ is

$$\mu(t) = \int_0^t g(t, t') f_\mathrm{m}(t') \mathrm{d}t',$$

where

$$g(t, t') = \frac{i\omega_1}{2\mathcal{L}} \sum_{j=1}^{4} \frac{\mathcal{S}_j + \omega_{21} - i\beta}{\Pi_{k \neq j}(\mathcal{S}_j - \mathcal{S}_k)} e^{i\mathcal{S}_j(t-t')}, \tag{2}$$

for $t \geqslant t'$. Here $\omega_{21} \equiv \omega_2 - \omega_1$, and the $\mathcal{S}$'s are the roots of the quartic equation

$$(\mathcal{S}^2 - \omega_\mathrm{m}^2 - 2i\beta_\mathrm{m}\mathcal{S})(\mathcal{S}^2 - \omega_{21}^2 - 2i\beta\mathcal{S}) = \omega_1 \omega_{21} \mathcal{L}^{-2}(U_1/2M). \tag{3}$$

The real parts of the $\mathcal{S}$'s give the detector's operating frequencies.

The $\mathcal{S}$'s change as the field in mode 1 is turned on.

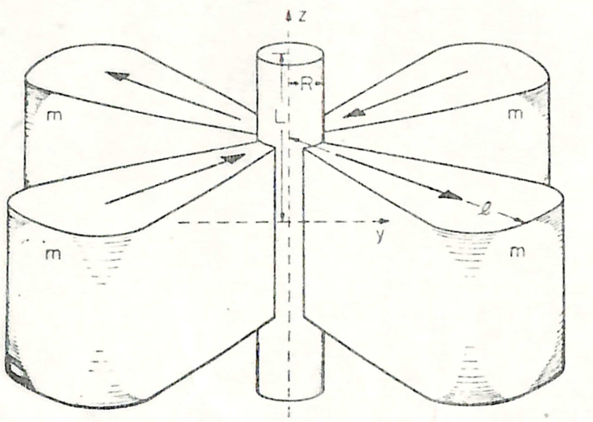

Fig. 1. Diagram of microwave cavity detector described in text.

For large fields, they can differ substantially from their zero-field values. Unless the entire system is designed carefully, one or more of the $\mathcal{S}$'s may have a large and negative imaginary part; and mode 2 will be unstable. This instability can be avoided by suitably arranging the mass in the cavity walls and by suitably choosing the cavity's zero-field shape.

One possible design for such a detector is shown in fig. 1. The microwave cavity is nearly cylindrical with radius $R \approx 10$ cm and length $2L \approx 500$ cm. The microwave modes are the two $TE_{1,1,1}$ modes with angular frequency $\omega_1 \approx 6 \times 10^9$ rad s^{-1}; for a perfect cylinder these two modes are degenerate. The magnetic field lies principally in the z-direction; in mode 1 it has a "cos φ" azimuthal dependence, and in mode 2 a "sin φ" dependence – where φ is measured from the x-axis. Mode 1 is driven to a magnetic field strength ≈ 750 G, which corresponds to total energy $U_1 \approx 9 \times 10^8$ erg and number of quanta $N_1 \approx 10^{26}$. Almost all the mass in the cavity walls is in four lobes – each of the mass $m \approx 10^6$ g – which extend a distance $l \approx 500$ cm from the axis of the cavity at angles midway between the x- and y-axes. The relevant mechanical mode is the one whose motion is indicated by the large arrows in fig. 1; its angular frequency is $\omega_m \approx 10^3$ rad s^{-1}. The zero-field shape of the cavity is chosen so that $\mathcal{S}_1 \approx 3.3\omega_m + 3i\beta$, $\mathcal{S}_2 \approx 2.8\omega_m - 2i\beta$, $\mathcal{S}_3 = -\mathcal{S}_2^*$, and $\mathcal{S}_4 = -\mathcal{S}_1^*$ ((operating frequency) ≈ 500 Hz)[‡1].

[‡1] Mode 2 is unstable on time scales of order the electromagnetic damping time $[\mathrm{Im}(\mathcal{S}_2) \approx -2\beta]$. This weak instability is of no concern for measurements made on much shorter time scales; it can be eliminated by using "artificial damping".

The lobes have been placed so that they couple strongly to a gravitational wave with "cross" polarization propagating along the axis of the microwave cavity and so that the resulting motion of the walls strongly couples the electromagnetic modes ($\mathcal{L} \sim R$). Such a wave, with dimensionless amplitude h, characteristic time $\tau_g \approx \mathcal{S}_1^{-1}$, and duration $\hat{\tau}$, changes the amplitude of μ by an amount

$$|\Delta\mu_0| \approx (1/6)h(\omega_1\hat{\tau})(l/R)(\hat{\tau}/\tau_g) \approx 3 \times 10^{-13},$$

for $h \approx 2 \times 10^{-21}$ and $\hat{\tau} \approx 3\tau_g \approx 10^{-3}$ s. This sensitivity goal is comparable to the most optimistic design goals for 1 km baseline laser systems and third-generation bar antennas.

To detect the wave, one must be able to monitor this small change in c_2. For example, one might probe the magnetic field in mode 2 using a small wire loop whose output is fed into a standard linear amplifier Such a linear system attempts to measure c_2 as a function of time, which means measuring both c_2 and $\dot{c}_2$; the uncertainty principle ($(\Delta c_2)(\Delta \dot{c}_2) \geqslant 2\pi\hbar$) guarantees that the system cannot determine μ with greater precision than $|\Delta\mu| \geqslant (2N_1)^{-1/2} \approx 6 \times 10^{-14}$. This limit is small enough that a standard linear system, provided it is nearly quantum-limited, can detect the desired change in c_2. It should be noted that systems which do not attempt to measure both c_2 and $\dot{c}_2$ (quantum-nondemolition systems [8]) can, in principle, achieve greater precision.

Another serious problem is Nyquist noise (thermal fluctuations) in the cavity walls. To achieve a signal-to-noise ratio of 5 for an integration time $\approx \hat{\tau}$ requires

$$h \gtrsim 10\,(\tau_g^2/l)\,(8kT_m/M\tau_m^*\hat{\tau})^{1/2} \approx 2 \times 10^{-21},$$

for wall temperature $T_m \approx 3 \times 10^{-3}$ K and mechanical damping time $\tau_m^* = \beta_m^{-1} \approx 2 \times 10^3$ s, which corresponds to a mechanical $Q \approx 10^6$. The mass, wall temperature, and mechanical Q assumed here are similar to those projected for third-generation aluminum-bar antennas. Thermal fluctuations in mode 2 itself produce, after a time $\approx \hat{\tau}$, a root-mean-square change in μ of

$$|\Delta\mu| \approx [(2kT_e/U_1)(\hat{\tau}/\tau^*)]^{1/2} \approx 2 \times 10^{-14},$$

for electromagnetic temperature $T_e \approx 4$ K and damping time $\tau^* = \beta^{-1} \approx 3$ s. The corresponding electromagnetic

$Q \approx 10^{10}$ has been attained and exceeded in small superconducting cavities excited in a fundamental mode [9]. This discussion of Nyquist noise assumes that one can couple to mode 2 strongly enough to measure the small change in c_2 in a time $\approx \hat{\tau}$. If a longer integration time is required, Nyquist noise will be a more serious problem.

It should not be difficult to design low-frequency microwave cavity detectors which operate over a wide range of frequencies as detectors of either pulsed or CW radiation. For example, by changing its zero-field shape and mechanical eigenfrequency, the detector described here can be modified to operate at lower frequencies ($\nu \sim 10$–100 Hz). Another possible design consists of two long cavities at right angles, weakly coupled and excited in a high-frequency mode in which the two cavities oscillate in phase. A gravitational wave propagating in the direction perpendicular to the plane of the cavities induces transitions into a mode in which the two cavities oscillate out of phase. Alternatively, one could omit the weak coupling and operate the two cavities as a Fabry–Perot interferometer. This design has been suggested by Pegoraro et al. [5] to detect CW radiation from known binary star systems; however, Nyquist noise in the walls and seismic noise (earth vibrations) would pose severe problems for such an attempt. Operated as a detector of pulses in the same frequency band as the detector described here, this design would have comparable sensitivity.

Although I have referred to these coupled electro-mechanical systems as microwave cavity detectors, they can also be regarded as purely mechanical detectors with a particular kind of electromagnetic transducer. Viewed in this way, they are similar to Braginsky's [10] proposal to instrument a bar detector with a microwave cavity transducer. In the Braginsky scheme a small microwave cavity, which narrows at one place to a small gap, sits on the end of the bar; the cavity's fundamental mode is excited off-resonance, and movement of the wall at the gap induces on-resonance excitation of the same mode. The main distinguishing features of the design considered here are that the cavity is much larger, the coupling to wall motion occurs over virtually the entire cavity boundary, and – perhaps most important – the signal to be detected appears in a mode which is spatially distinct from the highly excited mode. This last feature may be very important in reducing contamination due to the large field in mode 1 when one attempts to monitor the very weak field in mode 2.

For helpful suggestions I thank R.W. P. Drever, K.S. Thorne, and M. Zimmermann.

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