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§ 1. Introduction

Recent developments in experiments with artificial mesons have yielded much information regarding the nature of the π-meson, but its quantitative properties are not yet obvious, mainly because of difficulties with meson theory. The existence at present of more experimental data, especially the comprehensive analyses of cosmic ray phenomena at high energy, urges us to draw up a consistent picture of the meson. Unfortunately, however, there may be some ambiguity and inconsistency among the various experiments, a part of which will be eliminated by adequate corrections accounting for experimental conditions. Such experimental results, though lacking in perfection, should first be analyzed in the light of current meson theories since the discrepancy between experiment and theory will give a clue to later developments. In the present paper we attempt to give a brief summary of mesonic interactions in cosmic rays together with the future aspects of this field.

First of all we give some kinematical relations between an observed coordinate system and the center of mass system of two colliding nucleons with few assumptions about the mechanism of the meson production (§ 2). Next a brief account of our theory,[1] to which we tentatively refer, and its characteristic features are presented (§ 3). The greater part is devoted to the analysis and interpretation of several remarkable points of nucleonic showers (§ 4).

§ 2. Lorentz Transformation Between Laboratory and Center of Mass Systems

For the interpretation of meson showers produced by a nucleon-nucleon collision, it is necessary to provide the Lorentz transformation between the laboratory system (l. s.) and the center of mass system (c. m. s.), since it is more convenient to work in the latter system.[2),3),4),5),6),7)]

We label the quantities in c. m. s. and l. s. with and without asterisks, respectively. Suppose a nucleon with energy E and momentum P collides with a nucleon at rest of mass M. Then the Lorentz factor ξ, transforming one system into the other system, is given by

$$\xi=\sqrt{(E+M)/2M}=E^*/M, \tag{2.1}$$

where we set the velocity of light to unity.

The transformation formulae of momentum p and angle θ of a produced meson are expressed as

$$p^*=\{(\epsilon-\beta p\cos\theta)^2-(1-\beta^2)\mu^2\}^{1/2}\xi, \tag{2.2}$$

$$\tan\theta^*=p\sin\theta/(p\cos\theta-\beta\epsilon)\xi, \tag{2.3}$$

or

$$p=\{(\epsilon^*+\beta p^*\cos\theta^*)-(1-\beta^2)\mu^2\}^{1/2}\xi, \tag{2.2'}$$

$$\tan\theta=p^*\sin\theta^*/(p^*\cos\theta^*+\beta\epsilon^*)\xi, \tag{2.3'}$$

where ϵ and μ represent the energy and mass of the meson and β is the relative velocity between both systems, $\xi=1/\sqrt{1-\beta^2}$.

The momentum and angular distribution in 1. s. $F(p,\theta)\,dpd\theta$ and that in c.m.s $f(p^*,\theta^*)\,dp^*d\theta^*$ are connected by

$$F(p,\theta)\,dpd\theta=f(p^*,\theta^*)Jdpd\theta, \tag{2.4}$$

where J represents the Jacobian between both sets of variables:

$$J=p(\epsilon-\beta p\cos\theta)/\epsilon p^*. \tag{2.5}$$

In the case of an isotropic angular distribution in c.m.s., we obtain

$$F(p,\theta)=g(p^*)p^2(\epsilon-\beta p\cos\theta)\sin\theta/2\epsilon p^{*2}, \tag{2.6}$$

where $g(p^*)$ is the momentum distribution in c.m.s. The main behavior of the angular distribution is governed by p^* in the denominator of (2.6). For small θ and large p, the emitted angle of a meson in 1.s. is restricted as follows

$$(\mu/p)+(1/\xi)\gtrsim\theta\gtrsim|(\mu/p)-(1/\xi)|, \tag{2.7}$$

which means that the emitted mesons are restricted to a cone with the vertical angle $\theta\sim1/\xi$ for $p>\xi\mu$.

The angular or the momentum distribution is obtained by integrating $F(p,\theta)$ over p or θ, provided the function $g(p^*)$ is known. The simplest case is $g(p^*)=\delta(p^*-p_0^*)$, namely, the unique momentum p_0^* in c.m.s., as adopted by Rossi[6] and Gamba and Radicati.[4] The angular distribution is expressed as

$$\Phi(\theta)=\frac{\sin\theta\{\beta\epsilon_0^*\cos\theta+(\epsilon_0^*-\xi^2\beta^2\mu^2\sin^2\theta)^{1/2}\}^2}{2\xi^2(1-\beta^2\cos^2\theta)(p_0^{*2}-\xi^2\beta^2\mu^2\sin^2\theta)^{1/2}}, \tag{2.8}$$

where it should be noted that the angle of the emitted meson is restricted so as to have $\sin\theta\lesssim p_0^*/\xi\beta\mu$. This maximum angle corresponds to the meson emitted at an angle in the backward hemisphere. As inferred from (7), $\Phi(\theta)/\theta$ increases only slowly for $\theta\lesssim1/\xi$. The momentum distribution is

$$\Psi(p)=\{\xi^2\beta^2p^2-(\xi\epsilon-\epsilon_0{}^*)^2\}^{1/2}\epsilon_0{}^*/\xi^2\beta p_0{}^*\epsilon. \tag{2.9}$$

For larger p, $\Psi(p)$ has a maximum at $\xi\mu^2/\epsilon_0{}^*$ and p is restricted such that $2\xi(\epsilon_0{}^*-\mu)\gtrsim p\gtrsim\xi\mu^2/2(\epsilon_0{}^*-\mu)$. For smaller p, $\Psi(p)$ tends to $\propto p^{-1/2}$.

If $g(p^*)$ is represented by a power of p^* as assumed by Heisenberg,[3] $g(p^*)\propto(\mu/p^*)^n$, $\Phi(\theta)$ and $\Psi(p)$ are obtained as follows. Approximating p^* as $\theta-\beta p\cos\theta\approx(1-\beta\cos\theta)p+\mu^2/2p$, the integration is carried out for two ranges of p, $p\geq\bar{p}$ and $p\leq\bar{p}$, where $\bar{p}=\mu/\sqrt{2(1-\beta\cos\theta)}$. For $p>\bar{p}$, the upper limit of the integration must be put at p_0, the maximum acceptable momentum, but this does little to affect the result for $n\geq1$. Then we obtain

$$\Phi(\theta)\propto\begin{cases}\sin\theta/(1-\beta\cos\theta)^{\frac{n}{2}+2} & \text{for } p\geq\bar{p}, \quad (2.10\text{a})\\ (1-\beta\cos\theta)^{\frac{n}{2}+1}\sin\theta & \text{for } p\leq\bar{p}. \quad (2.10\text{b})\end{cases}$$

In this case also, the emitted mesons have a core with angular departure $\theta\sim1/\xi$. The momentum distribution is obtained as

$$\Psi(p)\propto\frac{p}{\beta\epsilon}\frac{\mu^n}{\{(\epsilon-\beta p)^2-\mu^2/\xi^2\}^{n/2}}\sim\begin{cases}(\mu/p)^{\frac{n}{2}}\xi^n & \text{for } p>\xi\mu,\\ (p/\mu)^{n/2} & \text{for } p<\xi\mu.\end{cases} \tag{2.11}$$

Thus $\Psi(p)$ has a maximum at $p=\xi\mu$.

For $n=0$, both $\Phi(\theta)$ and $\Psi(p)$ show logarithmic dependences on $(1-\beta\cos\theta)$ and p/μ, respectively, but are not shown here.

Generally speaking, almost all secondary mesons are restricted within the angle $\theta\lesssim1/\xi$ and their momenta concentrate about $p\sim\xi\mu$. Such properties are due to isotropic emission in c.m.s. If the emission is not isotropic, the results are more complicated. For example, there may exist double cones, if forward and backward emission are favored.[6),7)]

§ 3. Theory of Meson Production at High Energy

The cross-section for meson production in a nucleon-nucleon collision increases with increasing energy of the colliding nucleon, if the coupling between meson and nucleon contains a derivative of the wave function of the meson. It becomes larger than the area of the effective meson field, namely the geometrical cross-section, as long as we apply straightforward perturbation theory. This fact is hardly understandable and suggests that the higher order process is more effective than the lower one. Actually we can observe the multiple production of mesons at high energies. This experimental fact favors the existence of the coupling with derivative, as far as current meson theory is concerned. Therefore, we may ignore scalar meson theory and have only to consider the pseudovector coupling of pseudoscalar meson. We do not consider the spin 1 meson because it is highly improbable on the basis of various arguments.

The multiple production of mesons is extensively treated by Lewis et al. on semi-quantum mechanical grounds.[8] Their theory is extended by Fukuda and Takeda,[1] accounting for the charge exchange effect, which we refer hereafter as *I*. Their theories are essentially classical, since they take only the terms remaining as $\hbar \to 0$ and neglect the radiative correction. However, the classical treatment will be a good approximation, because the case where a considerable number of mesons is produced corresponds to the higher quantum state.

The probability for emitting n mesons is given in *I* as

$$\sigma(n) = \sigma_1 A(n), \tag{3.1}$$

$$A(n) = \left(\frac{g}{\pi}\right)^{2n} \frac{2^{2n}(n+2)}{3n!\,(2n)!} \left(\frac{w^*}{\mu}\right)^{2n}, \tag{3.2}$$

where σ_1 is the scattering cross-section of a nucleon, g the pseudovector coupling constant and w^* is the energy transferred to all mesons in c.m.s. The latter quantity can not be determined by the present theory and will include some unknown effect, such as damping. Here we may tentatively assume that w^* is proportional to the maximum available energy in a collision

$$w^* = \eta 2(E^* - M) \equiv \eta W^*. \tag{3.3}$$

Whether η is a constant or not will be decided by comparison with experiments.

Because of the unsatisfactory nature of our theory, the absolute value of the cross-section is not convincing, so that we mainly account for the relative probability for emitting n mesons· Comparing the probabilities for n and $n+1$, we obtain

$$R(n) = \frac{A(n+1)}{A(n)} = \frac{g^2}{\pi^2} \frac{2(n+3)}{(n+1)^2(n+2)(2n+1)} \left(\frac{w^*}{\mu}\right)^2, \tag{3.4}$$

which is approximated for high multiplicity by

$$R(n) \sim (g^2/\pi^2 n^3)(w^*/\mu)^2. \tag{3.5}$$

The most probable multiplicity and the root mean square deviation are found to be

$$\bar{n} = (g^2/\pi^2)^{1/3}(w^*/\mu)^{2/3}, \tag{3.6}$$

$$\Delta n = \sqrt{\bar{n}/3}\,. \tag{3.7}$$

These results may also express the classical nature of our theory, if one compares our results with the theory of Fermi,[9] who considers only the density of final state and ignores the interaction between nucleons and mesons. Both theories show an approximate coincidence, especially in the energy dependence.

These theories are seen to contradict the simplest assumption, that the energy of a produced meson is independent of the primary energy, as adopted by several authors.[4),5),6] The average energy of emitted mesons is

$$\bar{\epsilon}^* = w^*/\bar{n} = (\pi/g)^{3/2} w^{*1/3} \mu^{2/3}, \tag{3.8}$$

which increases slowly with incident energy. Which of the assumptions is better will be decided by comparison with experiments.

Our theory can further predict more physical quantities. The angular distribution of emitted mesons is nearly isotropic. As for final nucleons, they are concentrated forward and backward with the angle about μ/E^* if the scattering potential is vector or pseudovector type, whereas they are isotropic in other cases. The non-isotropic emission of the final nucleons may affect the angular distribution of mesons, and it may be possible that the fastest meson is emitted like the nucleons.

The charge dependence of produced meson is considerably dependent upon the type of scattering potential. The ratio of the numbers of neutral to charged mesons is estimated as 3/5 or 1/4 for the momentum of a nucleon $p^*=2M$ according to the symmetrical or the mixture of half symmetrical and half neutral potential. If we adopt the former case, we can obtain the total number of mesons by multiplying 1.36 by the number of charged mesons, N, in higher multiplicity.

$$n=1.36\ N. \tag{3.9}$$

§ 4. Analysis of Experiments

Various experimental data concerning our problem show seeming discrepancies with each other, when we deal with the raw data. Such discrepancies mainly come from the different experimental methods and mostly disappear with an appropriate correction as shown in what follows. The most convincing experimental results thus obtained are discussed in the light of the above theory.

1. *The energy of the agent nucleons.* The kinetic energy of nucleons effective to produce mesons seems to be greater than about 2 Bev. These are referred to as the A-component in our previous paper.[10] Some method to assign the energy described in *II* are not always applicable to such high energies discussed here. We confine ourselves to the following two methods.

One is the absolute intensity of the associated shower phenomena, which is closely related to the energy of the primary rays through their energy spectrum. Unfortunately, however, we have few cases where the intensity is estimated. In only two available cases, we can estimate the agent energies as $\gtrsim$ 10 Bev and $\gtrsim$20 Bev in the experiments of Fretter,[11] and Janossy and Rochester,[12] respectively. Thc latter experimenral arrangement is used by Butler et al.,[13] who estimated the average agent energy as$\sim$7 Bev. But this seems to be an underestimation, since the whole incident energy is assumed to be transferred to the penetrating particles emitted within one steradian.[14] If we assume that half of the available energy is converted into mesons, $\eta=0.5$ in (3.3), the transferred energy in one collision is about 7 Bev for 20 Bev incident energy. Actually, on the one hand, there may be plural collisions in a lead nucleus and, on the other hand, a considerable part of the released energy may escape observation so that our estimation may not be inconsistent with *BRB*.

The other method is to make use of the integral multiplicity-frequency relation. This relation is well approximated to $\sim N^{-2}$ in the experiments at lower altitude if N is smaller than ten. In Fretter's result, this relation is fairly distorted as discussed later. If we correct this distortion, N^{-2} relation is found valid $N \geq 6$ and the steep behavior for smaller N is explained as due to the selection by the triggering device. This leads to the agent energy $\gtrsim 9$ Bev, provided energy is equal to about 1.5 N as estimated in *II*. The most remarkable evidence for this assumption is obtained by *FLO*, who find a marked change in the slope of the multiplicity spectrum at $N=6$, which is attributed to the magnetic cut-off at about 8 Bev.[6]

2. *Momentum distribution of mesons.* We have not yet enough data about the momentum distribution for individual nuclear collisions, but have the statistical data. At higher energies, the cross-section for meson production is inferred as a function of the ratio of the momenta of meson and nucleon from the similarity between the energy distributions for both components.[15),16] The higher average energy of secondary mesons in *BRB* may be due to the absence of lower energy primaries. Their average momentum, ~ 0.8 Bev/c, is too high provided the power distribution in c.m.s. is assumed, (2.11), but consistent with the unique energy assumption, (2.9). Nevertheless, the experimental evidence seems to be not so certain as to decide in favor of either theory.

The momentum distribution obtained by the Bristol group[17] shows a surprising coincidence with that obtained by the indirect analysis of Sands,[18] although the energy spectra of primary nucleons are somewhat different for both experiments because of the different altitudes. This will result from the lower efficiency of meson production by the lower energy primaries, where the energy spectra will differ.

3. *Charge dependence.* Also, for the charge of produced mesons, we have little direct data. We are obliged to argue this problem from indirect knowledge.

The ratio of charged to neutral mesons can be estimated from the total energy of electronic rays, $W^{(e)}$, and μ-mesons, $W^{(m)}$, referring to the following decay schemes:

neutral π-meson $\rightarrow$ two photons,

charged π-meson $\rightarrow$ charged μ-meson + neutrino,

charged μ-meson $\rightarrow$ electron (positron) + two neutrinos.

The total energies of both components are given by Rossi[19] as

$$W^{(e)} = 0.285 \text{ Bev cm}^{-2} \text{ sec}^{-1},$$

$$W^{(m)} = 0.289 \text{ Bev cm}^{-2} \text{ sec}^{-1}.$$

From the above, we can estimate the total energies transferred to the charged and neutral π-mesons, W_c and W_n, as

$W_c = 0.37$ Bev cm^{-2} sec^{-1},

$W_n = 0.22$ Bev cm^{-2} sec^{-1}.

These figures are consistent with the result of the symmetrical theory, which gives neutral-charged ratio 0.60 for $n=1$ and 0.36 for larger n.

The ratio of positive to negative mesons is 2 : 1 for a proton incident in both cases of coupling mentioned in § 3. If the main contribution to the positive excess comes from the mesons produced in the upper atmosphere,[20] where the primary nucleons consist of 1/3 neutrons and 2/3 protons,[21] and the mesons are mostly produced singly as argued lately, we get a positive excess of about 20%, well in agreement with observation.

4. *Angular distribution.* The angular distribution is often used to assign the energy of an incident ray, but it seems to be inappropriate considering the different results by the different experimental methods. The average angles of forward secondaries obtained by some authors are compared in Table I, with the primary energies assigned by the simple relation $\theta = 1/\xi$, where the angle is corrected to the true one if the projected angle is measured.

Table I

Observer	*B*	*FLO* light	*FLO* heavy	*FLO* total	*F*	*BM*[22]	*W*[23]
Average angle	30°	22°	30°	28°	26°	38°	14°
Primary energy (Bev)	5.8	12	5.8	7.0	8.1	3.3	30

Among these, the first two, *B* and *FLO*, are observed by photographic plates and only thin tracks, following the nomenclature of *B*, are counted. *FLO* discriminate between showers occuring in light and heavy nuclei and notice that the angular distribution is considerably different for each case. For heavy nuclei the estimated energy ~6 Bev is too low to reconcile with the minimum energy of primaries of about 6 Bev (see § 4.1 and 6), considering the rather flat energy spectrum in this region. On the other hand, the estimated energy ~12 Bev for light nuclei is not inconsistent with the average energy of primaries. From this fact, the assignment of the absolute value of the energy by the average angular spread leads, for showers produced in heavy materials, to a serious error.

Furthermore, there are many sources of error in cloud chamber and counter work. The average angle of *F* is of course too large. Beside the above source, the error comes from the mixing of fast protons in penetrating particles. We can correct this effect accounting for the distribution of grey tracks in *B* and obtain the average angle ~13°. This seems to give a reasonable value, but one must not forget the fact that a considerable number of particles with large angular spread escape observation and these particles contribute greatly to the average angle. In the counter work, *W*, the average angle is surprisingly small, which means that almost all protons may be absorbed by the lead filter between the two lowest counter trays and the large angle ones may escape. Then the average angle except for photographic works serves only to give the relative order of the energy, such as the comparison of *F* and *BM*.

In photographic works, however, we can discuss detailed properties. One remarkable point is the forward restriction of the secondary particles of light stars in *FLO*. If the energy of emitted mesons is unique in c.m.s., the angle is limited by the relation $\sin\theta < p_0^*/\xi\beta\mu$. Putting the maximum angle as 60° and $\xi \sim 2$, $p_0^*/\mu > 3/2$. If a meson is emitted backward with this momentum, this track should be observed as a grey one. This contradiction suggests that we cannot accept either the unique energy emission or the single collision in a nucleus or both. There is another piece of evidence against the single collision even in a light nucleus. Some of the light stars have an asymmetric angular distribution, which is suggested by *FLO* as a possible indication of a spin dependent interaction in a single collision. But some of these stars have two or more groups of secondary particles. This fact may not be due to the spin dependent interaction, but may be due to the plural collisions in a nucleus, although the experimental evidence is not so clear as to permit the decision of the above alternative.

The angular distribution in the single collision is observed in a star obtained by Freier and Ney.[24] The produced mesons, 10 or 11, are noticeably grouped around the two emitting angles, ~1° and ~10°. These angles are transformed into ~12° and ~168° in c.m.s., if we adopt the energy of the incident nucleon as ~60 Bev as estimated by them on the basis of the relativistic increase of the ionization. This represents the narrow angle emission in forward and backward. However, this cannot be the evidence for the anisotropic emission, because there may be a considerable fluctuation and the method of estimation of the energy is not free from ambiguity.

The anisotropic emission is also inferred from the analysis about the core of extensive air showers.[25] But this does not necessarily mean the anisotropic emission of mesons, if we consider that the scattered nucleon plays an important role in the development of the core. Then the scattered angle of the secondary nucleon must be small, as is the case of vector or pseudovector potential (see §3).

5. *Multiplicity*. It has been disputed whether or not the genuine multiple production of mesons exists. The recent experiments seem to have settled this problem in such a way that genuine multiple production certainly occurs at extremely high energies together with plural production.[5),6),7),24] Even in the moderate energy region, the multiple process is not necessarily neglected as discussed by several authors.[6),34),26]

At what energy multiple production becomes predominant is an interesting problem. We shall see this from the distribution of observed multiplicity ν. In the photographic works[6),17),27] and in counter works,[23),38] we can find the integral ν distribution as $\sim\nu^{-2}$ for $\nu < 10$, but in other experiments one sees a considerable deviation from ν^{-2}. For example, we see $\sim\nu^{-2.5}$ in *BM* and ν^{-3} in *F*. Such deviation is supposed to be due to the bias in the experimental conditions, if we notice the fact that in W and P the factor of about 2 is multiplied by the observed ν in order to get the true number of shower particles. In the cloud chamber experiments, the particles with small and large divergent angles may be missed, because of the low optical resolving power in the former case and of the limited illuminating region and the counter selection in the latter case. If we tentatively multiply the factors 1.5 and 2 by the observed ν in BM and F, respectively, accounting for the looser counter selection in BM, the corrected ν-distribution agrees with $\sim\nu^{-2}$. The agreement becomes better, if the small contribution from grey tracks is accounted.

ν^{-2} distribution is regarded as representing the approximate proportionality between ν and the primary energy, though the energy distribution is slightly less steep than E^{-2}. Then we can use ν as indication of energy as seen in § 4.1. This behavior, including the slight difference as mentioned above, is well explained by the plural production theory of Heitler and Janossy.[29] Then we may consider that the meson is singly produced as far as ν^{-2} distribution holds. The ν-distribution becomes steeper for $\nu \gtrsim 10$, which means the set-in of the genuine multiple production. Actually the multiple production may begin at smaller ν than 10, since both the theory of Heitler and Janossy and the experiments give a gradual bend in this region. Then we adopt $\sim$10 Bev as the critical energy where multiple production becomes predominant.

If we substitute this critical energy, corresponding to $W^*=2.5$ Bev, in (3.4) for $n=1$, we get the relation

$$(g\eta)^2/4\pi=3\pi(\mu/W^*)^2=3.0\times10^{-2}. \tag{4.1}$$

To verify our theory in § 3, we see the relation between $\bar{n}$ and $\bar{\theta}$, assuming $\bar{\theta}=1/\xi$, in an individual star with many thin tracks. The most systematic data are given by *FLO*. We plot the ν-$(1/\bar{\theta}-1)$ relation in Fig. 1, together with the formula (3.6) and the result of Osborne's theory. The heavy and light stars are completely separated as emphasized by *FLO*, though they show the similar inclination. If we examine each star in detail, three light stars, which have the large values of $\nu^{3/2}/(\xi-1)$, seem to have two or more cores, and one heavy star, which has small $\nu^{3/2}/(\xi-1)$ show a single core, though other heavy stars may have two or more cores. If we adopt the lowest value of $\nu^{3/2}/(\xi-1)$, a good estimation of $g\eta$ will be obtained:

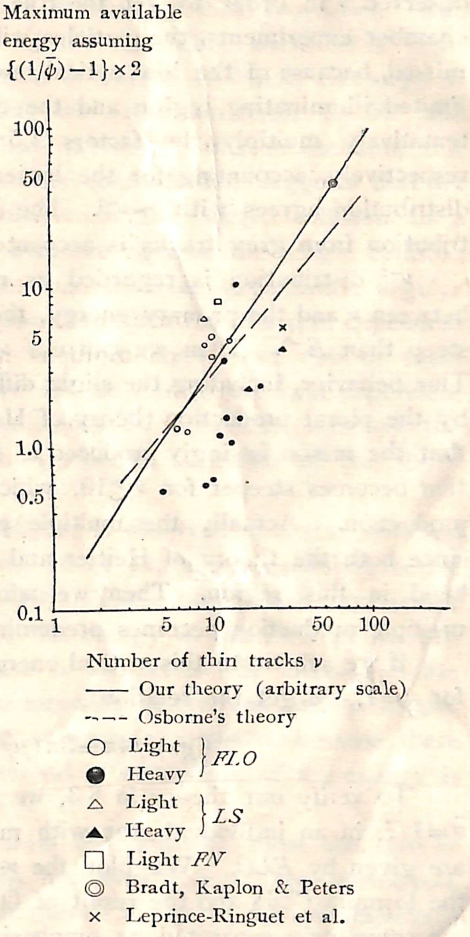
$$(g/\eta)^2/4\pi=(\pi/4)\bar{n}^{-3}(\mu/W^*)^2\approx1.0\sim2.2, \tag{4.2}$$

accounting for the contribution of neutral mesons as in (3.9). The estimated value $(g\eta)^2/4\pi$ is about 50 times larger than that in (4.1).

The reasons for such a discrepancy are considered as follows. (1) η is not a constant and increases with incident energy. (2) ν is still larger than N in the above estimation, so that the plural collision takes place almost always even in a light nucleus. (3) $\bar{\theta}$ is smaller than the above estimated value because of the same reason as in (2).

The magnitude of η can be estimated from the absorption of nucleons in the atmosphere. In order to explain the longer absorption mean free path than the collision mean free path, about a half of the energy of a nucleon must remain after a collision with an air nucleus.[16) This estimate holds for nucleon energies near 10 Bev, which form essentially the bulk of cosmic ray intensity. This amount of the surviving energy corresponds to $\eta \sim 1/2$, provided the average number of collisions in an air nucleus is two. Therefore, the increase of η affects the result at most 4 times, even if η become unity in the higher energy region.

Fig. 1. Relation between multiplicity and angular divergence



The other two causes are more sensitive, since both make the result larger in the same direction. If we divide the secondary particles into two groups, $\bar{n}$ decreases about one half and $\bar{\theta}$ decreases two or more times. Such a posibility is fairly plausible, if one looks at the distribution of shower particles in detail. For this reason, the estimate (4.2) is not reliable enough to obtain the magnitude of the coupling constant g. This presumption will be supported by considering the parallel behavior between heavy and light stars in Fig. 1. On the contrary, the estimate (4.1) has an ambiguity of at most a factor 2.

Referring to (4.1) and assuming $\eta = 1/2$, we obtain

$$g^2/4\pi \approx 0.12. \tag{4.3}$$

This figure is not inconsistent with that obtained from the analysis of the experiment of artificial mesons.[30)

6. *Nuclear interaction of secondary particles.* To interpret the behavior of secondary particles, their constitution is different from each other according to the experimental conditions. The energy range, in which the protons and mesons are identified as fast shower particles, is shown in Table II for each observer.

Table II. The energy range of fast particles

Observer	Proton		Meson	
	Minimum energy Mev	Minimum range in Pb g cm^{-2}	Minimum energy Mev	Minimum range in Pb g cm^{-2}
B_*	330	124	49	18.5
BM	230	78	34	11.6
F^{**}	120	14.5	41	14.5

The constitution of both components is much different according to the energy range. Fowler showed that about 80% of the thin tracks with energies greater than the above values are mesons but that in the lower energy region this ratio is reversed.[31),17)] Accordingly a large part of the penetrating secondary particles in the cloud chamber photographs may be protons. In the counter work of Piccioni,[28)] almost all should be mesons following his description, though there is some objection to his interpretation.[32)]

The mean free paths for the nuclear interaction of secondary particles obtained by several authors are summarized in *B*. They criticized the results of other works and stated that the mean free path is the geometrical one. One should be careful to draw any conclusion from this result about the nuclear interaction of mesons, since the interaction varies with the energy of the meson.

Firstly, we consider the nuclear scattering of a meson by a free nucleon. We may ignore the damping effect, since we consider meson energies of at most a few Bev.[33)]

Accounting for the straightforward perturbation calculation, the pseudoscalar coupling gives the larger cross-section than the geometrical one if we adopted $f^2/4\pi=16$, while for pseudovector coupling the cross-section approaches the geometrical one at $\epsilon_0 \sim M$ if we adopted $g^2/4\pi=0.1$. In the former case the splitting of a meson is possible even for the ordinary energy $\epsilon_0 \sim \mu$ to M, while in the latter case the splitting may not take place and the nuclear interaction should rarely be observed.

Nevertheless, the actual situation is much different, because a meson strikes a compound nucleus. If the momentum of an incident meson is smaller than 200 Mev/c, corresponding to the maximum momentum of a nucleon in a nucleus, the simple elastic scattering should almost be forbidden by the Pauli exclusion principle for a recoil nucleon. This effect will considerably reduce the scattering cross-section of mesons with energies of several hundred Mev.

However, the cross-section for nuclear interaction will not be so greatly reduced, since there is a competing process, the absorption of a meson by a

* Other photographic works, *LS* and *FLO*, use about the same discrimination.

** Most of cloud chamber works except *BM* discriminate between the tracks by the penetration of a lead plate without electronic interaction.

nucleus.* This process is just the inverse process of the production of a meson in a nucleon-nucleon collision.[34] This cross-section is larger than that for production for the same matrix element, because of the large density of final states, in which only two nucleons are present in the former case, whereas two nucleons and one meson are produced in the latter case. The absolute value of the cross-section is largely dependent on the magnitude of the coupling constant, but if we adopt the experimental value for the production cross-section, the absorption cross-section is the order of $\sim 10^{-27}$ cm^2. Here one must notice the fact that the magnitude of the coupling constant $f^2/4\pi$ determined from this experiment is much smaller than that obtained from the cross-section for meson production by γ-rays.[30] If we adopt the latter value of f, the cross-section becomes very large, about 10^{-25} cm^2. If we refer to the result of B, the absorption process is more probable than the others.

§ 5. Conclusion

Although we cannot draw definite conclusions from the above analysis because of poor statistics in the experiments, we may say, referring to II, that the mesonic interaction is dependent upon the energy of the colliding nucleon as follows.

Below 2 Bev elastic scattering is more predominant than meson production. Above 2 Bev meson production is still single, though we observe multiple mesons in a nuclear collision which are attributed to the plural collisions of a nucleon in a nucleus. At about 10 Bev genuine multiple meson production sets in. The behavior of the multiple production can be explained by our theory I, but we cannot rule out other theories at the present stage of our knowledge.

The existence of multiple production strongly suggests the necessity of pseudovector coupling of a pseudoscalar meson, which is supported by various arguments. Pseudoscalar coupling is not always necessary and may not exist or may be weak if it does exist. One of the arguments for this assumption is the contradiction, which appears when we estimate the magnitude of the pseudoscalar coupling constant in the meson production by γ-rays or protons. If we take into account only pseudovector coupling in the case of proton bombardment, the magnitude of the pseudovector coupling constant is estimated as $g^2/4\pi \sim 0.2$. This value is consistent with the case of γ-ray bombardment and with our estimate in the present paper. However, this magnitude seems to be too large to evaluate the probability of various processes by the simple perturbation theory. If we retain our above estimate, we may fall into self-inconsistency, because our theory is grounded on the perturbation theory. The development of other

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* This process was emphasized by Dr. Steinberger in a conversation with one of the authors (S. H.).

approximation methods, for example, the improvement of the strong coupling theory, is strongly required.

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