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A Note on the Development of the Nucleon Component of the Cosmic Radiation in Air When Ionization Losses Are Accounted For

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The development of the nucleon component of the cosmic radiation in air is considered in terms of a model previously discussed by Messel and Ritson, with the inclusion of ionization losses. The results obtained are compared with experiment, and good qualitative agreement is found. The good qualitative agreement found strongly suggests that the Heitler-Jánossy assumptions regarding the nucleon-nucleon collision cross section are valid. But from a quantitative standpoint it appears that a number of points require revision. There is little doubt that the development of the nucleonic component can in fact be well accounted for in in terms of a cascade process. Results are presented in graphic form for neutrons, protons, neutrons plus protons and their ratio.

I. INTRODUCTION

RECENTLY Messel and Ritson[1] discussed the development of the nucleonic component of the cosmic radiation in air using a model which was formulated by Messel[2] and which had been suggested by the work of Heitler and Jánossy.[3] The results obtained by Messel and Ritson were in fair agreement with those found experimentally even though ionization loss by the proton component was only roughly accounted for.

In the above work, since the ionization loss was largely neglected, only the total nucleonic component was considered and no differentiation made between protons and neutrons. Lately, however, the theory of the nucleon cascade has been treated more exhaustively by Messel.[4] Ionization loss was taken into consideration, protons distinguished from neutrons, and a solution satisfying the correct boundary conditions given which should be valid down to energies at which little is known regarding the cross section for nucleon-nucleon collisions. This work has verified that the effect of ionization loss at large depths is by no means negligible, and that its neglect does in fact lead to serious errors in any problem dealing with energies of the order of the ionization loss.

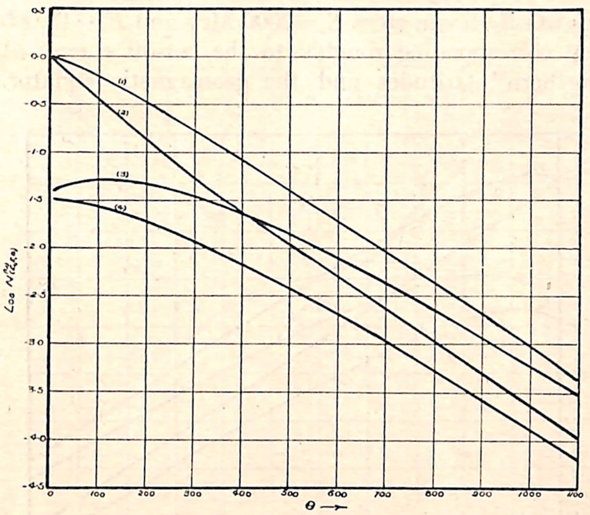

Fig. 1. Logarithm of the vertical intensity of protons with energies greater than E at an atmospheric depth of θ g/cm² due to an incident porton power law spectrum given by (4). In curve (1), $E=266$ Mev, $E_c=2000$ Mev, the cut-off energy corresponding to northern latitudes; curve (2), $E=2000$ Mev, $E_c=2000$ Mev; curve (3), $E=266$ Mev, $E_c=15{,}000$ Mev, the cut-off energy corresponding to the geomagnetic equator; curve (4), $E=2000$ Mev, $E_c=15{,}000$ Mev. The results have been normalized to an intensity of "one" proton at northern latitudes and $(2/15)^{1.7}$ protons at the geomagnetic equator.

There has also appeared lately[5–7] an overwhelming amount of experimental evidence supporting the hypothesis that the development of the nucleon component of the cosmic radiation must be described in terms of a cascade process. In view of the above, we have felt justified in investigating the application of the same model as previously[1] to the interpretation of experimental results on the nucleon component, but in this instance we have considered the effect of ionization loss and distinguished protons from neutrons.

The results given below are obtained using the Heitler-Jánossy assumptions regarding the cross section for nucleon-nucleon collisions. It may be countered that their assumptions are open to serious objections, that the form of the cross section obtained from their theory is not even approximately correct. We believe the answer to the above lies in the results obtained below. These results do show that the theory using the Heitler-Jánossy form of cross section can account, from a qualitative point, for the observed experimental results on cosmic-ray nucleons. That complete quantitative agreement is not found does not prove that the assumptions with regard to the nucleon-nucleon cross section are wrong, but may only point to the fact that the actual values of constants appearing in the cross section are not correct.

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[1] H. Messel and D. M. Ritson, Proc. Phys. Soc. (London) **A63**, 1359 (1950).

[2] H. Messel, Proc. Roy. Irish Acad. **A54**, 125 (1951).

[3] W. Heitler and L. Jánossy, Proc. Phys. Soc. (London) **A63**, 374 (1949).

[4] H. Messel, Phys. Rev. **83**, 21 (1950).

[5] M. Conversi, Phys. Rev. **79**, 749 (1950).

[6] Cocconi, Cocconi, Tongiorgi, and Widgoff, Phys. Rev. **79**, 768 (1950).

[7] Bernardini, Cortini, and Manfredini, Phys. Rev. **79**, 952 (1950).

II. THE MODEL, EVALUATION AND NUMERICAL RESULTS

For a detailed discussion of the model used see reference 1. The cross section was defined as follows. In a process in which a nucleon of energy E_0 loses energy and also gives rise to a recoil nucleon, it is assumed that the probability for a collision to occur is given by

$$\begin{aligned} w(E_0; E_1, E_2)dE_1dE_2 &= w(E_1/E_0, E_2/E_0)dE_1dE_2/E_0^2 \\ &= \sigma\epsilon_2^{\delta}(1-\epsilon_1)^{\alpha}d\epsilon_1 d\epsilon_2, \qquad (1) \end{aligned}$$

where $\sigma=15$, $\delta=2$, and $\alpha=1$. The model also incorporates an interaction mean free path of 65 g/cm², and an absorption mean free path of 130 g/cm² in air. The ionization loss β is assumed to remain constant throughout the course of the work. This assumption is reasonably correct for the energy region we shall be considering. Below a few hundred million electron volts β is no longer independent of the energy. In the work below β is taken to equal 130.10^6 ev per 65 g/cm² air. The depth θ is measured in units of 65 g/cm².

Using the above model it has been shown[4] that the number of particles $N^{(i,k)}(E_c, E, \theta)$ with energies greater than E at a depth θ in inhomogeneous matter (E_c is the cut-off energy) is given by the expression[7a]

$$N^{(i,k)}(E_c, E, \theta) = \sum_{n=0}^{\infty} N_n^{(i,k)}(E_c, E, \theta), \qquad (2)$$

(the suffix $i=1, 2$ refers to the primary particle and $k=1, 2$ to the secondary and the number "one" always refers to a proton and "two" to a neutron) where

$$\begin{aligned} &N_n^{(i,k)}(E_c, E, \theta) \\ &= \frac{1}{2\pi i}\int_{s_0-i\infty}^{s_0+i\infty}\left(\frac{E_c}{\beta}\right)^{s-1}\frac{\gamma}{\gamma-s+1}\left(\frac{\beta}{E+\beta g^{(i,k)}(s,\theta)}\right)^{s+n-1} \\ &\qquad \times\frac{\Gamma(s+n)}{\Gamma(s)}f_n^{(i,k)}(s,\theta)\frac{ds}{s+n-1} \qquad (3) \\ &\qquad\qquad (\gamma+1>s_0) \end{aligned}$$

with

$$N^{(i)}(E_c, E, 0) = \begin{cases} (E/E_c)^{-\gamma} & E>E_c \quad \gamma=1.7 \\ 1 & E<E_c. \end{cases} \qquad (4)$$

For the definitions of $g^{(i,k)}(s, \theta)$ and $f_n^{(i,k)}(s, \theta)$ we refer the readers to reference 4, as the expressions are rather too lengthy to be given here in full. It was also shown there that, to a fair approximation for $E\approx\beta$ and $E_c\ll\beta$, one obtains

$$\begin{aligned} N^{(i,k)}(E_c, E, \theta) &\approx \left(\frac{\beta}{E+\beta g^{(i,k)}(s,\theta)}\right)^{s-1}\frac{1}{2\pi i} \\ &\times\int_{s_0-i\infty}^{s_0+i\infty}\left(\frac{E_c}{\beta}\right)^{s-1}\frac{\gamma}{\gamma-s+1}f_0^{(i,k)}(s,\theta)\frac{ds}{s-1}. \qquad (5) \end{aligned}$$

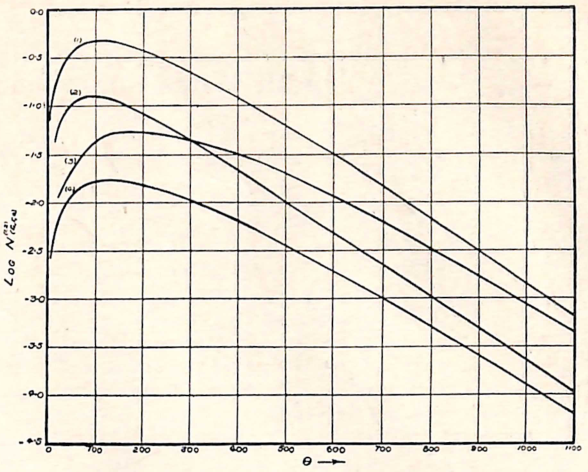

FIG. 2. Logarithm of the vertical intensity of neutrons with energies greater than E at an atmospheric depth of θ g/cm² due to an incident proton power law spectrum given by (4). In curve (1), $E=266$ Mev, $E_c=2000$ Mev, the cut-off energy corresponding to northern latitudes; curve (2), $E=2000$ Mev, $E_c=2000$ Mev; curve (3), $E=266$ Mev, $E_c=15{,}000$ Mev, the cut-off energy corresponding to the geomagnetic equator; curve (4), $E=2000$ Mev, $E_c=15{,}000$ Mev. The results have been normalized to an intensity of "one" proton at northern latitudes and $(2/15)^{1.7}$ protons at the geomagnetic equator.

Using (5) we have evaluated the curves giving the numbers of protons and neutrons with energies greater than $E=266$ Mev and $E=2000$ Mev for the incident proton power law spectrum given by (4). The values of the cut-off chosen were $E_c=2000$ Mev and $E_c=15{,}000$ Mev corresponding roughly to the cut-off energy at "northern" latitudes and the geomagnetic equator.

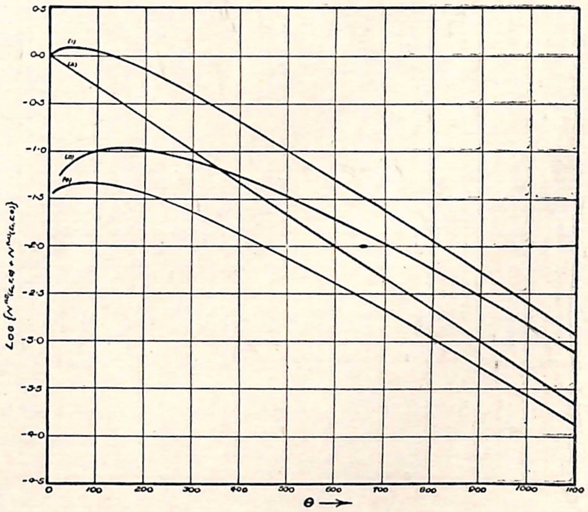

FIG. 3. Logarithm of the vertical intensity of neutrons plus protons with energies greater than E at an atmospheric depth of θ g/cm² due to an incident proton power law spectrum given by (4). In curve (1), $E=266$ Mev, $E_c=2000$ Mev, the cut-off energy corresponding to northern latitudes; curve (2), $E=2000$ Mev, $E_c=2000$ Mev; curve (3), $E=266$ Mev, $E_c=15{,}000$ Mev, the cut-off energy corresponding to the geomagnetic equator; curve (4), $E=2000$ Mev, $E_c=15{,}000$ Mev. The results have been normalized to an intensity of "one" proton at northern latitudes and $(2/15)^{1.7}$ protons at the geomagnetic equator.

[7a] The proof was given for a single incident particle; however, it may readily be shown by an almost identical proof that (2) holds when we consider an incident power law spectrum.

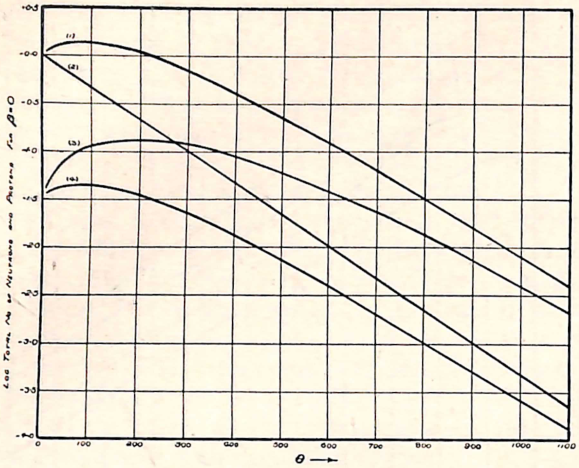

FIG. 4. Logarithm of the vertical intensity of neutrons plus protons with energies greater than E at an atmospheric depth of θ g/cm^2 due to an incident proton power law spectrum given by (4). In this case we assume the ionization loss β equal to zero. In curve (1), $E=266$ Mev, $E_c=2000$ Mev, the cut-off energy corresponding to northern latitudes; curve (2), $E=2000$ Mev, $E_c=2000$ Mev; curve (3), $E=266$ Mev, $E_c=15{,}000$ Mev, the cut-off energy corresponding to the geomagnetic equator; curve (4), $E=2000$ Mev, $E_c=15{,}000$ Mev. The results have been normalized to an intensity of "one" proton at northern latitudes and $(2/15)^{1.7}$ protons at the geomagnetic equator.

Figures 1, 2, and 3 give the vertical intensities of protons, neutrons and protons plus neutrons for the various energies above. For the sake of comparison we have reproduced[1] in Fig. 4 the curves giving the vertical intensity of protons plus neutrons when ionization losses were neglected. It will be noted that the errors due to the neglect of ionization loss are indeed considerable for the case of $E=266$ Mev.

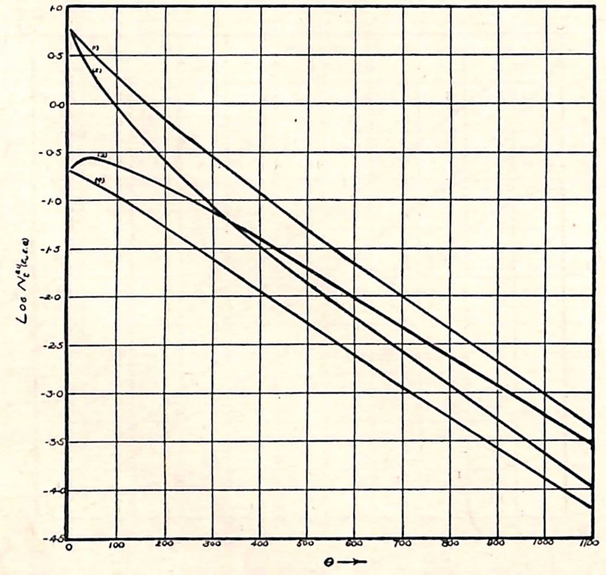

FIG. 5. Logarithm of the "total" intensity of protons with energies greater than E at an atmospheric depth of θ g/cm^2 due to an incident proton power law spectrum given by (4). In curve (1), $E=266$ Mev, $E_c=2000$ Mev, the cut-off energy corresponding to northern latitudes; curve (2), $E=2000$ Mev, $E_c=2000$ Mev; curve (3), $E=266$ Mev; $E_c=15{,}000$ Mev, the cut-off energy corresponding to the geomagnetic equator; curve (4), $E=2000$ Mev, $E_c=15{,}000$ Mev. The results have been normalized to an intensity of 2π protons at northern latitudes and $2\pi(2/15)^{1.7}$ protons at the geomagnetic equator.

An inverse Gross transformation was also carried out in order that we be able to compare our results with experimental data, which are practically all of a nondirectional variety. The results of the transformation are given in Figs. 5, 6, and 7. Finally, in Fig. 8 we have plotted the ratio of neutrons to protons for the various energies.

III. COMPARISON OF THEORETICAL RESULTS WITH EXPERIMENT

(a) Latitude Effect

In Tables I and II we give the latitude effect both for the vertical and total intensity of protons (P) and neutrons (N) above an energy $E=266$ Mev. The results in all cases show an increasing latitude effect with increasing altitude. The results of Simpson,[8] Yagoda,[9] and Yuan[10] on the slow neutron component of the cosmic radiation indicate an increasing latitude effect with increasing altitude. Their results indicate a value of 3.5 for this effect at an atmospheric depth of 312 g/cm^2 and 2.3 at 600 g/cm^2. These results appear to be in good agreement with those predicted in Table II. It should be noted, however, that the above authors were dealing with neutron energies considerably smaller than ours, and hence one would expect to find the experimental values higher than the predicted values of Table II.

TABLE I. Predicted latitude effect for the vertical intensity of the proton and neutron component of the cosmic radiation, between northern latitudes $E_c=2$ Bev and the geomagnetic equator $E_c=15$ Bev. The particles have energies greater than $E=266$ Mev.

Atmospheric depth in g/cm^2	200		310		650		1030	
	P	N	P	N	P	N	P	N
$E=266$ Mev	7.1	6.8	4.7	4.6	2.2	2.4	1.7	1.6

TABLE II. Predicted latitude effect for the total intensity of the proton and neutron component of the cosmic radiation betwen northern latitudes and the geomagnetic equator. Only particles with energies greater than 266 Mev are considered.

Atmospheric depth in g/cm^2	200		310		650		1030	
	P	N	P	N	P	N	P	N
$E=266$ Mev	5.1	5.1	3.5	3.5	2.1	2.1	1.6	1.6

The work of Conversi[5] on protons with energies of the order of 500 Mev gives a latitude effect of 3.2±0.5 between the latitudes 0° and 60°N, at an atmospheric

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[8] S. A. Simpson, Proc. Echo Lake Conf., p. 175 (1949) (unpublished).

[9] H. Yagoda, Proc. Echo Lake Conf., p. 169 (1949) (unpublished).

[10] L. Yuan, Proc. Echo Lake Conf., p. 181 (1949) (unpublished),

depth of 312 g/cm². McMahon *et al.*[11] working with ionization chambers give a latitude effect of 1.17±0.04 for protons with energies of the order of 5 Bev and 1.96 ±0.18 for protons with energies of the order of 0.4 Bev, at an atmospheric depth of 300 g/cm², between latitudes 55°N and 20°N. The results of Conversi[5] and McMahon *et al.*[11] appear to be in general agreement, but the values predicted above appear too high in comparison with theirs, especially for protons with energies greater than 2 Bev. (See Figs. 1 and 5, curves (2) and (4).) The work of the above authors also indicates a decrease in latitude effect for the protons with greater energies, at small atmospheric depths. It should be noted that our model shows only a slight variation in latitude effect for particles of different energies. Winckler *et al.*[12] give a latitude effect of 6.3 between 0° and 50°N at an atmospheric depth of 15 g/cm² for the vertical flux, which can quite safely be assumed equal to the primary vertical flux. In agreement with Winckler *et al.*, Whyte,[13] from ionization chamber observations on bursts at high altitudes, finds a latitude effect of 6.0 at 20 g/cm² and 3.0 at 200 g/cm² between latitudes 0° and 52°. These values are much smaller than those given in Figs. 1-7.

TABLE III. Predicted values for the AMFP in g/cm² air at the geomagnetic equator and northern latitudes, for the vertical intensity of the proton and neutron component of the cosmic radiation. Only particles with energies greater than 266 Mev are considered.

Atmospheric depth in g/cm²	200-400		500-700	
	P	*N*	*P*	*N*
Geomagnetic equator	275	360	172	171
Northern latitudes	145	176	135	135

TABLE IV. Predicted values for the AMFP in g/cm² at the geomagnetic equator and northern latitudes, for the total intensity of the proton and neutron component of the cosmic radiation. Energies larger than 266 Mev are considered.

Atmospheric depth in g/cm² air	200-400		500-700	
	P	*N*	*P*	*N*
Geomagnetic equator	159	175	143	135
Northern latitudes	108	115	125	123

TABLE V. Predicted values for the AMFP in g/cm² air at the geomagnetic equator and northern latitudes for the total nucleon intensity. Energies greater than 266 Mev are considered.

Atmospheric depth in g/cm²	200-400	500-700
Geomagnetic equator	157	143
Northern latitudes	109	123

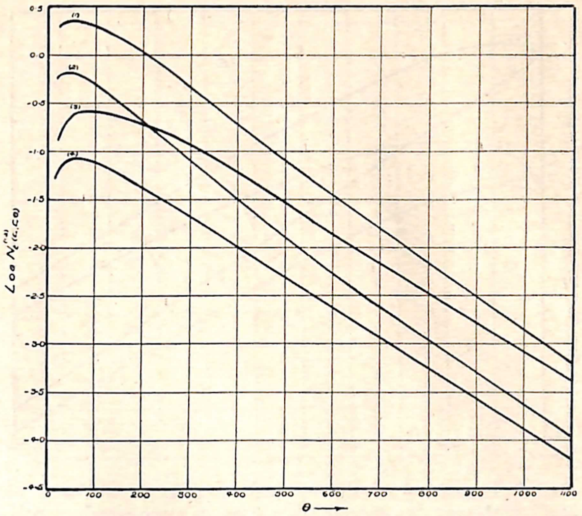

FIG. 6. Logarithm of the "total" intensity of neutrons with energies greater than E at an atmospheric depth of θ g/cm² due to an incident proton power law spectrum given by (4). In curve (1), $E=266$ Mev, $E_c=2000$ Mev, the cut-off energy corresponding to northern latitudes; curve (2), $E=2000$ Mev, $E_c=2000$ Mev; curve (3), $E=266$ Mev, $E_c=15{,}000$ Mev, the cut-off energy corresponding to the geomagnetic equator; curve (4), $E=2000$ Mev, $E_c=15{,}000$ Mev. The results have been normalized to an intensity of 2π protons at northern latitudes and $2\pi(2/15)^{1.7}$ protons at the geomagnetic equator.

(b) Absorption Mean Free Path (AMFP)

In Tables III, IV, and V, we have collected the values of the predicted AMFP for the vertical and total proton and neutron and total nucleon intensity, between va-

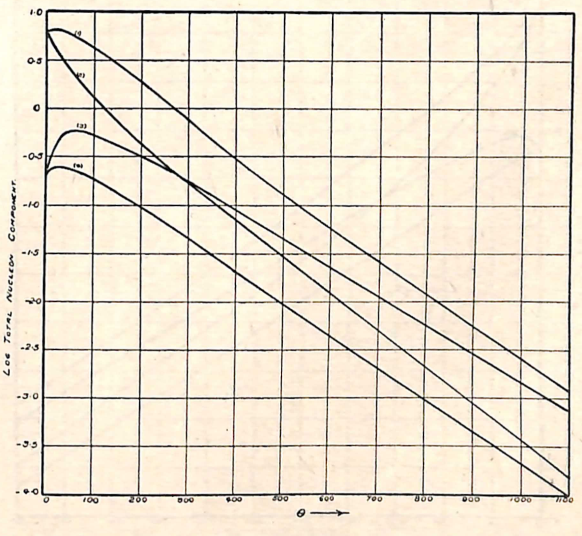

FIG. 7. Logarithms of the "total" intensity of neutrons plus protons with energies greater than E at an atmospheric depth of θ g/cm² due to an incident proton power law spectrum given by (4). In curve (1), $E=266$ Mev, $E_c=2000$ Mev, the cut-off energy corresponding to northern latitudes; curve (2), $E=2000$ Mev, $E_c=2000$ Mev; curve (3), $E=266$ Mev, $E_c=15{,}000$ Mev, the cut-off energy corresponding to the geomagnetic equator; curve (4), $E=2000$ Mev, $E_c=15{,}000$ Mev. The results have been normalized to an intensity of 2π protons at northern latitudes and $2\pi(2/15)^{1.7}$ protons at the geomagnetic equator.

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[11] McMahon, Rossi, and Burdett, Phys. Rev. **80**, 157 (1950).

[12] Winckler, Stix, Dwight, and Sabin, Phys. Rev. **79**, 656 (1950).

[13] G. N. Whyte, Phys. Rev. **82**, 204 (1951).

rious atmospheric depths. The absorption was assumed to be exponential for the depths considered. The AMFP in air has been measured by various people, both for the proton and slow neutron component of the cosmic radiation. It has been shown to increase with increasing altitude and to vary from northern latitudes to the geomagnetic equator at an equivalent altitude.

McMahon *et al.*,[11] Tinlot,[14] and Bridge and Rossi[15] find an AMFP for high energy protons, of approximately 125 g/cm². Our results as seen from the tables and Figs. 1, 5, and 7 are in general agreement with those found by these authors.

A number of features appear on closer examination of Tables III, IV, and V.

(1) In all cases, at the geomagnetic equator the AMFP decreases with decreasing depth.

(2) At northern latitudes the AMFP for the vertical proton and neutron intensity decreases with decreasing depth, but for the total neutron, proton and nucleon intensity, the AMFP increases with decreasing atmospheric depth.

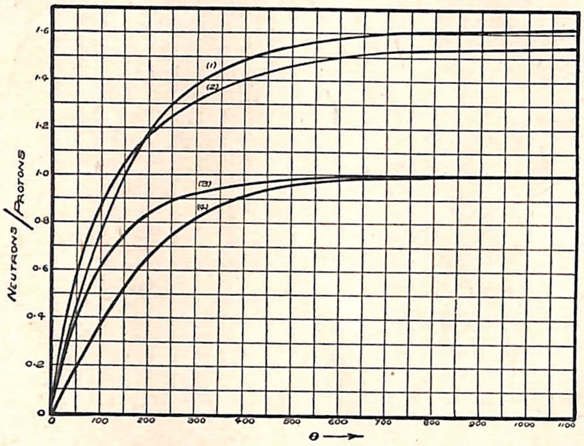

FIG. 8. The ratio of neutrons to protons at an atmospheric depth of θ g/cm². In curve (1), $E=266$ Mev, $E_c=2000$ Mev, the cut-off energy corresponding to northern latitudes; curve (2), $E=2000$ Mev, $E_c=2000$ Mev; curve (3), $E=266$ Mev, $E_c=15{,}000$ Mev, the cut-off energy corresponding to the geomagnetic equator; curve (4), $E=2000$ Mev, $E_c=15{,}000$ Mev.

(3) At small atmospheric depths the AMFP is considerably larger for neutrons than protons.

(4) AMFP increases as one passes from northern latitudes to the geomagnetic equator.

(c) Position of Maximum Density

The results of Staker[16] on atmospheric neutron density indicate that the maximum density of atmospheric neutrons occurs at a greater absorption depth the closer to the geomagnetic equator the measurements are made. His results give a maximum density at 110 g/cm² for 30.4°N and at 100 g/cm² at 54.7°N. These results appear to be in good agreement with those predicted theoretically. From Fig. 2 it will be seen that the maximum density does occur at a greater absorption depth, the closer to the geomagnetic equator the measurements are made. A maximum neutron density at 100 g/cm² for northern latitudes, and at 170 g/cm² at the geomagnetic equator, is predicted. (See curves (1) and (3).)

Whyte's[13] results on bursts appear to behave similarly. He finds that the position of the maximum for the burst producing component of the cosmic radiation varies with latitude. At the geomagnetic equator he finds that the maximum intensity occurs at an atmospheric depth of 100 g/cm², while at northern latitudes a small maximum (if any) occurs at 50 g/cm². This type of behavior is also indicated by our theoretical results (see Fig. 7). A very small maximum for northern latitudes at 25 g/cm² and a pronounced maximum at 80 g/cm² for the geomagnetic equator is predicted, for nucleons with energies greater than 266 Mev.

(d) Neutron-Proton Ratio

Measurements with the photographic plate by Page[17] and by Brown *et al.*,[18] and measurement with counters by Cocconi *et al.*,[6] give an indication of the proton-neutron ratio in the cosmic radiation.

Cocconi *et al.*, dealing with very high energy nucleons, find a neutron-proton ratio varying from about 1 to 2. The measurements by means of the photographic plate indicate a neutron-proton ratio of approximately 4 for the energies we are concerned with. It will be noted that these values are much higher than those indicated by our theory. Figure 8 shows a maximum neutron-proton ratio of 1.7.

IV. CONCLUSION

The results above appear to leave little room for doubt that the observed experimental behavior of the nucleon component of the cosmic radiation can be accounted for in terms of a nucleon cascade. The quantitative agreement between theory and experiment is still not entirely satisfactory, even for the limited energy range we have been considering. Recent experimental results strongly indicate a much smaller power law exponent than that which we have been using. A revision of the actual cross section used is therefore required.

Lately we have reformulated our model and it is hoped to present the results shortly. Much stronger quantitative agreement is indicated.

My thanks are due Professor E. Schrödinger for encouragement to carry on with the above research, to Professor C. B. A. McCusker, Dr. E. P. George, and Mr. D. D. Millar for helpful discussion. I would also like to express may thanks to the School of Theoretical Physics at the Dublin Institute for Advanced Studies for the provision of a Research Scholarship.

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[14] J. Tinlot, Phys. Rev. **73**, 1476 (1948); **74**, 1197 (1948).

[15] H. Bridge and B. Rossi, Phys. Rev. **75**, 810 (1949).

[16] W. P. Staker, Phys. Rev. **80**, 52 (1950).

[17] N. Page, Proc. Phys. Soc. (London) **A63**, 250 (1950).

[18] Brown, Camerini, Fowler, Heitler, King, and Powell, Phil. Mag. **40**, 862 (1949).