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On Some Recent Calculations on Cascade Shower Theory

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THERE have recently appeared a number of works[1-3] on the electron-photon cascade theory. In each case the authors were concerned with the solution of the diffusion equations of the cascade theory of electron showers when the ionization loss term was included. Bhabha and Chakrabarty[4] developed an elegant and simple series solution of the diffusion equations and showed that the first two terms of the series gave results which were quite satisfactory for all practical purposes. It was also pointed out by Jánossy and Messel[5] that the solutions given by B.C. could be quickly and easily evaluated. Their solutions have also the added advantages:

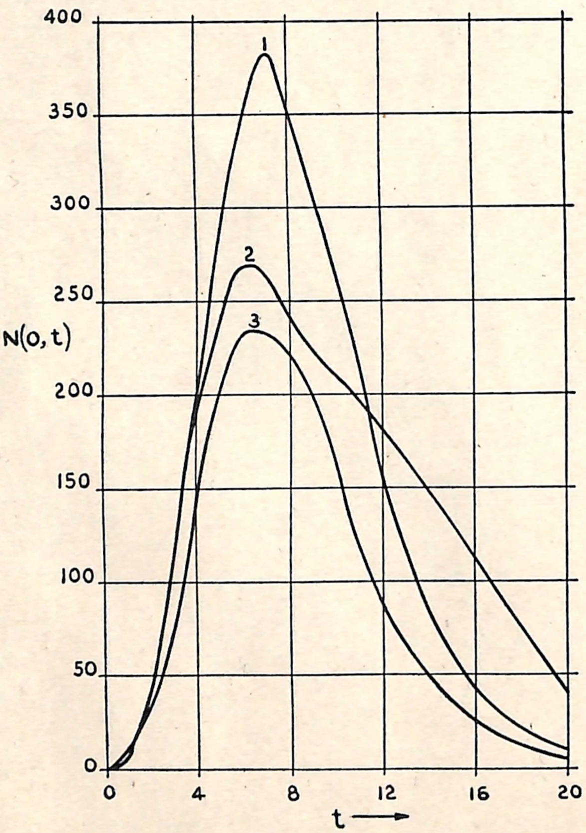

FIG. 1. $N(0, t)$, the average number of electrons induced by an incident electron of energy E_0, plotted against the depth t in cascade units. We have marked the curve given by Snyder's solution 1, that given by Bernstein for lead 2, and that by B.C. 3. In each case $\ln E_0/\beta = 8$.

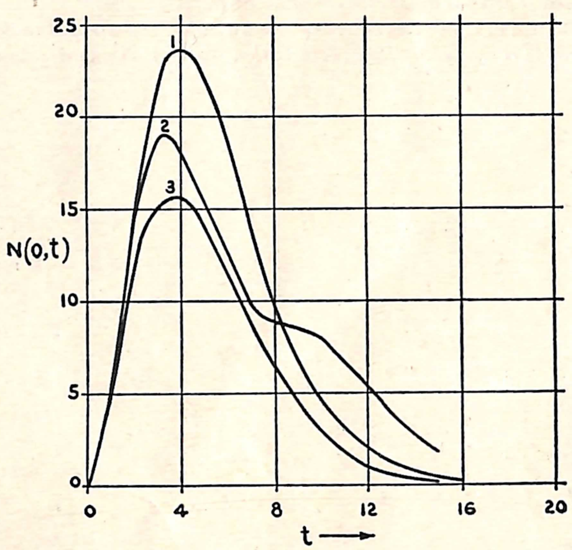

FIG. 2. $N(0, t)$, the average number of electrons induced by an incident electron of energy E_0, plotted against the depth t in cascade units. We have marked the curve given by Snyder's solution 1, that given by Bernstein for lead 2, and that by B.C. 3. In each case $\ln E_0/\beta = 5$.

(a) the first-order correction for the effects of ionization loss is expressed as a shift of the energy spectrum by an amount of the order of the critical energy,

(b) the series may be used either to evaluate the number of electrons $[N(E, t)]$ or photons $[\gamma(E, t)]$ above a specified energy E (and this is the physically important quantity), or to evaluate the total number of electrons $[N(0, t)]$ or photons $[\gamma(0, t)]$ at various depths t.

In the meantime Snyder[2] developed solutions of the cascade equations using a different approach. He showed that the solution of the cascade equations could be reduced to the solution of certain difference equations. The method is fairly long and is far from yielding solutions which lend themselves to easy computation. In fact, the solutions given by Snyder are of practical value only for computing the total number of electrons or photons for $E = 0$, at various depths. In order to obtain results for any other value of E, the evaluation of a triple complex integral is required. This in itself is a serious drawback. Snyder also pointed out that his solutions yielded values for $N(0, t)$ at the cascade maximum which were about 35 percent higher than those obtained by B.C., who used the first two terms of their series. Hence, it was concluded that the results of B.C. were inaccurate. At the same time Snyder pointed out that for a shower initiated by an electron of energy E_0

$$\int_0^t N(0, t)dt = E_0/\beta, \quad (1)$$

where β is the ionization loss. He then mentioned that the first two terms of the B.C. solution contribute only 70 to 85 percent of E_0/β and hence these authors did not use a sufficient number of terms of their series. B.C. had given in their paper a very satisfactory explanation for the above, which appears to have been overlooked by both Snyder and Bernstein.[3] We quote from reference 1.

"From the physical point of view, however, Eq. (1) must be taken with caution, especially in substances of high atomic number where the critical energy is low. It is not true that all the energy of a cascade is dissipated by the collision loss of cascade electrons alone. A good deal of energy is lost in the form of quanta of energy less than 2 mc^2 which are incapable of further pair creation. Thus the complete series for $N(0, t)$ must give too many cascade electrons of low energy at large thicknesses, and the first two or three terms of the series may well give a truer picture of the physical process in substances of high atomic number."

Bernstein lately has taken Snyder's solution and applied to it a correction by means of a perturbation method. He used a more refined approximation to the Bethe-Heitler cross sections than had hitherto been used. The method employed by Bernstein is straightforward but naturally even more tedious than that of Snyder—and again of the same limited applicability. From Bernstein's results there evolves an interesting feature. He finds that Snyder's results give a value for $N(0, t)$ which is much too high at the cascade maximum. We have plotted in Figs. 1 and 2 the results found by Bernstein, Snyder, and B.C. for $N(0, t)$ for $\ln E_0/\beta = 8$ and 5. The curves taken from Bernstein are those for lead.

It becomes immediately evident that the B.C. solutions (which can be used with ease) are not as inaccurate as hitherto made out. We also wish to point out that Bernstein, like Snyder, has failed to take into account the facts given in the quotation above. When this is taken into consideration, it appears likely that the pronounced "tail" of the curves given by Bernstein, beyond the cascade maximum, will be decreased to a value close to that given by B.C. It thus now appears that the series solution of B.C. can be used with confidence.

I am indebted to Professor H. J. Bhabha for valuable discussion of the above.

[1] H. J. Bhabha and S. K. Chakrabarty, Phys. Rev. **74**, 1352 (1948).
[2] H. S. Snyder, Phys. Rev. **76**, 1563 (1949).
[3] I. B. Bernstein, Phys. Rev. **80**, 995 (1950).
[4] Reference 1, henceforth shortened to B.C.
[5] L. Jánossy and H. Messel, Proc. Phys. Soc. (London) **A64**, 1 (1951).