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On the Multiple Production of Mesons

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SOME years ago we proposed a description of the multiple creation of mesons by the high energy collisions of nucleons on the basis of few assumptions.[1] Today we can compare this description with the observations and supplement it with few a remarks. From the experimental data we can deduce: (a) The cross section for the meson production by a primary proton (or by a secondary nucleon) is $\sim 5\times 10^{-25}$ cm^2 per air nucleus (4×10^{-26} cm^2 per nucleon), varying only slowly with the energy. (b) In every collision the incident particle loses a large fraction of its energy. (c) The multiple production of mesons (π, μ, neutrettos, etc.), is a fundamental process characteristic of high energy collisions of two nucleons or two nuclei.[2] The average multiplicity is given by the following rule: in the center of mass system, the mesons and the nucleons have, after the collision, kinetic energies $\sim\mu c^2\sim 10^8$ ev.[1] (d) The variation with atmospheric depth of the number of fast nucleons, producing the local penetrating showers, obeys an exponential law.[3] If b denotes the coefficient of absorption of a fast nucleon due to the creation of mesons, and b^* is the average coefficient of production of fast secondary nucleons, then the intensity of the flux of fast nucleons as function of the depth x (in g/cm^2) is $I=I_0\exp[-(b-b^*)x]$, where $(b-b^*)=0.01$ g/cm^2. Measurements of the transition effect[4] and of the saturation point give us directly the value of b (because locally produced nucleons give rise to showers simultaneous to the incident nucleon and thus are not registered separately). We find $b^{-1}\sim 50$ and $(b^*)^{-1}\sim 100$ g/cm^2. Thus primary protons are very rare at sea level, and most of the fast nucleons at sea level are produced by the mechanism of "cascades of nucleons and meson showers."

Let us consider the collision of two nucleons in the center of mass system, and let their energies before the collision be $E_{01}=E_{02}>Mc^2$. Let $E_{f1}E_{f2}$ be the energies of the nucleons after the collision, and let n_i denote the number of mesons of mass μ_i and of average energy ϵ_i created in the collision. Putting $E_{f1}\sim E_{f2}\sim 1.1\times Mc^2$ and neglecting the eventual emission of other particles, we have

$$E_{01}+E_{02}-2.2\times Mc^2=\Sigma n_i\epsilon_i=n\epsilon.$$

Now we assume that the distribution of the momenta of created mesons and of nucleons is spherically symmetrical and that the average kinetic energy ϵ per particle is $\sim\mu c^2$. Performing a Lorentz transformation from the center of mass system to the terrestrial frame, where one of the nuclei is at rest and the other has an energy E_P, we obtain:

$$E_p=2Mc^2(an^2+bn+d),$$

TABLE I. Average multiplicity n and average energy of mesons E_μ are given in function of the primary energy E_P (in ev).

	$A=1$		$A=r=14$	
E_P	n	E_μ	n	E_μ
5.0×10^9	3	5×10^8	9	3×10^8
1.8×10^{10}	12	9×10^8	27	4×10^8
1.3×10^{12}	1.6×10^2	8×10^9	6×10^2	2×10^9
1.3×10^{14}	1.6×10^3	8×10^{10}	6×10^3	2×10^{10}
1.9×10^{17}	6×10^4	3×10^{12}	2×10^5	8×10^{11}

where $a\sim 0.03$, $b\sim 0.4$, and $d\sim 1.1$. This formula gives the dependence of n from E_P. In Table I are indicated some typical values of E_P, n and of the average energy of the mesons E_μ.[5] The observations seem to support the correctness of the picture of the meson showers given above. Indeed, if, e.g., $n\gtrsim 100$, we have $n\sim(E_P)^{\frac{1}{2}}$ in accord with the recent results of G. F. Chew,[6] who derived this relation from the study of fast mesons made by Gill, Schein, and Yngve.[7] [Energetic mesons are created in showers with high n.] For primaries, sensitive to the earth's magnetic field ($E_p\sim 10^{10}$ ev), we obtain plausible values for the E_μ. It seems also noteworthy that Schein and Steinberger obtained remarkable accord between observed and theoretical spectrum of mesons starting from assumptions which, for $E_p\sim 10^{10}$, are similar to ours.[8] The best proof can be derived in the following way: let us indicate with $dN=kE_P^{-\gamma}dE_P$ and $dn=k'\epsilon^{-s}d\epsilon$ the spectral distributions of the primary protons and of the μ-mesons, respectively. If the average multiplicity n of a meson shower is proportional to $E_P{}^r$, then, assuming $E_P\sim n\epsilon$ (in accord with our description if $n>100$), one has $r=(s-\gamma)/(s-2)$.[9] Experimentally one finds[10] $\gamma=2.45$, from the azimuthal effect for high energy mesons, and one has $s=2.9$, from the meson intensity at great depths. Then, substituting these values in r, we obtain $r=0.5$ for high values of E_P, in accord with our theory.

Let us consider the assemblage of n-created mesons and of two nucleons after the collision (in the center of mass system) at the moment when they still occupy a volume of linear dimensions $l\sim h/\mu c$. We assume that the interaction between these particles at this moment is still sufficiently strong in order to give rise to a statistical distribution of energy and momenta. Then the most probable distribution is $n_i=g_i/(\exp(\alpha+\beta E_i)\pm 1)$, where $g_i=8\pi l^3h^{-3}p_i{}^2dp_i$ and other symbols have their usual meaning. From the uncertainty reaction the momenta must be $\gtrsim h/l$, and thus $\beta^{-1}\gtrless\mu c^2$. Now, either we assume $\beta^{-1}=\mu c^2$, or we introduce a convenient cut-off factor; in both cases we obtain the properties of the showers specified above. Very similar calculations can be made in the case of the collision of a proton with a nucleus of mass number A. We can suppose that r nucleons ($r \leqslant A$) suffer simultaneous collision with the incident proton, the remaining $(A-r)$ nucleons suffering only a negligible exchange of energy during the collision. Some results of these calculations (for $r=A=14$) are given in Table I. The high energy mesons must also be produced in groups. If $E_P \sim 10^{17}$ ev, we find multiplicities $\sim 10^5$ mesons, and meson energies $\sim 10^{12}$ ev sufficient to explain the penetration of mesons at great depths.[11]

[1] Symposium on Cosmic Rays, Acad. Bras. Ciencias 129 (1941); G. Wataghin, Phys. Rev. **70**, 787 (1946); Comptes Rendus **207**, 358 (1938).

[2] In accord with Heisenberg's general idea of explosion showers.

[3] G. Wataghin, Phys. Rev. **71**, 453 (1947); E. P. George and A. C. Jason, Nature **161**, 248 (1948); J. Tinlot, Phys. Rev. **73**, 1476 (1948).

[4] L. Janossy, Proc. Roy. Soc. **A179**, 361 (1941); L. Janossy and Rochester, Proc. Roy. Soc. **A183**, 181 (1944); V. H. Regener, Phys. Rev. **64**, 252 (1943).

[5] More data about these showers can be found in our paper (see reference 1).

[6] G. F. Chew, Phys. Rev. **73**, 1128 (1948).

[7] P. S. Gill, M. Schein, and V. Yngve, Phys. Rev. **72**, 733 (1948).

[8] M. Schein and J. Steinberger, Phys. Rev. **72**, 734 (1948).

[9] H. W. Lewis, J. R. Oppenheimer, and S. A. Wouthuysen, Phys. Rev. **73**, 140 (1948).

[10] P. S. Gill and G. H. Vaze, Phys. Rev. **73**, 1395 (1948).

[11] See Professor A. H. Compton's remarks at the Symposium (reference 1).