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Elimination of Divergences in the Meson Theory. I

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The divergences of the meson theory are avoided by using a method introduced by the author in quantum electrodynamics; the method is worked out for the case of pseudoscalar charged mesons. The quasi-static forces between the nucleons are treated as relativistic actions at a distance; two independent complex pseudoscalars are used in the description of the meson field. It is shown that it is possible to choose separately the type of nuclear forces and the interaction with the meson field, because of the action at a distance treatment. The formalism, associated to special rules of interpretation, leads to finite transition probabilities; it can also be applied to self-energy problems and leads to finite self-energies and magnetic moments. In the computation of the self-energies and magnetic moments two constants appear, which may be chosen in such a way as to give different masses to the proton and neutron.

I. INTRODUCTION

In this paper we shall extend to the meson theory the methods applied by us to the quantum theory of the electromagnetic field.[1,2] Our treatment of the electromagnetic field is based on a combination of the ideas of actions at the distance of Tetrode[3] and Fokker[4] with the field concepts of Faraday and Maxwell[5] (in quantum theory these concepts correspond to the quantum theory of fields of Heisenberg and Pauli[6]). The separation of the actions at a distance from the radiative actions (actions through the field) is closely related to the separation of the static forces from the radiative actions, which is usually done by means of a contact transformation (in the meson theory such a treatment was systematically developed by Möller and Rosenfeld[7]). The actions at a distance are a relativistic generalization of the static forces; they are half-advanced, half-retarded forces, not instantaneous ones. The actions at a distance include the static actions plus a part of the forces which are usually derived from the interaction between the particles and the radiation field (interaction with the transverse waves, in the electromagnetic case). In order to separate the actions at a distance from the true radiation field, which is classically one-half the difference between the retarded and advanced fields, it is necessary to modify the method of quantization of the fields by using two sets of field quantities, which correspond to the quantities of the classical retarded and advanced fields. The commutation rules for the quantities of the two independent fields, which correspond to the classical retarded and advanced fields, are not the same, because of the structure of the classical Lagrangian, but they are not very different. On the other hand, the commutation rules for the quantities of the true radiation field are essentially different from those of the usual forms of the field theories. Since the particles interact only with the true radiation field, because the actions at a distance are introduced separately, the description of the interaction between the particles and the fields, given by our formalism, is essentially different from the usual one and allows us to avoid the divergences.

In order to describe the actions at a distance we define particle operators for the half-retarded, half-advanced quantities analogous to the half-retarded, half-advanced classical potentials. The method of defining those operators, in the quantum formalism, is similar to that used in the classical theory; it is analogous to the method used by Möller and Rosenfeld in their definition of the static potentials. Instead of the Green function of the static Yukawa equation, used by Möller and Rosenfeld, we use the Green function

[1] M. Schönberg, "Quantum theory of the point electron. I." Phys. Rev. 74, 738 (1948).
[2] M. Schönberg, "Quantum theory of the electromagnetic field. I." Anais Acad. Brasil. Cienc. (in print).
[3] H. Tetrode, Zeits. f. Physik 10, 317 (1922).
[4] A. D. Fokker, Zeits. f. Physik 58, 386 (1929).
[5] M. Schönberg, "Classical theory of the point electron," Phys. Rev. 69, 211 (1946).
[6] W. Heisenberg und W. Pauli, Zeits. f. Physik 56, 1 (1929), and 59, 168 (1930).
[7] C. Möller and L. Rosenfeld, Kgl. Danske Vid. Sels. Math.-Fys. Medd 17, 8 (1940).

of the Klein-Gordon equation which corresponds to half-advanced, half-retarded solutions. The essential difference between our treatment and the Möller and Rosenfeld one lies in the fact that they can obtain the static forces by means of a contact transformation applied to the Hamiltonian of the system nucleons plus meson field, whereas we do not get the actions at a distance from the interaction between the nucleons and the meson field, but do introduce them directly and independently. Our treatment allows us to associate actions at a distance corresponding to the symmetrical Möller-Rosenfeld mixture to an arbitrary type of meson field. In this part of the paper we consider only a field of pseudoscalar charged mesons, for the sake of simplicity, but we could associate the same actions at a distance with a charged vector meson field or to a field of charged mesons with spin $\frac{1}{2}$. The interaction constants which appear in the actions at a distance can be taken independently. Our particle operators for the potentials allow us to get the static forces, by neglecting all the relativistic corrections; they lead to relativistic interactions of the type considered by Leite Lopes[8] and Tamm,[9] by taking into account the retardation effects in a first approximation.

The treatment of the nuclear forces as actions at a distance offers new possibilities. From the point of view of the actions at a distance it is not necessary to use the Green functions of the Klein-Gordon equation in the derivation of the nuclear forces. We may replace those Green functions by other relativistic functions in such a way as to modify the form of the nuclear forces for small distances and thus avoid the difficulties arising from the high singularity of the dipole interactions as well as from those involved in the non-static part of the interaction. Thus we get a relativistic "radius" for the nucleons which does not come in the interactions between the nucleons and the meson or electromagnetic fields but does introduce a relativistic cut-off of the forces at small distances.

The special nature of the commutation rules of the radiation meson field allows us to get exact solutions of the Schrödinger equation of the system nucleons plus meson field, in which the nucleons interact only with the degrees of freedom corresponding to some arbitrarily chosen momenta of the mesons. We call those solutions "frozen" solutions and the degrees of freedom of the meson field, which do not interact with the nucleons in those "frozen" solutions, the "frozen" degrees of freedom. By using "frozen" solutions it becomes possible to get rid of the divergences both in the computation of the transition probabilities and the self-energies. The possibility of the "freezing" results from the fact that, in our formalism, the field has twice as many degrees of freedom as in the usual treatment, because we introduce two sets of field quantities. Our method of quantization has some points in common with Dirac's method,[10,11] but the two methods differ fundamentally because in Dirac's treatment the particles interact with a field which does not have the same commutation rules as our radiation field, so that it is not possible to "freeze" the irrelevant degrees of freedom of the field. The duplication of the degrees of freedom of the meson field leads to the existence of mesons with positive and negative energies. It is important to notice that the negative energy mesons which appear in our quantum formalism correspond to the waves with negative energy of the underlying c-number theory. The existence of the negative energy mesons is not due to a special method of quantization, but it follows from the fact that the c-number energy of the radiation field is not necessarily positive as the total energy of the field-including the potential energy of the actions at a distance—in the usual c-number field theories.

The existence of negative energy mesons and the necessity of using "frozen" solutions, in order to avoid divergences, require a physical interpretation rule and a "freezing" criterion. The physical interpretation rule can be chosen in many different ways—as it happens in Dirac's formalism with an indefinite metric in Hilbert space (see W. Pauli[12]). In the electromagnetic case it is possible to justify the choice of the simplest interpretation rule by correspondence considerations, presumably the same thing can

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[9] Ig. Tamm, J. Physics, **9**, 449 (1945).

[10] P. A. M. Dirac, Proc. Roy. Soc. **A180**, 1 (1942).

[11] P. A. M. Dirac, Com. Dublin Inst. Adv. Stud. A, No. 3 (1946).

[12] W. Pauli, Rev. Mod. Phys. **15**, 175 (1943).

also be done in meson theory, so that our choice of a definite interpretation rule seems justified. In the computation of transition probabilities there is a very natural "freezing" criterion; the degrees of freedom which must be kept "unfrozen" are indicated by the mesons which appear before and after the transition. Thus the application of our formalism to the computation of transition probabilities is quite straightforward and does not lead to any real arbitrariness, because the "freezing" criterion is natural and the physical interpretation rule has a classical analog.

The application of our formalism to self-energy problems is less satisfactory than in the case of the transition probabilities, because we lack a natural "freezing" criterion. Presumably a further development of the theory will improve the situation. We applied the formalism to the computation of the self-energies and magnetic moments of the nucleons, by assuming that the degrees of freedom of the field corresponding to momenta of the mesons above certain limits, in the rest system of the nucleon, are "frozen." The "freezing" in the case of the meson field presents a difference, with respect to the electromagnetic case, arising from the existence of positive and negative mesons: It is possible to introduce two radiation meson fields, one giving rise to the emission and absorption of positive mesons with positive energy and negative mesons with negative energy, another giving rise to the emission and absorption of negative mesons with positive energy and positive mesons with negative energy. The two radiation meson fields can be "frozen" separately in the computation of the self-energies and magnetic moments, with the introduction of two different upper limits for the "unfrozen" meson momenta. If the two limits are equal, the self-energies of the proton and the neutron will become equal, and the contributions of the meson field to the magnetic moments will be opposite. By taking two different limits, the self-energy of the neutron can be made larger than the self-energy of the proton, in agreement with the experimental evidence.

II. QUANTIZATION OF THE MESON FIELD

We shall consider a system of n point nucleons interacting with a field of pseudoscalar charged mesons, described by two independent pseudoscalars, $\phi^{(+)}$ and $\phi^{(-)}$. The equations of motion of the field will be derived from the action principle

$$\delta \int L d_4 x = 0, \tag{1}$$

$$\begin{aligned} L = & \frac{1}{2}\left(\frac{\partial\phi^{(+)*}}{\partial x^\mu}\frac{\partial\phi^{(+)}}{\partial x_\mu} - K^2\phi^{(+)*}\phi^{(+)}\right) \\ & -\frac{1}{2}\left(\frac{\partial\phi^{(-)*}}{\partial x^\mu}\frac{\partial\phi^{(-)}}{\partial x_\mu} - K^2\phi^{(-)*}\phi^{(-)}\right) \\ & +\tfrac{1}{2}g_0 \sum_{j=1}^{n} \Gamma_j \delta(\mathbf{x}-\mathbf{x}_j)[T_j(\phi^{(+)}-\phi^{(-)}) \\ & +T_j^*(\phi^{(+)*}-\phi^{(-)*})] + \frac{i}{2} f_0 \sum_{j=1}^{n} \Gamma_j \gamma_j{}^\mu \delta(\mathbf{x}-\mathbf{x}_j) \\ & \times\left[T_j\left(\frac{\partial\phi^{(+)}}{\partial x^\mu} - \frac{\partial\phi^{(-)}}{\partial x^\mu}\right)\right. \\ & \left. +T_j^*\left(\frac{\partial\phi^{(+)*}}{\partial x^\mu} - \frac{\partial\phi^{(-)*}}{\partial x^\mu}\right)\right]. \end{aligned} \tag{2}$$

K is related to the mass of the meson m.

$$K = mc/\hbar. \tag{3}$$

g_0 and f_0 are interaction constants. The index j denotes the nucleons. The tensor index μ takes the values 0, 1, 2, 3. The operators $\gamma_j{}^\mu$ and Γ_j are

$$\gamma_j{}^0 = \beta_j, \quad \gamma_j{}^a = \beta_j \alpha_j{}^a, \quad (a = 1, 2, 3). \tag{4}$$

$$\Gamma_j = \gamma_j{}^1\gamma_j{}^2\gamma_j{}^3 = -\beta_j\alpha_j{}^1\alpha_j{}^2\alpha_j{}^3. \tag{5}$$

The α and β are the well-known operators of the Dirac wave equation. The operators T and T^* transform neutrons into protons and protons into neutrons, respectively. We are taking as metrical tensor the tensor $g_{\mu\nu}$ whose components are

$$g_{00} = 1, \quad g_{11} = g_{22} = g_{33} = -1, \quad g_{\mu\nu} = 0, \quad (\mu \neq \nu). \tag{6}$$

$\phi^{(+)}$ and $\phi^{(-)}$ satisfy the same wave equation

$$\begin{aligned} \left(\frac{\partial^2}{\partial x^\mu \partial x_\mu} + K^2\right)\phi = & \; g_0 \sum_{j=1}^{n} \Gamma_j T_j^* \delta(\mathbf{x}-\mathbf{x}_j) \\ & - i f_0 \sum_{j=1}^{n} \frac{\partial}{\partial x^\mu}[\Gamma_j \gamma_j{}^\mu T_j^* \delta(\mathbf{x}-\mathbf{x}_j)]. \end{aligned} \tag{7}$$

The conjugated momenta of the ϕ are the Π

$$\Pi^{(\pm)}(x)=\pm\frac{1}{2c}\frac{\partial}{\partial x_0}\phi^{(\pm)*}(x)$$
$$\pm\frac{i}{2c}f_0\sum_j\Gamma_j\beta_jT_j\delta(\mathbf{x}-\mathbf{x}_j). \quad (8)$$

The only non-vanishing commutators between the ϕ and their derivatives are the following ones

$$\left[\phi^{(+)}(\mathbf{x},x_0),\frac{\partial}{\partial x_0}\phi^{(+)*}(\mathbf{x}',x_0)\right]=\left[\phi^{(+)*}(\mathbf{x},x_0),\frac{\partial}{\partial x_0}\phi^{(+)}(\mathbf{x}',x_0)\right]=2i\hbar c\delta(\mathbf{x}-\mathbf{x}'). \quad (9)$$

$$\left[\phi^{(-)}(\mathbf{x},x_0),\frac{\partial}{\partial x_0}\phi^{(-)*}(\mathbf{x}',x_0)\right]=\left[\phi^{(-)*}(\mathbf{x},x_0),\frac{\partial}{\partial x_0}\phi^{(-)}(\mathbf{x}',x_0)\right]=-2i\hbar c\delta(\mathbf{x}-\mathbf{x}'). \quad (10)$$

The commutation rules between quantities taken at different times can be derived in the usual way

$$[\phi^{(+)}(x),\phi^{(+)*}(x')]=-2i\hbar cD(x,x'), \quad (11)$$
$$[\phi^{(-)}(x),\phi^{(-)*}(x')]=2i\hbar cD(x,x'). \quad (12)$$

$D(x,x')$ is the symbolic function defined by the following equations:

$$\left(\frac{\partial^2}{\partial x^\mu\partial x_\mu}+K^2\right)D(x,x')=0, \quad (13)$$

$$D(x,x')_{(x_0=x_0')}=0,$$
$$\left[\frac{\partial}{\partial x_0}D(x,x')\right]_{(x_0=x_0')}=\delta(\mathbf{x}-\mathbf{x}'). \quad (14)$$

Let us put

$$\phi_{\mathrm{rad}}=\tfrac{1}{2}[\phi^{(+)}-\phi^{(-)}]. \quad (15)$$

The radiation field ϕ_{rad} satisfies the homogeneous Klein-Gordon equation

$$\left(\frac{\partial^2}{\partial x^\mu\partial x_\mu}+K^2\right)\phi_{\mathrm{rad}}=0. \quad (16)$$

The commutation rules of the radiation field and its complex conjugated are

$$[\phi_{\mathrm{rad}}(x),\phi_{\mathrm{rad}}(x')]=[\phi_{\mathrm{rad}}^*(x),\phi_{\mathrm{rad}}^*(x')]=0 \quad (17)$$

and

$$[\phi_{\mathrm{rad}}(x),\phi_{\mathrm{rad}}^*(x')]=0. \quad (18)$$

We shall enclose the field in a cubic box of volume V and impose periodic boundary conditions. It results from (16) that we may expand ϕ_{rad} in free waves

$$\phi_{\mathrm{rad}}(x)=(\hbar c/V)^{\frac{1}{2}}\sum_k k_0^{-\frac{1}{2}}\{A_{\mathrm{rad}}^p(k)\exp(-ik^\rho x_\rho)+A_{\mathrm{rad}}^{n*}(k)\exp(ik^\rho x_\rho)\}. \quad (19)$$

$$k_0=(|\mathbf{k}|^2+K^2)^{\frac{1}{2}}. \quad (20)$$

We shall now introduce the two partial radiation fields ϕ_{rad}^p and ϕ_{rad}^n

$$\phi_{\mathrm{rad}}^{p,n}=(\hbar c/V)^{\frac{1}{2}}\sum_k k_0^{-\frac{1}{2}}A_{\mathrm{rad}}^{p,n}(k)\times\exp(-ik^\rho x_\rho). \quad (21)$$

We have

$$[(\partial^2/\partial x^\mu\partial x_\mu)+K^2]\phi_{\mathrm{rad}}^{p,n}=0, \quad (22)$$

and

$$[\phi_{\mathrm{rad}}^{p,n}(x),\phi_{\mathrm{rad}}^{p,n}(x')]=[\phi_{\mathrm{rad}}^{p,n}(x),\phi_{\mathrm{rad}}^{p,n*}(x')]=0. \quad (23)$$

The energy of the meson waves is H_m.

$$H_m=\tfrac{1}{2}\int\left(4c^2\Pi^{(+)*}\Pi^{(+)}-\frac{\partial\phi^{(+)*}}{\partial x^a}\frac{\partial\phi^{(+)}}{\partial x_a}+K^2\phi^{(+)*}\phi^{(+)}\right)d_3x-\tfrac{1}{2}\int\left(4c^2\Pi^{(-)*}\Pi^{(-)}-\frac{\partial\phi^{(-)*}}{\partial x^a}\frac{\partial\phi^{(-)}}{\partial x_a}+K^2\phi^{(-)*}\phi^{(-)}\right)d_3x. \quad (24)$$

The interaction energy between the nucleons and the mesons is H'.

$$H'=-g_0\sum_j[\Gamma_j\{T_j\phi_{\mathrm{rad}}(x_j)+T_j^*\phi_{\mathrm{rad}}^*(x_j)\}]-if_0\sum_j\left[\Gamma_j\gamma_j^\mu\left\{T_j\frac{\partial}{\partial x_j^\mu}\phi_{\mathrm{rad}}(x_j)+T_j^*\frac{\partial}{\partial x_j^\mu}\phi_{\mathrm{rad}}^*(x_j)\right\}\right]. \quad (25)$$

In our formalism the contact forces between nucleons arising from the two fields $\phi^{(\pm)}$ cancel each other, and there is no ambiguity as in the one field theories (see Kemmer[13]). The total momentum of the meson field is $\mathbf{G}_m$

$$\mathbf{G}_m=-\int[\text{grad}\phi^{(+)*}\cdot\Pi^{(+)*}+\Pi^{(+)}\,\text{grad}\phi^{(+)}]d_3x-\int[\text{grad}\phi^{(-)*}\Pi^{(-)*}+\Pi^{(-)}\,\text{grad}\phi^{(-)}]d_3x. \quad (26)$$

The four vector of charge and current density of the meson field is S_m^{μ}.

$$S_m^{\mu}=\frac{ie}{2\hbar c}\left(\phi^{(+)*}\frac{\partial\phi^{(+)}}{\partial x_\mu}-\phi^{(+)}\frac{\partial\phi^{(+)*}}{\partial x_\mu}\right)-\frac{ie}{2\hbar c}\left(\phi^{(-)*}\frac{\partial\phi^{(-)}}{\partial x_\mu}-\phi^{(-)}\frac{\partial\phi^{(-)*}}{\partial x_\mu}\right)+\frac{e}{\hbar c}f_0\sum_j\Gamma_j\gamma_j^{\mu}\delta(\mathbf{x}-\mathbf{x}_j)\times[T_j\phi_{\text{rad}}(x)-T_j^*\phi_{\text{rad}}^*(x)]. \quad (27)$$

III. THE WENTZEL MESON FIELDS

In quantum theory it is necessary to introduce different time variables for the particles and the field, as it was shown in the case of the electromagnetic field by Dirac, Fock, and Podolsky.[14] We shall introduce only two time variables, x_m^0 and x_N^0, the time variables of the meson field and the nucleons, respectively. The field quantities $\phi_W^{(\pm)}$ which depend on the two time variables are the Wentzel meson fields; the $\phi_W^{(\pm)}$ go over into the $\phi^{(\pm)}$ by putting $x_m^0=x_N^0$. For the sake of simplicity we shall denote the time variable of the meson field by x_0 and the time of the whole system nucleons plus meson field by t. Let us put

$$H_p=\sum_j H_j+H'+H_{\text{int}}, \quad (28)$$

$$H_j=c(\boldsymbol{\alpha}_j\cdot\mathbf{p}_j)+\beta_j m_j c^2, \quad (29)$$

$$m_j=\tfrac{1}{2}[M_N(1+\tau_j^3)+M_P(1-\tau_j^3)]. \quad (30)$$

[13] N. Kemmer, Proc. Roy. Soc. A166, 127 (1938).
[14] P. A. M. Dirac, V. Fock, and B. Podolski, Physik. Zeits. Sowjetunion 2, 468 (1932).

$\mathbf{p}_j$ is the momentum of the jth particle, and τ_j^3 the third component of its isotopic spin. We assume that the isotopic spin component τ^3 has the value 1 in the neutron state and -1 in the proton state. H_{int} is the potential energy of the actions at a distance between the nucleons; it commutes with the meson field quantities. M_N and M_P are the rest masses of the neutron and the proton, respectively. We have the equations of motion of the Wentzel fields $\phi_W^{(\pm)}$:

$$i\hbar c\frac{\partial\phi_W^{(\pm)}}{\partial x^0}=[\phi_W^{(\pm)},H_m(x_0)], \quad (31)$$

$$i\hbar c\frac{\partial\phi_W^{(\pm)}}{\partial x_N^0}=[\phi_W^{(\pm)},H_p(x_N^0)]=[\phi_W^{(\pm)},H'(x_N^0)]=i\hbar c\left\{\sum_j\Gamma_jT_j^*\left[g_0-if_0\gamma_j^{\mu}\frac{\partial}{\partial x^{\mu}}\right]\times D(x,x_j)\right\}_{(x_j^0=x_N^0)}, \quad (32)$$

$$i\hbar c\frac{\partial\phi_{\text{rad},W}}{\partial x_N^0}=[\phi_{\text{rad},W},H_p(x_N^0)]=0, \quad (32a)$$

$$i\hbar c\frac{\partial^2\phi_W^{(\pm)}}{\partial x_0^2}=\left[\frac{\partial\phi_W^{(\pm)}}{\partial x_0},H_m\right]=-i\hbar c\left(\frac{\partial^2}{\partial x^a\partial x_a}+K^2\right)\phi_W^{(\pm)}. \quad (33)$$

From now on we shall denote the Wentzel fields by $\phi^{(\pm)}$. It results from Eq. (33) that we can expand the Wentzel fields in terms of free meson waves. We shall assume that the field is enclosed in a cubic box of volume V, and impose periodic conditions in order to get Fourier series

$$\phi^{(+)}=(\hbar c/V)^{\frac{1}{2}}\sum_k k_0^{-\frac{1}{2}}\{A_p^{(+)}(k)\exp[-ik^{\rho}x_{\rho}]+A_n^{(+)*}(k)\exp[ik^{\rho}x_{\rho}]\}. \quad (34)$$

$$\phi^{(-)}=(\hbar c/V)\sum_k k_0^{-\frac{1}{2}}\{A_n^{(-)}(k)\exp[-ik^{\rho}x_{\rho}]+A_p^{(-)*}(k)\exp[ik^{\rho}x_{\rho}]\}. \quad (35)$$

It results from (32a) that the radiation field does not depend on the time of the nucleons so that

$\phi_{\text{rad}, W} = \phi_{\text{rad}}$ and

$$A_{\text{rad}}^{p}(k) = \tfrac{1}{2}[A_p^{(+)}(k) - A_n^{(-)}(k)],$$
$$A_{\text{rad}}^{n}(k) = \tfrac{1}{2}[A_n^{(+)}(k) - A_p^{(-)}(k)]. \quad (36)$$

The only non-vanishing commutators are those of a Fourier coefficient and its conjugate

$$[A_p^{(+)}(k), A_p^{(+)*}(k')] = [A_n^{(+)}(k), A_n^{(+)*}(k')] = \delta_{kk'}, \quad (37)$$

$$[A_p^{(-)}(k), A_p^{(-)*}(k')] = [A_n^{(-)}(k), A_n^{(-)*}(k')] = -\delta_{kk'}. \quad (38)$$

Let us introduce the operators N which give the numbers of mesons

$$N_{p,n}^{(+)}(k) = A_{p,n}^{(+)*}(k) A_{p,n}^{(+)}(k),$$
$$N_{p,n}^{(-)}(k) = A_{p,n}^{(-)}(k) A_{p,n}^{(-)*}(k). \quad (39)$$

We have

$$H_m = hc \sum_k k_0[N_p^{(+)}(k) + N_n^{(+)}(k) - N_p^{(-)}(k) - N_n^{(-)}(k)]. \quad (40)$$

$$Q_m = \int S_0 d_3x = e \sum_k [N_p^{(+)}(k) - N_n^{(+)}(k) + N_p^{(-)}(k) - N_n^{(-)}(k)]. \quad (41)$$

Q_m is the total charge of the meson field. Equation (40) shows that the mesons of the field $\phi^{(+)}$ have positive energies and those of the field $\phi^{(-)}$ negative energies. Mesons with the same wave number vector $\mathbf{k}$ belonging to different fields have opposite momenta, since

$$\mathbf{G}_m = h \sum_k \mathbf{k}[N_p^{(+)}(k) + N_n^{(+)}(k) - N_p^{(-)}(k) - N_n^{(-)}(k)]. \quad (42)$$

IV. THE ACTIONS AT A DISTANCE BETWEEN THE NUCLEONS

In the preceding section we did not give the explicit form of the potential energy of the actions at a distance H_{int}. In our theory the nature of the actions at a distance is not determined by the choice of the meson field interacting with the nucleons. We shall investigate the actions at a distance which would result from theories of the Heisenberg-Pauli type corresponding to pseudoscalar and vector symmetrical fields (charged and neutral mesons).

In order to get the actions at a distance of the symmetrical pseudoscalar type let us consider the following equations

$$\left(\frac{\partial^2}{\partial x^\mu \partial x_\mu} + K_0^2\right)\phi_j^s = \tfrac{1}{2} g_0^\dagger \tau_j^s \Gamma_j \delta(\mathbf{x} - \mathbf{x}_j) - \frac{i}{2} f_0^\dagger \frac{\partial}{\partial x^\mu}[\Gamma_j \gamma_j^\mu \tau_j^s \delta(\mathbf{x} - \mathbf{x}_j)]; \quad (s = 1, 2, 3). \quad (43)$$

K_0 is a constant related to the range of the static forces; $g_0^\dagger$ and $f_0^\dagger$ are real interaction constants. The τ_j^s are the components of the isotopic spins of the jth nucleon. We shall consider the half-retarded, half-advanced solution of Eqs. (43):

$$\phi_j^s(x) = \tfrac{1}{2} g_0^\dagger \int_{-\infty}^{+\infty} \Gamma_j(x_j^0)\tau_j^s(x_j^0) \times G(x, x_j(x_j^0); K_0) dx_j^0 + \frac{i}{2} f_0^\dagger \int_{-\infty}^{+\infty} \Gamma_j(x_j^0)\gamma_j^\mu(x_j^0)\tau_j^s(x_j^0) \times \frac{\partial}{\partial x_j^\mu} G(x, x_j(x_j^0); K_0) dx_j^0. \quad (44)$$

$G(x, x'; K_0)$ is the Green function of the Klein-Gordon equation corresponding to the value K_0 of the constant. G is related to the function D corresponding to the same value of the constant K_0.[15,16]

$$G(x, x'; K_0) = \tfrac{1}{2}\,\text{sign}(x_0 - x_0') D(x, x')_{(K=K_0)}. \quad (45)$$

We shall use the expression of G as a Fourier integral

$$G(x, x'; K_0) = \frac{1}{(2\pi)^4} \int_{-\infty}^{+\infty} \exp[iu^\rho(x_\rho - x_\rho')] \frac{d_4u}{\omega_0^2 - u_0^2}, \quad (46)$$

$$\omega_0 = (K_0^2 + |\mathbf{u}|^2)^{\frac{1}{2}}. \quad (47)$$

Let χ be any function of the type of the wave function of the system in consideration. We

[15] M. Schönberg, Physica, 5, 553, 961 (1938).
[16] M. Schönberg, Rev. Union Matem. Argentina 12, 238 (1947).

shall take

$$G(x, x_j(x_j^0); K_0)\chi = \frac{1}{(2\pi)^4}\int_{-\infty}^{+\infty} \exp[iu^\rho(x_\rho - x_{j,\rho}(x_j^0))]\chi \frac{d_4u}{\omega_0^2 - u_0^2}, \tag{48}$$

$$\frac{\partial}{\partial x_j^\mu} G(x, x_j(x_j^0); K_0)\chi = -\frac{i}{(2\pi)^4}\int_{-\infty}^{+\infty} \exp[iu^\rho(x_\rho - x_{j,\rho}(x_j^0))]\chi \frac{u_\mu d_4u}{\omega_0^2 - u_0^2}. \tag{49}$$

In a similar way we can define the operators $G(x_j(x_j^0), x_l(x_l^0); K_0)$ and $(\partial/\partial x_l^\mu)G(x_j(x_j^0), x_l(x_l^0); K_0)$, which we need in order to compute the operators $\phi_l^s(x_j)$ by means of (44).

The potential energy of the actions at a distance of the symmetrical pseudoscalar type is $H_{\text{int}}^{(0)}$.

$$H_{\text{int}}^{(0)}(x_0) = -\tfrac{1}{2}g_0^\dagger \sum_j \sum_{l\neq j} \sum_{s=1}^{3} [\Gamma_j(x_0)\tau_j^s(x_0)\phi_l^s(x_j)] - \frac{i}{2}f_0^\dagger \sum_j \sum_{l\neq j} \sum_{s=1}^{3} \left[\Gamma_j(x_0)\gamma_j^\mu(x_0)\tau_j^s(x_0)\frac{\partial}{\partial x_j^\mu}\phi_l^s(x_j)\right]. \tag{50}$$

In order to compute the potential energy of the actions at a distance of the symmetrical vector type let us consider the real four vectors $\phi_j^{(s)\mu}$ which are half-retarded, half-advanced solutions of the equations

$$\left(\frac{\partial^2}{\partial x^\mu \partial x_\mu} + K_1^2\right)\phi_j^{(s)\mu} = \tfrac{1}{2}g_1^\dagger \alpha_j^\mu \delta(\mathbf{x} - \mathbf{x}_j) + \frac{i}{2}f_1^\dagger \sum_{\nu\neq\mu} \frac{\partial}{\partial x^\nu}[\alpha_j^\nu \gamma_j^\mu \tau_j^s \delta(\mathbf{x} - \mathbf{x}_j)], \tag{51}$$

$$\phi_j^{(s)\mu}(x) = \tfrac{1}{2}g_1^\dagger \int_{-\infty}^{+\infty} \alpha_j^\mu(x_j^0)\tau_j^s(x_j^0) G(x, x_j(x_j^0); K_1)dx_j^0 - \frac{i}{2}f_1^\dagger \sum_{\nu\neq\mu} \int_{-\infty}^{+\infty} \alpha_j^\nu(x_j^0)\gamma_j^\mu(x_j^0)\tau_j^s(x_j^0)\frac{\partial}{\partial x_j^\nu} G(x, x_j(x_j^0); K_1)dx_j^0. \tag{52}$$

$g_1^\dagger$ and $f_1^\dagger$ are interaction constants. K_1 is the range constant.

The potential energy of the actions at a distance of the symmetrical vector type is $H_{\text{int}}^{(1)}$.

$$H_{\text{int}}^{(1)}(x_0) = -\tfrac{1}{2}g_1^\dagger \sum_j \sum_{l\neq j} \sum_{s=1}^{3} \tau_j^s(x_0)\alpha_{j,\mu}(x_0)\phi_l^{(s)\mu}(x_j) - \frac{i}{4}f_1^\dagger \sum_j \sum_{l\neq j} \sum_{s=1}^{3} \tau_j^s(x_0)\beta_j(x_0)\gamma_j^\mu(x_0)\gamma_j^\nu(x_0)\phi_{l,\mu\nu}^{(s)}(x_j), \tag{53}$$

$$\phi_l^{(s)\mu\nu}(x) = \frac{\partial}{\partial x_\mu}\phi_l^{(s)\nu}(x) - \frac{\partial}{\partial x_\nu}\phi_l^{(s)\mu}(x). \tag{54}$$

V. STATIC FORCES BETWEEN NUCLEONS

In order to get the static forces between nucleons we assume in (44) and (52) that the variables of the nucleons are constant. Since

$$\int_{-\infty}^{+\infty} G(x, x'; K_0)dx_0' = \frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} \exp[i(\mathbf{u}\cdot\{\mathbf{x} - \mathbf{x}'\})]\frac{d_3u}{\omega_0^2} = \frac{1}{4\pi}|\mathbf{x} - \mathbf{x}'|^{-1}\exp(-K_0|\mathbf{x} - \mathbf{x}'|) \tag{55}$$

and

$$\int_{-\infty}^{+\infty} \frac{\partial}{\partial x'^0} G(x, x'; K_0)dx_0' = 0, \tag{56}$$

we have in the static case

$$\phi_j^s(x_l) = \frac{g_0^\dagger}{8\pi}\tau_j^s\Gamma_j R_{jl}^{-1}\exp(-K_0R_{jl}) - \frac{f_0^\dagger}{8\pi}\tau_j^s(\boldsymbol{\sigma}_j\cdot\text{grad}_j)R_{jl}^{-1}\exp(-K_0R_{jl}), \tag{57}$$

$$R_{jl} = |\mathbf{x}_j - \mathbf{x}_l|. \tag{58}$$

$\boldsymbol{\sigma}_j$ denotes, as usual, the spin of the jth particle divided by $\frac{1}{2}\hbar$. We have also

$$\phi_j^{(s)\mu}(x_l)=\frac{g_1^\dagger}{8\pi}\tau_j^s\alpha_j^\mu R_{jl}^{-1}\exp(-K_1R_{jl})$$

$$-\frac{if_1^\dagger}{8\pi}\tau_j^s\sum_{\tau\neq\mu}\alpha_j^\nu\gamma_j^\mu\frac{\partial}{\partial x_j^\nu}[R_{jl}^{-1}\exp(-K_1R_{jl})]. \quad (59)$$

By neglecting all the relativistic corrections we get

$$\phi_j^s(x_l)_{st}=\frac{f_0^\dagger}{8\pi}\tau_j^s(\boldsymbol{\sigma}_j\cdot\mathrm{grad}_j)$$

$$\times[R_{jl}^{-1}\exp(-K_0R_{jl})], \quad (60)$$

$$\phi_j^{(s)0}(x_l)_{st}=\frac{g_1^\dagger}{8\pi}\tau_j^sR_{jl}^{-1}\exp(-K_1R_{jl}), \quad (61)$$

$$\boldsymbol{\phi}_j^{(s)}(x_l)_{st}=-\frac{f_1^\dagger}{8\pi}\tau_j^s[\boldsymbol{\sigma}_j\times\mathrm{grad}_j]$$

$$\times[R_{jl}^{-1}\exp(-K_1R_{jl})]. \quad (62)$$

By introducing the preceding expressions of the ϕ in Eqs. (50) and (53), we get the usual expressions for the static interactions.

We shall now derive explicit expressions of the ϕ_j^s which take into account the retardation effects in a first approximation. In the Heisenberg picture the operators which represent a non-variable physical quantity at two different times are related by a contact transformation

$$B(x_0')=U^{-1}(x_0',x_0)B(x_0)U(x_0',x_0). \quad (63)$$

U is an unitary operator related to the Hamiltonian. When the Hamiltonian does not depend explicitly on the time,

$$U(x_0',x_0)=\exp\left[-\frac{i}{\hbar c}H(x_0'-x_0)\right]. \quad (64)$$

By taking into account (63), we may write

$$\phi_j^s(x)=\tfrac{1}{2}g_0^\dagger\int_{-\infty}^{+\infty}U^{-1}(x_j^0,0)[\tau_j^s(0)\Gamma_j(0)$$

$$\times G(x,x_j(0);K_0)_{(x_j^0(0)=x_j^0)}]U(x_j^0,0)dx_j^0$$

$$+\frac{i}{2}f_0^\dagger\int_{-\infty}^{+\infty}U^{-1}(x_j^0,0)\bigg[\tau_j^s(x_j^0)\Gamma_j(x_j^0)$$

$$\times\gamma_j^\mu(x_j^0)\frac{\partial}{\partial x_j^\mu}G(x,x_j(0);K_0)\bigg]$$

$$\times U(x_j^0,0)dx_j^0. \quad (65)$$

In our approximation treatment we shall use the operator U corresponding to an unperturbed motion in which the nucleons do not interact with the meson fields and the actions at a distance are neglected. We shall denote the operator U of the unperturbed motion by U_0:

$$U_0(x_0',x_0)=\exp\left[-\frac{i}{\hbar c}H_m(x_0'-x_0)\right]$$

$$\times\exp\left[-\frac{i}{\hbar c}\{\textstyle\sum_j H_j\}(x_0'-x_0)\right], \quad (66)$$

so that

$$\phi_j^s(x)=\tfrac{1}{2}g_0^\dagger\int_{-\infty}^{+\infty}\exp\left[\frac{i}{\hbar c}H_jx_j^0\right]$$

$$\times\{\tau_j^s(0)\Gamma_j(0)G(x,x_j(0);K_0)\}$$

$$\times\exp\left[-\frac{i}{\hbar c}H_jx_j^0\right]dx_j^0+\frac{i}{2}f_0^\dagger\int_{-\infty}^{+\infty}$$

$$\times\exp\left[\frac{i}{\hbar c}H_jx_j^0\right]\bigg\{\tau_j^s(0)\Gamma_j(0)\gamma_j^\mu(0)\frac{\partial}{\partial x_j^\mu}$$

$$\times G(x,x_j(0);K_0)\bigg\}\exp\left[-\frac{i}{\hbar c}H_jx_j^0\right]dx_j^0. \quad (67)$$

The operator which represents a physical quantity in the Schrödinger picture is related to the corresponding operator in the Heisenberg picture

$$B(x_0)_{\mathrm{Schröd}}=U(x_0,0)B(x_0)U^{-1}(x_0,0). \quad (68)$$

In our approximation

$$\phi_j{}^s(x)_{\text{Schröd}} = \exp\left[-\frac{i}{\hbar c}H_j x_0\right]\phi_j{}^s(x)\exp\left[\frac{i}{\hbar c}H_j x_0\right]$$

$$= \tfrac{1}{2}g_0{}^\dagger \int_{-\infty}^{+\infty} \exp\left[\frac{i}{\hbar c}H_j z_j{}^0\right]\{\tau_j{}^s\Gamma_j G(x, x_j; K_0)\}_{\text{Schröd}} \exp\left[-\frac{i}{\hbar c}H_j z_j{}^0\right] dz_j{}^0$$

$$+\frac{i}{2}f_0{}^\dagger \int_{-\infty}^{+\infty} \exp\left[\frac{i}{\hbar c}H_j z_j{}^0\right]\left\{\tau_j{}^s\Gamma_j\gamma_j{}^\mu \frac{\partial}{\partial x_j{}^\mu}G(x, x_j; K_0)\right\}_{\text{Schröd}} \exp\left[-\frac{i}{\hbar c}H_j z_j{}^0\right] dz_j{}^0, \quad (69)$$

$$z_j{}^0 = x_j{}^0 - x^0. \quad (70)$$

It is worth while to remark that $\phi_j{}^s(x)_{\text{Schröd}}$ does not depend on x_0, because $G(x, x'; K_0)$ is a function of $x_0 - x_0'$. The matrix element $(I|H_{\text{int}}{}^{(0)}|II)_{\text{Schröd}}$ of $H_{\text{int}}{}^{(0)}$ between two states in which the nucleons are free and have the momenta $(\mathbf{p}_j{}^I, \mathbf{p}_j{}^{II})$ and the energies $(cp_j{}^{I0}, cp_j{}^{II0})$ can be easily computed by using (69).

$$(I|H_{\text{int}}{}^{(0)}|II)_{\text{Schröd}} = \tfrac{1}{4}h^{3(n-1)} \sum_j \sum_{l\neq j} \sum_s \delta(\mathbf{p}_j{}^I - \mathbf{p}_j{}^{II} + \mathbf{p}_l{}^I - \mathbf{p}_l{}^{II}) \prod_{r\neq l, j} \delta(\mathbf{p}_r{}^I - \mathbf{p}_r{}^{II})$$

$$\times\{\hbar^{-2}(p_l{}^{I\rho} - p_l{}^{II\rho})(p_{l,\rho}{}^I - p_{l,\rho}{}^{II}) - K_0{}^2\}^{-1}[(g_0{}^\dagger)^2(I|\tau_j{}^s\tau_l{}^s\Gamma_j\Gamma_l|II)$$

$$-(f_0{}^\dagger)^2\hbar^{-2}(I|\tau_j{}^s\tau_l{}^s\Gamma_j\gamma_j{}^\mu\Gamma_l\gamma_l{}^\mu|II)(p_{l,\mu}{}^I - p_{l,\mu}{}^{II})(p_{l,\nu}{}^I - p_{l,\nu}{}^{II})$$

$$+g_0{}^\dagger f_0{}^\dagger\hbar^{-1}(p_{l,\mu}{}^I - p_{l,\mu}{}^{II})(I|\tau_j{}^s\tau_l{}^s\Gamma_j\Gamma_l(\gamma_l{}^\mu - \gamma_j{}^\mu)|II)]. \quad (71)$$

This expression of the matrix element of the interaction corresponds to Möller's formula[17] in the electromagnetic case. Leite Lopes[8] and Tamm[9] have derived expressions of the matrix element of the relativistic interactions between nucleons analogous to (71), by using the usual field theories of the nuclear forces. The nature of the retardation effects does not appear clearly in their derivations because they come in through the recoil of the nucleons in the virtual emissions and absorptions of mesons.

VI. DISCUSSION

In the preceding section we derived approximate expressions for the relativistic interactions, of the same type as those derived from the field theory of the nuclear forces. Nevertheless our formalism and the field theory do not lead to the same description of the nuclear forces. We obtained linear operators for the relativistic interactions analogous to those of Leite Lopes and Tamm, in an approximation treatment. *But, in an exact treatment of the relativistic interactions, the fundamental equations we must solve are not linear.*

The fundamental equations of the exact theory are those which determine the operator U:

$$i\hbar c\frac{\partial}{\partial x_0}U(x_0, x_0') = U(x_0, x_0')H(x_0), \quad (72)$$

$$U(x_0, x_0) = 1, \quad (73)$$

because we cannot use the Schrödinger equation unless we know beforehand the explicit expression of H_{int}, and we cannot compute H_{int} without knowing U.

We do not know whether the exact expression of $[H_{\text{int}}]_{\text{Schröd}}$ is time independent, as it is in the approximation considered at the end of the preceding section. Thus it is not certain whether it is possible to obtain stationary states in the exact treatment, at least with the usual definition of a stationary state. It may be necessary to use indirect methods, such as that of the Heisenberg S-matrix, in order to get the energy levels of the closed stationary states.

The approximation method we have used to investigate the retardation effects differs essen-

[17] C. Möller, Zeits. f. Physik 70, 786 (1931).

tially from the perturbation method. That method may be considered satisfactory in the case of collisions of high energy nucleons, but it is not certain whether it describes correctly the retardation effects in the case of the nuclei, because of the small energy of the nucleons. It is important to take into account the preceding remark because of the difficulties which exist in the relativistic treatment of the nuclear forces; in this case we ought rather to start from an unperturbed motion in which the retardation effects are neglected and use its operator U in the computation of the first approximation values of the $\phi_j{}^s$ and $\phi_j{}^{(s)\mu}$, instead of the operator U_0 which corresponds to non-interacting nucleons.

We have developed the formalism of the actions at a distance between nucleons by analogy with the corresponding formalism for the electromagnetic case (references 1 and 2). We have replaced the inhomogeneous d'Alembert equation by the inhomogeneous Klein-Gordon equations (43) and (51) and, by taking their half-retarded, half-advanced solutions, we obtained actions at a distance which coincide in a first approximation with those given by the field theory. It is important to notice that the method of the actions at a distance allows many other possibilities. For instance, we may replace the Green function $G(x, x'; K_0)$ by some other invariant function in Eqs. (44) and (52). By a convenient choice of that function we can give a finite radius to the nucleons, from the point of view of the actions at a distance, without changing their point interaction with the meson field or the electromagnetic field. We shall now examine in some detail the introduction of such a radius. Let us replace the Green function G by the function G_Λ:

$$G_\Lambda(x, x'; K_0) = \frac{1}{4\pi} F_\Lambda'(s^2) J_0(K_0 s) - \frac{K_0}{8\pi s} F_\Lambda(s^2) J_1(K_0 s), \quad (74)$$

$$s = [(x^\mu - x'^\mu)(x_\mu - x_\mu')]^{\frac{1}{2}}, \quad (75)$$

$$F_\Lambda(s^2) = \frac{1}{\Lambda^2 \pi^{\frac{1}{2}}} \int_{-\infty}^{s^2} \exp\left(-\frac{t^2}{\Lambda^4}\right) dt. \quad (76)$$

J_0 and J_1 are Bessel functions, Λ is a constant with the dimension of a length. We have

$$\lim_{\Lambda \to 0} G_\Lambda(x, x'; K_0) = G(x, x'; K_0). \quad (77)$$

$G_\Lambda(x, x'; K_0)$ depends only on the variable s^2 and is free of singularities. In the computation of the static forces G_Λ can be replaced by $\mathbf{G}_\Lambda$.

$$\mathbf{G}_\Lambda(x, x'; K_0) = \delta(x_0 - x_0') K_\Lambda(\mathbf{x}, \mathbf{x}'; K_0), \quad (78)$$

$$K_\Lambda(\mathbf{x}, \mathbf{x}'; K_0) = \int_{-\infty}^{+\infty} G_\Lambda(x, x'; K_0) dx_0. \quad (79)$$

$K_\Lambda(\mathbf{x}, \mathbf{x}', K_0)$ and its derivatives of the first and second order are finite for $\mathbf{x} = \mathbf{x}'$. Thus we can get rid of the singularities in the interaction between nucleons.

VII. THE "FREEZING" OF THE DEGREES OF FREEDOM OF THE MESON FIELD

Let us introduce new variables for the description of the meson field:

$$P_{p,n}{}^2(k) + iP_{p,n}{}^1(k) = [A_{p,n}{}^{(+)}(k) + A_{n,p}{}^{(-)}(k)], \quad (80)$$

$$P_{p,n}{}^2(k) - iP_{p,n}{}^1(k) = [A_{p,n}{}^{(+)*}(k) + A_{n,p}{}^{(-)*}(k)], \quad (81)$$

$$Q_{p,n}{}^1(k) - iQ_{p,n}{}^2(k) = [A_{p,n}{}^{(+)}(k) - A_{n,p}{}^{(-)}(k)], \quad (82)$$

$$Q_{p,n}{}^1(k) + iQ_{p,n}{}^2(k) = [A_{p,n}{}^{(+)*}(k) - A_{n,p}{}^{(-)*}(k)]. \quad (83)$$

The only non-vanishing commutators between the new variables are those between a P and the corresponding Q:

$$[P_p{}^r(k), Q_p{}^{r'}(k')] = [P_n{}^r(k), Q_n{}^{r'}(k')] = -i\delta_{rr'}\delta_{kk'}. \quad (84)$$

We have

$$N_{p,n}{}^{(+)}(k) - N_{n,p}{}^{(-)}(k) = Q_{p,n}{}^1(k) P_{p,n}{}^2(k) - Q_{p,n}{}^2(k) P_{p,n}{}^1(k). \quad (85)$$

Let us consider the Fourier expansions of the fields $\phi_{\text{rad}}{}^{p,\,n}$.

$$\phi_{\text{rad}}{}^{p,\,n}(x) = (hc/V)^{\frac{1}{2}} \sum_k k_0{}^{-\frac{1}{2}} A_{\text{rad}}{}^{p,\,n}(k) \times \exp(-ik^\nu x_\nu). \quad (86)$$

It results from (82) and (83) that

$$A_{\text{rad}}{}^p(k) = \tfrac{1}{2}[Q_p{}^1(k) - iQ_p{}^2(k)], \quad (87)$$

$$A_{\text{rad}}{}^n(k) = \tfrac{1}{2}[Q_n{}^1(k) - iQ_n{}^2(k)]. \quad (88)$$

Since the only fields that come in the interaction between nucleons and mesons are the $\phi_{\text{rad}}{}^{p,\,n}$, it follows that the interaction can be expressed in terms of the field variables Q.

The wave function χ of the system nucleons plus field is a function of the variables of the nucleons and variables of the field which we may take as the P. If χ does not depend on the P corresponding to a given wave vector $\mathbf{k}$, we have

$$[N_{p,\,n}{}^{(+)}(k) - N_{n,\,p}{}^{(-)}(k)]\chi = 0, \quad (89)$$

$$A_{\text{rad}}{}^{p,\,n}(k)\chi = 0, \quad A_{\text{rad}}{}^{p,\,n*}(k)\chi = 0. \quad (90)$$

Therefore:

In the state of the system represented by the wave function we consider those degrees of freedom of the field do not give any contribution to the energy of the field and to the interaction energy between the nucleons and the field.

We shall call "frozen" degrees of freedom those which correspond to values of $\mathbf{k}$ whose P are not involved in the wave function of the state in considerations. *By using the "freezing" method we can get rid of the divergences.*

The simplest application of the "freezing" is the elimination of the infinite self-energies. *If we assume that the wave function of the system free nucleon plus meson field is described, in the rest system of the nucleon, by a wave function in which all the degrees of freedom corresponding to values of $|\mathbf{k}|$ above a given limit are "frozen", we will get a finite self-energy of the nucleon.*

It is important to remark that there are exact solutions of the Schrödinger equation of the system nucleons plus field in which the degrees of freedom of the field corresponding to any given set of vectors $\mathbf{k}$ *are "frozen."* Indeed the freezing conditions

$$Q_{p,\,n}{}^r(k)\chi = 0 \quad (91)$$

will be fulfilled at any time if they are satisfied at the initial time.

It is possible to introduce a more general kind of "freezing" which we shall need later. The freezing conditions (91) are equivalent to (90) which can be split into two groups of conditions

$$A_{\text{rad}}{}^p(k)\chi = A_{\text{rad}}{}^{p*}(k)\chi = 0, \quad (92a)$$

$$A_{\text{rad}}{}^n(k)\chi = A_{\text{rad}}{}^{n*}(k)\chi = 0. \quad (92b)$$

The conditions (92a) and (92b) "freeze" the interaction between the nucleons and the degrees of freedom corresponding to the wave vector $\mathbf{k}$ in the two radiation fields $\phi_{\text{rad}}{}^p$ and $\phi_{\text{rad}}{}^n$, respectively. We can associate with the conditions (92a) and (92b) the following ones, respectively,

$$[N_p{}^{(+)}(k) - N_n{}^{(-)}(k)]\chi = 0, \quad (93a)$$

$$[N_n{}^{(+)}(k) - N_p{}^{(-)}(k)]\chi = 0, \quad (93b)$$

which will be satisfied as a consequence of conditions (92a) and (92b). Therefore, if we impose the conditions (92a)–(93a) the degrees of freedom of the field $\phi_{\text{rad}}{}^p$ will not give any contribution to the energy of the system nucleons plus total meson field in the state in consideration. It is therefore possible to "freeze" separately the degrees of freedom of the two partial radiation fields $\phi_{\text{rad}}{}^p$ and $\phi_{\text{rad}}{}^n$. The possibility of "freezing" separately degrees of freedom of the two radiation fields is not necessary in order to eliminate the divergences, but it may play an important part in the explanation of the anomaly of the magnetic moments of the proton and neutron as well as in the explanation of the difference of their masses.

VIII. THE PHYSICAL INTERPRETATION RULE

Let us assume that we want to compute the probability of a transition in which are involved l_p positive mesons and l_n negative mesons with the momenta $\hbar\mathbf{k}_r$ $(r=1,\ 2,\ \cdots l_p)$ and $\hbar\mathbf{k}_s$ $(s=1,\ 2,\ \cdots l_n)$, respectively. We shall use a wave function χ_{fr} of the system particles plus field involving only the P_p corresponding to the $\mathbf{k}_r$ in the ranges $(\mathbf{k}_r - \frac{1}{2}\Delta\mathbf{k},\ \mathbf{k}_r + \frac{1}{2}\Delta\mathbf{k})$ and the P_n corresponding to the $\mathbf{k}$ in the ranges $(\mathbf{k}_s - \frac{1}{2}\Delta\mathbf{k},\ \mathbf{k}_s + \frac{1}{2}\Delta\mathbf{k})$ and $\Delta\mathbf{k}$ is a small vector with a length of the order of $V^{-\frac{1}{3}}$. We can satisfy the Schrödinger equation of the system nucleons plus meson field with such a wave function because the terms of the Hamiltonian of the system, which involve field variables corresponding to **k**'s outside the relevant ranges, give vanishing contributions when applied to our "frozen" wave function. The "frozen" wave function χ_{fr} satisfies a reduced Schrödinger equation which results from the exact one by dropping all the terms involving the frozen degrees of freedom. We shall now make an assumption:

The reduced Schrödinger equation must be treated in the same way as the usual Schrödinger equation of a dynamical system with a finite number of degrees of freedom, the degrees of freedom of the nucleons, and the meson field which it involves.

It results from this general assumption that the wave function χ_{fr} must be normalized as the wave function of a system with a finite number of degrees of freedom. It would not be possible to normalize χ_{fr}, if we would consider it as the wave function of the nucleons plus the "unfrozen" meson field. In order to formulate the rule of physical interpretation we shall take as variables, in the reduced Schrödinger equation, the numbers of mesons $N_{p,n}^{(\pm)}(k)$. The initial state of the observable meson fields is characterized by the occupation numbers $\mathfrak{N}_p(k_r)_I$ and $\mathfrak{N}_n(k_s)_I$; the final state of the observable meson field is characterized by the occupation numbers $\mathfrak{N}_p(k_r)_F$ and $\mathfrak{N}_n(k_s)_F$. We shall associate with these observable states the states of our formalism characterized by the occupation numbers $N_p^{(+)}(k_r)_I$, $N_n^{(-)}(k_r)_I$, $N_n^{(+)}(k_s)_I$, $N_p^{(-)}(k_s)_I$ (initial state) and $N_p^{(+)}(k_r)_F$, $N_n^{(-)}(k_r)_F$, $N_n^{(+)}(k_s)_F$, $N_p^{(-)}(k_s)_F$ (final state).

$$N_p^{(+)}(k_r)_I=\mathfrak{N}_p(k_r)_I,\quad N_n^{(+)}(k_s)_I=\mathfrak{N}_n(k_s)_I,\quad N_n^{(-)}(k_r)_I=N_p^{(-)}(k_s)_I=0, \tag{94a}$$

$$N_p^{(+)}(k_r)_F=\mathfrak{N}_p(k_r)_F,\quad N_n^{(+)}(k_s)_F=\mathfrak{N}_n(k_s)_F,\quad N_n^{(-)}(k_n)_F=N_p^{(-)}(k_s)_F=0. \tag{94b}$$

Let us denote by Δ the total number of absorbed and emitted mesons in the physical world (observed number). We shall assume the following rule of physical interpretation:

The probability of the transition $\mathfrak{N}_I\rightarrow\mathfrak{N}_F$, in the physical world, is equal to the product of 2^Δ by the probability of the associated transition computed with the reduced Schrödinger equation.

This interpretation rule is similar to the rule introduced in the electromagnetic case by Schönberg,[1,2] it is also similar to one of the interpretation rules given by Pauli[12] in his extension of Dirac's method of quantization[10] to the meson field. There are other possible interpretation rules, similar to those considered by Pauli, but they are more complicated than the one we have formulated. Moreover, Schönberg has shown that the corresponding interpretation rule for the electromagnetic case can be justified by considerations of correspondence with the classical theory; a similar justification could also be given if we would develop a formalism analogous to the one given in this paper for pseudoscalar neutral mesons, by considering the classical theory of point nucleons interacting with a pseudoscalar neutral meson field. The interpretation rules which do not use the negative energy particles, in our type of quantum formalism, correspond to the time boundary conditions of the classical theory of point particles which exclude the indefinitely self-accelerated motions of the particles.[5]

IX. SELF-ENERGIES AND MAGNETIC MOMENTS

The application of the "freezing" method to the computation of the self-energies and magnetic moments is not so straightforward as its application to the computation of transition probabilities. We shall introduce the following "freezing" criterion:

In the computation of the self-energies, magnetic moments, and other similar effects in which a nucleon interacts with the meson field in vacuum, we shall use wave functions of the system nucleon plus meson field in which the degrees of freedom of the two radiation fields, ϕ_{rad}^p and ϕ_{rad}^n, corresponding to momenta of the mesons above hK_p and hK_n, respectively, are "frozen," in the rest system of the nucleon. K_p and K_n are two constants of the order of K.

The quantities we are considering can be computed as average values of operators O of the form $O^{(+)}+O^{(-)}$, $O^{(+)}$ involving only the field $\phi^{(+)}$, and $O^{(-)}$ involving only the field $\phi^{(-)}$. In the case of the self-energy the operator O is

$$\tfrac{1}{2}H' = \tfrac{1}{2}(H'^{(+)} + H'^{(-)}).$$

$$H'^{(\pm)} = \mp\tfrac{1}{2}g_0\Gamma\{T\phi^{(\pm)}(x_N) + T^*\phi^{(\pm)*}(x_N)\} \mp \frac{i}{2}f_0\Gamma\gamma^\mu\left\{T\frac{\partial}{\partial x_N{}^\mu}\phi^{(\pm)}(x_N) + T^*\frac{\partial}{\partial x_N{}^\mu}\phi^{(\pm)*}(x_N)\right\}. \quad (95)$$

The x_N are the coordinates of the nucleon in consideration. In the case of the magnetic moments the operators $O^{(\pm)}$ are the $M_{\mu\nu}{}^{(\pm)}$.

$$M_{\mu\nu}{}^{(\pm)} = \tfrac{1}{2}\int (x_\mu S_\nu{}^{(\pm)} - x_\nu S_\mu{}^{(\pm)})d_3x, \quad (96)$$

$$S_\mu{}^{(\pm)}(x) = \pm\frac{ie}{2\hbar c}\left(\phi^{(\pm)*}(x)\frac{\partial\phi^{(\pm)}(x)}{\partial x^\mu} - \phi^{(\pm)}(x)\frac{\partial\phi^{(\pm)*}(x)}{\partial x^\mu}\right) \pm\frac{ef_0}{2\hbar c}\Gamma\gamma^\mu\delta(\mathbf{x}-\mathbf{x}_N) \times\{T\phi^{(\pm)}(x) - T^*\phi^{(\pm)*}(x)\}. \quad (97)$$

Besides the preceding "freezing" rule, we shall use the following computation rule which is obviously related to the physical interpretation rule of the preceding section:

In the computation of quantities such as the self-energies and magnetic moments of the nucleons, we must take the average of $2O^{(+)}$ *instead of the average of* $O^{(+)}+O^{(-)}$, *in the state of the system nucleon plus "frozen" meson field in which there are no mesons in the "unfrozen" states.*

Thus the self-energies are given by the following formula

$$\Delta E = (H'^{(+)})_{\mathrm{Av}}, \quad (100)$$

and the corrections of the magnetic moments by a formula analogous to that given by Jauch[18] in the one-field theory

$$\Delta\mu_3 = 2(M_{12}{}^{(+)})_{\mathrm{Av}}, \quad (101)$$

assuming that the spin of the nucleon is directed along the x^3 axis. For the sake of simplicity we shall put $g_0=0$ and neglect the recoil of the nucleons. Thus we get

$$(\Delta M)_P = -\frac{f_0{}^2}{4\pi^2c^2}\int_0^{K_P}\left(\frac{k}{k_0}\right)^2 k^2dk, \quad (102)$$

$$(\Delta M)_N = -\frac{f_0{}^2}{4\pi^2c^2}\int_0^{K_N}\left(\frac{k}{k_0}\right)^2 k^2dk, \quad (103)$$

$$(\Delta\mu_3)_P = \frac{ef_0{}^2}{12\pi^2\hbar c}\int_0^{K_P}\left(\frac{k}{k_0}\right)^4 dk, \quad (104)$$

$$(\Delta\mu_3)_N = -\frac{ef_0{}^2}{12\pi^2\hbar c}\int_0^{K_N}\left(\frac{k}{k_0}\right)^4 dk. \quad (105)$$

The existence of the two "freezing" limits, K_P *and* K_N *allows us to get different self-energies for the proton and the neutron.*

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[18] J. M. Jauch, Phys. Rev. **63**, 334 (1943).

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