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ON THE CALCULATION OF AVERAGE NUMBERS FOR THE ELECTRON-PHOTON CASCADE.

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Introduction.

Showers observed in cosmic rays were found to consist largely of electrons and photons. The mechanism of such showers was clearly recognised simultaneously in the well-known papers by Bhabha and Heitler (1937) and Carlson and Oppenheimer (1937). The concept of the cascade gave rise to an interesting mathematical problem.

Diffusion equations governing the average numbers of particles in a shower and numerical solutions using simplified models were given in the original papers. Great progress in the mathematical treatment of the cascade process was achieved when it was pointed out by Landau and Rumer (1938) that the diffusion equations can be greatly simplified by application of a Mellin transformation with respect to the energy parameter.

The mathematical problem of the electron-photon cascade is summarised. e.g. in Jánossy's " Cosmic Rays," Oxford, 1950, §370–393. It will be found there that solutions in " approximation A " can be obtained in the form of complex integrals. In approximation "A" all the cross-sections are taken to be homogeneous functions of the energies ; this means that the cross-sections for all energies are replaced by the assymptotic cross-sections exactly valid for high energies only. The effects of ionization loss are also neglected.

We have computed detailed tables for the average numbers of electrons or photons above a given energy at various depths in this approximation.

In figures 1 to 5 we have presented the results graphically, convenient for practical use. The average numbers of electrons or photons arising from a primary electron or photon were computed. The four families of curves are very similar in appearance and, therefore, we have preferred to plot only one of the families, i.e. the electrons produced by electrons and express the other families as ratios between the remaining three average number families and the electron—electron family.

The final results are given with three figure accuracy. It was found, however, that in order to achieve this moderate accuracy it was necessary to start calculating with considerably greater accuracy ; the auxiliary functions appearing had to be computed to ten figures. These auxiliary

functions turn up frequently in connexion with many types of cascade problems ; therefore, in order to avoid duplicating effort we present the detailed tables which we computed below. Some of these tables have been reproduced in an abbreviated form in ref. 4 (Appendix).

Calculation of Average Numbers.

The number of electrons or photons with energies exceeding w produced by an electron or photon primary of energy w_0 under a layer of thickness ζ cascade units, neglecting ionization loss and assuming cross sections for full screening are given by the following integrals :—

$$Q_0^{(i)}\ (w_0,\ w\ ;\ \zeta) = \frac{1}{2\pi i} \int_{s_0 - i\infty}^{s_0 + i\infty} \left(\frac{w_0}{w}\right)^{s-1} \frac{R^{(i)}\ (s,\ \zeta)}{s-1}\ ds$$

$$P_0^{(i)}\ (w_0,\ w\ ;\ \zeta) = \frac{1}{2\pi i} \int_{s_0 - i\infty}^{s_0 + i\infty} \left(\frac{w_0}{w}\right)^{s-1} \frac{S^{(i)}\ (s,\ \zeta)}{s-1}\ ds$$

where Q_0 represents the number of electrons and P_0 the number of photons ; suffix i indicates the nature of the primary : $i = 1$ refers to an electron primary and $i = 2$ refers to a photon primary, and

$$R^{(1)}\ (s,\ \zeta) = \frac{D - a_1}{a_2 - a_1}\ e^{-a_1\zeta} + \frac{a_2 - D}{a_2 - a_1}\ e^{-a_2\zeta}$$

$$R^{(2)}\ (s,\ \zeta) = \frac{B}{a_2 - a_1}\ e^{-a_1\zeta} - \frac{B}{a_2 - a_1}\ e^{-a_2\zeta}$$

$$S^{(1)}\ (s,\ \zeta) = \frac{C}{a_2 - a_1}\ e^{-a_1\zeta} - \frac{C}{a_2 - a_1}\ e^{-a_2\zeta}$$

$$S^{(2)}\ (s,\ \zeta) = \frac{a_2 - D}{a_2 - a_1}\ e^{-a_1\zeta} + \frac{D - a_1}{a_2 - a_1}\ e^{-a_2\zeta}$$

$$a_1 = \tfrac{1}{2}\ \{A\ (s) + D\} - \tfrac{1}{2}\ \sqrt{(A\ (s) - D)^2 + 4\ B\ (s)\ C\ (s)}$$

$$a_2 = \tfrac{1}{2}\ \{A\ (s) + D\} + \tfrac{1}{2}\ \sqrt{(A\ (s) - D)^2 + 4\ B\ (s)\ C\ (s)}$$

$$A(s) = \int_0^1 \left\{v - (4/3 + a_0)\left(1 - \frac{1}{v}\right)\right\} \{1 - (1 - v)^{s-1}\}\ dv$$

$$= (4/3 + a_0)\ \{\psi\ (s) - \psi\ (1)\} + \tfrac{1}{2} - \frac{1}{s(s+1)}$$

$$\psi(s) = \frac{d}{ds} \log (s!)$$

$$B(s) = 2 \int_0^1 \{1 - (4/3 + a_0)(v - v^2)\} v^{s-1} dv$$

$$= 2 \left\{ \frac{1}{s} - (4/3 + a_0) \frac{1}{(s+1)(s+2)} \right\}$$

$$C(s) = \int_0^1 \left\{ 1 - (4/3 + a_0) \left(\frac{1}{v} - \frac{1}{v^2} \right) v^s \right\} dv$$

$$= \frac{1}{s+1} + (4/3 + a_0) \frac{1}{s(s-1)}$$

$$D = \int_0^1 \{1 - (4/3 + a_0)(v - v^2)\} dv = 7/9 - \tfrac{1}{6} a_0.$$

Tables of $\log_{10} Q_0(w_0, w; \zeta)$ and $\log_{10} P_0(w_0, w; \zeta)$ for primary electrons and photons have been computed using the above formulae.

The integrals for Q_0 and P_0 were evaluated by the saddle point method, neglecting the second exponential term in each case. However for small thicknesses of absorber the second exponential is not entirely negligible, and in order to take care of the effect of this term in a first approximation the following procedure was adopted. Each of the expressions was first computed in the usual way without the second exponential and then in order to correct for the neglect of the second exponential the expression was multiplied by the following factor :—

$$K = (1 + M e^{(a_2 - a_1)\zeta})_{s = s_0}$$

where $M = -1$ for primary electrons producing photons or primary photons producing electrons. Further

$$M \begin{cases} = \dfrac{a_2 - D}{D - a_1} & \text{for electrons producing electrons} \\ = \dfrac{D - a_1}{a_2 - D} & \text{for photons producing photons} \end{cases}$$

s_0 is the value of s for which the uncorrected integrand has its minimum. In the tables below the correction never exceeds 10%.

Ionization Loss.

According to Bhabha and Chakrabarty ionization loss can be taken into account approximately by introducing into the expression for approximation "A" a modified value for the energy. Thus the second approximation can be written

$$Q_1\ (w_0,\ w\ ;\ \zeta) = Q_0\ (w_0,\ w + gw_c,\ \zeta)$$

w_c is the critical energy and g is a function of w/w_0 and ζ. For the case of electrons giving rise to electrons we have computed the g-values corresponding to the entries of the main table.

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TABLE I.

D = 0·773677778

s	A	B	C
1·1	0·152040516	1·400997998	12·821038960
1·2	0·286213372	1·280890151	6·112601009
1·3	0·406253059	1·180639842	3·916662949
1·4	0·514864274	1·095744631	2·841547618
1·5	0·614060233	1·022948571	2·210577777
1·6	0·705374815	0·959843305	1·799129272
1·7	0·790000562	0·904612063	1·511490818
1·8	0·868881122	0·855860485	1·300152116
1·9	0·942774956	0·812501900	1·138940646
2·0	1·012300000	0·773677778	1·012299999
2·1	1·077966037	0·738701435	0·910430572
2·2	1·140198219	0·707017496	0·826868686
2·3	1·199354710	0·678172218	0·757188608
2·4	1·255739857	0·651791444	0·698264472
2·5	1·309614519	0·627564021	0·647829840
2·6	1·361203875	0·605229159	0·604204059
2·7	1·410703656	0·584566694	0·566116312
2·8	1·458285048	0·565389515	0·532589110
2·9	1·504098703	0·547537607	0·502859136
3·0	1·548277777	0·530873333	0·476322222
3·1	1·590940673	0·515277662	0·452494348
3·2	1·632193079	0·500647131	0·430983495
3·3	1·672129801	0·486891380	0·411468986
3·4	1·710836113	0·473931142	0·393686125
3·5	1·748389121	0·461696585	0·377414602
3·6	1·784858710	0·450125949	0·362469651
3·7	1·820308453	0·439164405	0·348695219
3·8	1·854796329	0·428763107	0·335958646
3·9	1·888375443	0·418878394	0·324146471
4·0	1·921094444	0·409471111	0·313161111
4·1	1·952998123	0·400506046	0·302918190
4·2	1·984127755	0·391951435	0·293344399
4·3	2·014521520	0·383778553	0·284375745
4·4	2·044214703	0·375961349	0·275956130
4·5	2·073240167	0·368476146	0·268036171
4·6	2·101628206	0·361301368	0·260572234
4·7	2·129407390	0·354417312	0·253525620
4·8	2·156604128	0·347805950	0·246861893
4·9	2·183243211	0·341450746	0·240550308
5·0	2·209347776	0·335336508	0·234563333
5·1	2·234939563	0·329449251	0·228876240
5·2	2·260038957	0·323776078	0·223466757
5·3	2·284665164	0·318305073	0·218314772
5·4	2·308836195	0·313025213	0·213402076
5·5	2·332569089	0·307926278	0·208712147
5·6	2·355879900	0·302998785	0·204229955
5·7	2·378783804	0·298233916	0·199941799
5·8	2·401295135	0·293623464	0·195835165
5·9	2·423427521	0·289159777	0·191898596
6·0	2·445193808	0·284835714	0·188121587

TABLE II.

s	$\frac{dA}{ds}$	$\frac{dB}{ds}$	$\frac{dC}{ds}$	$\frac{d^2A}{ds^2}$
1·1	1·4237547	− 1·3196582	− 134·8978317	− 1·7739342
1·2	1·2658374	− 1·0929808	− 33·2119356	− 1·4049710
1·3	1·1394087	− 0·9194265	− 14·4736732	− 1·1372150
1·4	1·0361357	− 0·7838401	− 7·9678713	− 0·9376102
1·5	0·9503175	− 0·6760536	− 4·9882074	− 0·7853419
1·6	0·8779465	− 0·5890522	− 3·3895233	− 0·6668405
1·7	0·8161294	− 0·5178779	− 2·4385936	− 0·5729935
1·8	0·7627375	− 0·4589518	− 1·8302066	− 0·4975189
1·9	0·7161638	− 0·4096416	− 1·4192082	− 0·4359851
2·0	0·6751831	− 0·3679787	− 1·1295611	− 0·3852022
2·1	0·6388436	− 0·3324689	− 0·9183958	− 0·3428305
2·2	0·6063951	− 0·3019625	− 0·7601008	− 0·3071270
2·3	0·5772391	− 0·2755640	− 0·6386400	− 0·2767721
2·4	0·5508964	− 0·2525680	− 0·5435760	− 0·2507535
2·5	0·5269711	− 0·2324135	− 0·4678892	− 0·2282866
2·6	0·5051412	− 0·2146495	− 0·4067255	− 0·2087541
2·7	0·4851376	− 0·1989107	− 0·3566457	− 0·1916665
2·8	0·4667398	− 0·1848988	− 0·3151615	− 0·1766322
2·9	0·4497549	− 0·1723681	− 0·2804385	− 0·1633335
3·0	0·4340238	− 0·1611152	− 0·2511019	− 0·1515127
3·1	0·4194101	− 0·1509702	− 0·2261056	− 0·1409571
3·2	0·4057959	− 0·1417907	− 0·2046434	− 0·1314919
3·3	0·3930791	− 0·1334562	− 0·1860860	− 0·1229709
3·4	0·3811745	− 0·1258647	− 0·1699369	− 0·1152708
3·5	0·3700019	− 0·1189292	− 0·1558003	− 0·1082894
3·6	0·3594954	− 0·1125748	− 0·1433579	− 0·1019386
3·7	0·3495945	− 0·1067374	− 0·1323512	− 0·0961441
3·8	0·3402504	− 0·1013613	− 0·1225689	− 0·0908422
3·9	0·3314130	− 0·0963982	− 0·1138368	− 0·0859775
4·0	0·3230419	− 0·0918061	− 0·1060106	− 0·0815032

TABLE III.

s	a_1	$\frac{d a_1}{ds}$	$\frac{d^2 a_1}{ds^2}$	$\frac{d^2 B}{ds^2}$
1·1	− 3·786712377	24·9912071	− 355·5517840	2·6010703
1·2	− 2·278786491	9·4501223	− 65·3883330	1·9704605
1·3	− 1·568255737	5·4113309	− 24·6757500	1·5253789
1·4	− 1·125010369	3·6527173	− 12·4759201	1·2030034
1·5	− 0·812011740	2·6908033	− 7·4003632	0·9642419
1·6	− 0·575026293	2·0909327	− 4·8578467	0·7839408
1·7	− 0·387510836	1·6831696	− 3·4217814	0·6454400
1·8	− 0·234663134	1·3884316	− 2·5390358	0·5374233
1·9	− 0·107454690	1·1653443	− 1·9606339	0·4520312
2·0		0·9903792	− 1·5617682	0·3836955
2·1	0·091745049	0·8492974	− 1·2746040	0·3284018
2·2	0·170684263	0·7330586	− 1·0599255	0·2832087
2·3	0·238983977	0·6356970	− 0·8939767	0·2459298
2·4	0·298315928	0·5531410	− 0·7618652	0·2149186
2·5	0·350009143	0·4825190	− 0·6540183	0·1889198
2·6	0·395147257	0·4217396	− 0·5641722	0·1669661
2·7	0·434632177	0·3692246	− 0·4881569	0·1483041
2·8	0·469226376	0·3237421	− 0·4231475	0·1323416
2·9	0·499581609	0·2842952	− 0·3671626	0·1186091
3·0	0·526258898	0·2500569	− 0·3187676	0·1067312
3·1	0·549743110	0·2203254	− 0·2768665	0·0964053
3·2	0·570453981	0·1944966	− 0·2405810	0·0873860
3·3	0·588754980	0·1720464	− 0·2091772	0·0794729
3·4	0·604960663	0·1525195	− 0·1820231	0·0725012
3·5	0·619343038	0·1355188	− 0·1585642	0·0663345
3·6	0·632137102	0·1206999	− 0·1383115	0·0608595
3·7	0·643545741	0·1077640	− 0·1208329	0·0559815
3·8	0·653744053	0·0964535	− 0·1057495	0·0516209
3·9	0·662883156	0·0865456	− 0·0927273	0·0477104
4·0	0·671093511	0·0778490	− 0·0814769	0·0441930
4·1	0·678487830	0·0701995		
4·2	0·685163590	0·0634561		
4·3	0·691205197	0·0574978		
4·4	0·696685853	0·0522211		
4·5	0·701669155	0·0475370		
4·6	0·706210452	0·0433689		
4·7	0·710358016	0·0396522		
4·8	0·714154030	0·0363295		
4·9	0·717635442	0·0333526		
5·0	0·720834676	0·0306795		
5·1	0·723780255	0·0282739		
5·2	0·726497324	0·0261044		
5·3	0·729008100	0·0241438		
5·4	0·731332255	0·0223683		
5·5	0·733487242	0·0207573		
5·6	0·735488590	0·0192927		
5·7	0·737350129	0·0179587		
5·8	0·739084215			
5·9	0·740701900			
6·0	0·742213095			

TABLE IV.

s	a_2	$\frac{da_2}{ds}$	$\frac{d^2a_2}{ds^2}$	$\frac{d^2B}{ds^2}$
1·1	4·712430671	−23·5674524	353·7778498	2714·0421556
1·2	3·338677641	− 8·1842849	63·9833620	338·0994801
1·3	2·748186575	− 4·2719222	23·5385350	99·5158628
1·4	2·413552421	− 2·6165816	11·5383099	41·5903453
1·5	2·199749750	− 1·7404858	6·6150213	21·0502321
1·6	2·054078887	− 1·2129862	4·1910062	12·0241949
1·7	1·951189176	− 0·8670402	2·8487879	7·4667972
1·8	1·877222034	− 0·6256941	2·0415169	4·9298514
1·9	1·823907424	− 0·4491805	1·5246488	3·4115162
2·0	1·785977778	− 0·3151961	1·1765660	2·4504574
2·1	1·759898767	− 0·2104538	0·9317735	1·8143467
2·2	1·743191735	− 0·1266635	0·7527985	1·3776584
2·3	1·734048511	− 0·0584579	0·6172046	1·0686077
2·4	1·731101707	− 0·0022446	0·5111117	0·8441725
2·5	1·733283155	0·0444521	0·4257317	0·6775330
2·6	1·739734397	− 0·0834016	0·3554181	0·5513990
2·7	1·749749257	0·1159130	0·2964904	0·4542959
2·8	1·762736450	0·1429977	0·2465153	0·3784136
2·9	1·778194874	0·1654597	0·2038291	0·3183164
3·0	1·795696657	0·1839669	0·1672549	0·2701457
3·1	1·814875341	0·1990847	0·1359094	0·2311132
3·2	1·835416877	0·2112993	0·1090891	0·1991721
3·3	1·857052600	0·2210327	0·0862063	0·1727978
3·4	1·879553229	0·2286550	0·0667523	0·1508399
3·5	1·902723862	0·2344831	0·0502748	0·1324195
3·6	1·926399386	0·2387955	0·0363729	0·1168583
3·7	1·950440491	0·2418305	0·0246888	0·1036268
3·8	1·974730055	0·2437969	0·0149073	0·0923083
3·9	1·999170065	0·2448674	0·0067498	0·0825718
4·0	2·023678710	0·2451929	− 0·0000263	0·0741522
4·1	2·048188072			
4·2	2·072641942			
4·3	2·096994101			
4·4	2·121206629			
4·5	2·145248729			
4·6	2·169095532			
4·7	2·192727152			
4·8	2·216127876			
4·9	2·239285548			
5·0	2·262190878			
5·1	2·284837085			
5·2	2·307219412			
5·3	2·329334842			
5·4	2·351181717			
5·5	2·372759626			
5·6	2·394069088			
5·7	2·415111453			
5·8	2·435888697			
5·9	2·456403398			
6·0	2·476658491			

TABLE V.

s	$\log \frac{D-a_1}{a_2-a_1}$	$\log \frac{D-a_2}{a_2-a_1}$	$\log \frac{a_2-a_1}{B}$	$\log \frac{C}{a_2-a_1}$
1·1	− 0·2703731	− 0·3340164	0·7829376	0·1785481
1·2	− 0·2648898	− 0·3404530	0·6420284	0·0366857
1·3	− 0·2655514	− 0·3396669	0·5630085	− 0·0422097
1·4	− 0·2703733	− 0·3340163	0·5091175	− 0·0952720
1·5	− 0·2786024	− 0·3246791	0·4689668	− 0·1343147
1·6	− 0·2898913	− 0·3124619	0·4376076	− 0·1647456
1·7	− 0·3040718	− 0·2980094	0·4125121	− 0·1895698
1·8	− 0·3210629	− 0·2818806	0·3922674	− 0·2106761
1·9	− 0·3408225	− 0·2645795	0·3760394	− 0·2293626
2·0	− 0·3633160	− 0·2465668	0·3633160	− 0·2465668
2·1	− 0·3884945	− 0·2282618	0·3537671	− 0·2629893
2·2	− 0·4162800	− 0·2100386	0·3471625	− 0·2791561
2·3	− 0·4465548	− 0·1922210	0·3433199	− 0·2954558
2·4	− 0·4791569	− 0·1750770	0·3420726	− 0·3121613
2·5	− 0·5138819	− 0·1588156	0·3432502	− 0·3294471
2·6	− 0·5504880	− 0·1435864	0·3466691	− 0·3474053
2·7	− 0·5887063	− 0·1294828	0·3521304	− 0·3660587
2·8	− 0·6282518	− 0·1165478	0·3594221	− 0·3853776
2·9	− 0·6688362	− 0·1047818	0·3683253	− 0·4052928
3·0	− 0·7101786	− 0·0941525	0·3786205	− 0·4257106
3·1	− 0·7520146	− 0·0846028	0·3900946	− 0·4465227
3·2	− 0·7941032	− 0·0760600	0·4025461	− 0·4676171
3·3	− 0·8362307	− 0·0684424	0·4157891	− 0·4888840
3·4	− 0·8782122	− 0·0616651	0·4296561	− 0·5102213
3·5	− 0·9198918	− 0·0556439	0·4439989	− 0·5315369
3·6	− 0·9611410	− 0·0502979	0·4586883	− 0·5527507
3·7	− 1·0018564	− 0·0455517	0·4736134	− 0·5737946
3·8	− 1·0419569	− 0·0413363	0·4886808	− 0·5946124
3·9	− 1·0813810	− 0·0375891	0·5038117	− 0·6151584
4·0	− 1·1200839	− 0·0342543	0·5189414	− 0·6353968

TABLE VI.

s	$\frac{d}{ds}\ln\frac{D-a_1}{a_2-a_1}$	$\frac{d}{ds}\ln\frac{a_2-D}{a_2-a_1}$	$\frac{d}{ds}\ln\frac{B}{a_2-a_1}$	$\frac{d}{ds}\ln\frac{C}{a_2-a_1}$
1·1	0·2333006	− 0·2701213	4·7714180	− 4·8082395
1·2	0·0433120	− 0·0515432	2·2859136	− 2·2941443
1·3	− 0·0672840	0·0798046	1·4645885	− 1·4520682
1·4	− 0·1521034	0·1761091	1·0563584	− 1·0323528
1·5	− 0·2256015	0·2508526	0·8104409	− 0·7851894
1·6	− 0·2936568	0·3093219	0·6429743	− 0·6273093
1·7	− 0·3590840	0·3541063	0·5179529	− 0·5229307
1·8	− 0·4232368	0·3867239	0·4174638	− 0·4539767
1·9	− 0·4866015	0·4082539	0·3317783	− 0·2113283
2·0	− 0·5490783	0·4196480	0·2553916	− 0·3848220
2·1	− 0·6101431	0·4218897	0·1852118	− 0·3734649
2·2	− 0·6689785	0·4160741	0·1196271	− 0·3725316
2·3	− 0·7246015	0·4034275	0·0579643	− 0·3791382
2·4	− 0·7759944	0·3852820	0·0001282	− 0·3908408
2·5	− 0·8222182	0·3630118	− 0·0536539	− 0·4055525
2·6	− 0·8625199	0·3379617	− 0·1030285	− 0·4215295
2·7	− 0·8963964	0·3113699	− 0·1476551	− 0·4373714
2·8	− 0·9236304	0·2843113	− 0·1872973	− 0·4520218
2·9	− 0·9442686	0·2576566	− 0·2218650	− 0·4647471
3·0	− 0·9585997	0·2320658	− 0·2514285	− 0·4751057
3·1	− 0·9670930	0·2079967	− 0·2761987	− 0·4828979
3·2	− 0·9703394	0·1857293	− 0·2964980	− 0·4881120
3·3	− 0·9689924	0·1653987	− 0·3127222	− 0·4908716
3·4	− 0·9637287	0·1470306	− 0·3253091	− 0·4913890
3·5	− 0·9551960	0·1305703	− 0·3347039	− 0·4899216
3·6	− 0·9440030	0·1159125	− 0·3413417	− 0·4867486
3·7	− 0·9306970	0·1029209	− 0·3456306	− 0·4821453
3·8	− 0·9157640	0·0914456	− 0·3479444	−
3·9	− 0·8996144	0·0813326	− 0·3486130	− 0·4696683
4·0	− 0·8825998	0·0724327	− 0·3479281	− 0·4622393

TABLE VII.

s	$\frac{d}{ds}\log\frac{D-a_1}{a_2-a_1}$	$\frac{d}{ds}\log\frac{a_2-D}{a_2-a_1}$	$\frac{d}{ds}\log\frac{B}{a_2-a_1}$	$\frac{d}{ds}\log\frac{C}{a_2-a_1}$
1·1	0·1013212	−0·1173122	2·0722005	−2·0881919
1·2	0·0188102	−0·0223849	0·9927597	−0·9963342
1·3	−0·0292211	0·0346587	0·6360627	−0·6306252
1·4	−0·0660577	0·0764832	0·4587706	−0·4483451
1·5	−0·0979775	0·1089439	0·3519700	−0·3410034
1·6	−0·1275335	0·1343368	0·2792402	−0·2724370
1·7	−0·1559482	0·1537864	0·2249441	−0·2271059
1·8	−0·1838094	0·1679521	0·1813022	−0·1971596
1·9	−0·2113283	0·1773024	0·1440895	−0·1781155
2·0	−0·2384617	0·1822508	0·1109152	−0·1671261
2·1	−0·2649818	0·1832244	0·0804365	−0·1621937
2·2	−0·2905337	0·1806987	0·0519534	−0·1617884
2·3	−0·3146904	0·1752063	0·0251736	−0·1646576
2·4	−0·3370101	0·1673258	0·0000557	−0·1697400
2·5	−0·3570848	0·1576540	−0·0233016	−0·1761292
2·6	−0·3745876	0·1467749	−0·0447447	−0·1830679
2·7	−0·3893000	0·1352262	−0·0641258	−0·1899480
2·8	−0·4011276	0·1234748	−0·0813422	−0·1963106
2·9	−0·4100906	0·1118988	−0·0963547	−0·2018371
3·0	−0·4163146	0·1007849	−0·1091940	−0·2063358
3·1	−0·4200032	0·0903318	−0·1199516	−0·2097199
3·2	−0·4214130	0·0806612	−0·1287674	−0·2119843
3·3	−0·4208281	0·0718317	−0·1358135	−0·2131828
3·4	−0·4185421	0·0638546	−0·1412799	−0·2134075
3·5	−0·4148364	0·0567060	−0·1453601	−0·2127702
3·6	−0·4099753	0·0503402	−0·1482428	−0·2113922
3·7	−0·4041966	0·0446980	−0·1501055	−0·2093930
3·8	−0·3977113	0·0397143	−0·1511103	−0·2068865
3·9	−0·3906976	0·0353223	−0·1514007	−0·2039744
4·0	−0·3833082	0·0314571	−0·1511033	−0·2007480

TABLE VIII.

s	$\frac{d^2}{ds^2} \ln \frac{D-a_1}{a_2-a_1}$	$\frac{d^2}{ds^2} \ln \frac{a_2-D}{a_2-a_1}$	$\frac{d^2}{ds^2} \ln \frac{B}{a_2-a_1}$	$\frac{d^2}{ds^2} \ln \frac{C}{a_2-a_1}$
1·1	3·2012369	− 2·8823175	− 49·8471529	50·1660711
1·2	1·5882493	− 1·3387215	− 12·3653830	12·6149109
1·3	1·1029867	− 0·9398365	− 5·4517952	5·6149457
1·4	0·8426700	− 0·7777264	− 3·0613258	3·1262688
1·5	0·6603260	− 0·7013455	− 1·9829061	1·9418867
1·6	0·5131497	− 0·6642307	− 1·4224635	1·2713825
1·7	0·3849809	− 0·6464875	− 1·1064039	0·8448968
1·8	0·2691130	− 0·6373267	− 0·9190043	0·5507906
1·9	0·1630462	− 0·6297743	− 0·8036034	0·3368748
2·0	0·0664621	− 0·6188673	− 0·7291388	0·1767333
2·1	− 0·0198061	− 0·6010439	− 0·6770593	0·0562096
2·2	− 0·0944554	− 0·5740083	− 0·6356975	− 0·0327662
2·3	− 0·1562396	− 0·5367474	− 0·5976783	− 0·0953083
2·4	− 0·2043736	− 0·4895164	− 0·5586283	− 0·1352617
2·5	− 0·2387769	− 0·4336899	− 0·5164007	− 0·1560656
2·6	− 0·2601498	− 0·3715047	− 0·4705126	− 0·1611420
2·7	− 0·2698800	− 0·3056853	− 0·4216209	− 0·1539413
2·8	− 0·2698458	− 0·2390551	− 0·3710612	− 0·1378401
2·9	− 0·2621519	− 0·1741976	− 0·3204132	− 0·1159359
3·0	− 0·2489036	− 0·1132197	− 0·2712123	− 0·0909111
3·1	− 0·2320176	− 0·0576420	− 0·2247372	− 0·0649226
3·2	− 0·2131111	− 0·0083844	− 0·1819154	− 0·0395794
3·3	− 0·1934589	0·0341677	− 0·1433107	− 0·0159806
3·4	− 0·1740020	0·0700456	− 0·1091644	0·0052087
3·5	− 0·1553828	0·0995941	− 0·0794574	0·0236685
3·6	− 0·1380031	0·1233474	− 0·0539852	0·0393294
3·7	− 0·1220777	0·1419442	− 0·0324245	0·0522920
3·8	− 0·1076886	0·1560605	− 0·0143890	0·0627604
3·9	− 0·0948225	0·1663514	0·0005330	0·0709964
4·0	− 0·0834090	0·1734356	0·0127469	0·0772803

TABLE IX.

Electrons produced by Electrons.

$\log_{10} Q_0^{(1)}(w_0, w; \zeta)$ $\epsilon = \log_{10}(w_0/w)$

ζ	$\epsilon = 1{\cdot}0$	2·0	3·0	4·0	5·0
0·1					
0·2					
0·3					
0·4					
0·5				0·448	0·582
0·6		0·236	0·422	0·563	0·692
0·7		0·304	0·512	0·670	0·801
0·8		0·364	0·594	0·768	0·911
0·9		0·418	0·668	0·858	1·015
1·0		0·467	0·737	0·943	1·116
1·1		0·511	0·801	1·024	1·214
1·2		0·551	0·861	1·100	1·302
1·3		0·587	0·917	1·172	1·386
1·4		0·621	0·971	1·240	1·465
1·5		0·651	1·021	1·306	1·543
1·6		0·679	1·068	1·368	1·615
1·7		0·705	1·113	1·427	1·687
1·8		0·728	1·155	1·485	1·755
1·9		0·749	1·196	1·539	1·821
2·0		0·768	1·234	1·591	1·885
2·2		0·801	1·303	1·689	2·008
2·4		0·827	1·366	1·780	2·122
2·6	−0·096	0·848	1·422	1·864	2·230
2·8	−0·132	0·863	1·473	1·941	2·330
3·0	−0·171	0·874	1·518	2·013	2·42
3·2	−0·212	0·880	1·559	2·079	2·508
3·4	−0·254	0·883	1·594	2·140	2·590
3·6	−0·299	0·882	1·626	2·197	2·667
3·8	−0·344	0·878	1·654	2·249	2·739
4·0	−0·391	0·871	1·678	2·297	2·807
4·2	−0·439	0·861	1·699	2·342	2·872
4·4	−0·488	0·849	1·717	2·383	2·933
4·6	−0·537	0·834	1·731	2·421	2·989
4·8	−0·587	0·817	1·743	2·456	3·043
5·0	−0·638	0·799	1·753	2·487	3·093
5·5	−0·768	0·744	1·766	2·555	3·206
6·0	−0·899	0·681	1·765	2·606	3·302
6·5	−1·033	0·609	1·753	2·645	3·382
7·0	−1·168	0·532	1·731	2·671	3·450
7·5	−1·305	0·448	1·700	2·686	3·505
8·0	−1·443	0·360	1·661	2·691	3·549
8·5	−1·581	0·267	1·616	2·688	3·584
9·0	−1·721	0·171	1·564	2·676	3·608
9·5	−1·861	0·072	1·506	2·658	3·625
10·0	−2·002	−0·030	1·443	2·632	3·633
12·0	−2·573	−0·463	1·151	2·475	3·602
14·0		−0·925	0·807	2·248	3·486
16·0		−1·408	0·427	1·969	3·304
18·0		−1·908	0·018	1·649	3·073
20·0		−2·420	−0·414	1·298	2·801
25·0			−1·566	0·317	1·989
30·0			−2·793	−0·769	1·041
35·0				−1·928	0·001
40·0				−3·142	−1·109
45·0					−2·272

TABLE IXa.

Electrons produced by Electrons (continued).

$\log_{10} Q_0^{(1)}(w_0, w; \zeta)$ $\epsilon = \log_{10}(w_0/w)$.

ζ	$\epsilon = 6{\cdot}0$	7·0	8·0	9·0	10·0
0·2		0·166	0·250	0·331	
0·4		0·537	0·603	0·675	
0·6		0·861	0·942	1·019	1·088
0·8	1·033	1·142	1·241	1·333	1·422
1·0	1·263	1·393	1·511	1·618	1·721
1·2	1·470	1·621	1·757	1·881	1·995
1·4	1·659	1·831	1·983	2·125	2·252
1·6	1·832	2·023	2·192	2·348	2·495
1·8	1·993	2·201	2·388	2·559	2·717
2·0	2·143	2·369	2·573	2·760	2·930
2·2	2·283	2·528	2·749	2·948	3·133
2·4	2·416	2·678	2·912	3·126	3·323
2·6	2·542	2·820	3·068	3·297	3·507
2·8	2·660	2·953	3·218	3·458	3·680
3·0	2·771	3·080	3·359	3·612	3·847
3·2	2·876	3·201	3·494	3·763	4·007
3·4	2·977	3·318	3·624	3·902	4·160
3·6	3·071	3·427	3·747	4·039	4·309
3·8	3·161	3·532	3·865	4·170	4·450
4·0	3·246	3·633	3·980	4·295	4·589
4·2	3·327	3·728	4·088	4·417	4·718
4·4	3·404	3·819	4·193	4·533	4·847
4·6	3·477	3·907	4·294	4·646	4·969
4·8	3·547	3·991	4·390	4·753	5·089
5·0	3·613	4·071	4·484	4·858	5·203
5·5	3·765	4·258	4·700	5·103	5·475
6·0	3·899	4·425	4·899	5·329	5·726
6·5	4·017	4·576	5·078	5·536	5·957
7·0	4·120	4·711	5·242	5·726	6·171
7·5	4·211	4·833	5·393	5·901	6·370
8·0	4·289	4·942	5·529	6·063	6·555
8·5	4·356	5·040	5·653	6·213	6·727
9·0	4·414	5·126	5·767	6·351	6·887
9·5	4·462	5·202	5·869	6·477	7·036
10·0	4·501	5·270	5·962	6·593	7·174
12·0	4·586	5·460	6·249	6·970	7·634
14·0	4·573	5·544	6·423	7·227	7·970
16·0	4·485	5·545	6·508	7·390	8·207
18·0	4·338	5·479	6·519	7·474	8·360
20·0	4·144	5·359	6·470	7·494	8·444
25·0	3·498	4·875	6·144	7·320	8·417
30·0	2·688	4·201	5·603	6·909	8·133
35·0	1·764	2·394	4·910	6·329	7·664
40·0	0·757	2·487	4·104	5·622	7·055
45·0	−0·315	1·505	3·210	4·817	6·337
50·0	−1·439	0·463	2·248	3·934	5·533
60·0	−3·805	−1·762	0·164	1·989	3·727
70·0			−2·080	−0·136	1·719
80·0				−2·395	−0·437
90·0					−2·707

TABLE X.

Ionization Loss.

$100g\ (\zeta,\ w)$

$w' = w + g\ (\zeta,\ w)\ w_0.$

ζ	$\epsilon = 1$	2	3	4	5	6	7	8	9	10
1	–	51	43	37	33	32	30	28	27	25
2	–	67	55	48	43	40	37	35	33	32
3	98	77	65	57	51	47	43	41	39	37
4	101	86	72	63	57	52	49	45	43	41
5	–	92	78	69	62	57	53	50	47	45
6	–	96	84	74	67	62	57	53	51	48
7	–	100	88	79	71	65	61	57	54	52
8	–	102	92	83	75	69	64	60	57	54
9	–	105	95	86	79	72	67	63	60	57
10	–	107	97	89	82	75	71	66	63	60
12	–	109	101	94	87	81	76	71	68	65
14	–	–	105	98	91	86	80	76	72	69
16	–	–	108	101	95	89	85	80	76	73
18	–	–	110	104	98	92	88	83	79	76
20	–	–	112	106	100	95	90	87	83	79
25	–	–	–	111	105	101	97	93	89	86
30	–	–	–	114	109	105	101	97	94	91
35	–	–	–	–	112	108	104	101	98	95
40	–	–	–	–	114	111	108	104	101	98
50	–	–	–	–	–	115	112	109	106	103
60	–	–	–	–	–	–	114	112	110	108
70	–	–	–	–	–	–	–	–	112	111
80	–	–	–	–	–	–	–	–	115	113

TABLE XI.

Photons produced by Electrons.

$\log_{10} P_0^{(1)}(w_0, w; \zeta)$ $\epsilon = \log_{10}(w_0/w)$.

ζ	$\epsilon = 1{\cdot}0$	2·0	3·0	4·0	5·0
0·1					
0·2					
0·3			0·357	0·472	
0·4			0·457	0·610	
0·5			0·557	0·731	0·891
0·6		0·384	0·646	0·840	1·006
0·7		0·439	0·725	0·938	1·110
0·8		0·492	0·797	1·026	1·209
0·9		0·539	0·863	1·106	1·303
1·0		0·582	0·922	1·180	1·390
1·1		0·621	0·978	1·252	1·475
1·2	0·094	0·655	1·031	1·318	1·554
1·3	0·103	0·687	1·080	1·382	1·630
1·4	0·109	0·716	1·127	1·443	1·703
1·5	0·112	0·743	1·171	1·501	1·772
1·6	0·114	0·767	1·212	1·556	1·840
1·7	0·113	0·789	1·252	1·610	1·904
1·8	0·111	0·810	1·290	1·660	1·968
1·9	0·107	0·828	1·325	1·710	2·027
2·0	0·101	0·845	1·358	1·756	2·086
2·2	0·086	0·875	1·421	1·845	2·197
2·4	0·066	0·900	1·477	1·927	2·301
2·6	0·043	0·919	1·528	2·003	2·398
2·8	0·016	0·934	1·575	2·074	2·489
3·0	−0·014	0·946	1·616	2·140	2·575
3·2	−0·046	0·953	1·653	2·201	2·656
3·4	−0·080	0·958	1·687	2·257	2·732
3·6	−0·116	0·959	1·716	2·310	2·804
3·8	−0·154	0·958	1·742	2·359	2·872
4·0	−0·193	0·953	1·766	2·404	2·936
4·2	−0·233	0·947	1·786	2·446	2·996
4·4	−0·274	0·938	1·803	2·485	3·054
4·6	−0·317	0·927	1·818	2·521	3·107
4·8	−0·360	0·915	1·830	2·554	3·157
5·0	−0·404	0·900	1·840	2·584	3·205
5·5	−0·517	0·856	1·855	2·649	3·313
6·0	−0·634	0·804	1·859	2·700	3·405
6·5	−0·754	0·743	1·851	2·738	3·483
7·0	−0·877	0·676	1·834	2·765	3·548
7·5	−1·002	0·603	1·809	2·782	3·602
8·0	−1·129	0·525	1·776	2·789	3·646
8·5	−1·257	0·442	1·736	2·789	3·681
9·0	−1.387	0·355	1·690	2·781	3·706
9·5	−1·518	0·264	1·638	2·765	3·724
10·0	−1·651	0·171	1·582	2·744	3·734
12·0	−2·192	−0·231	1·314	2·603	3·713
14·0	−2·747	−0·667	0·993	2·393	3·608
16·0		−1·127	0·634	2·131	3·440
18·0		−1·607	0·243	1·828	3·222
20·0		−2·101	−0·171	1·493	2·963
25·0			−1·285	0·546	2·182
30·0				−0·510	1·263
35·0				−1·643	0·247
40·0				−2·834	−0·840
45·0					−1·964

TABLE XIa.

Photons produced by Electrons (continued).

$\log_{10} P_0^{(1)}(w_0, w; \zeta)$ $\epsilon = \log_{10}(w_0/w)$.

ζ	$\epsilon = 6{\cdot}0$	7·0	8·0	9·0	10·0
0·2		0·434	0·628	0·755	0·857
0·4		0·869	1·011	1·118	1·203
0·6	1·155	1·211	1·334	1·442	1·539
0·8	1·369	1·493	1·615	1·731	1·843
1·0	1·571	1·730	1·867	1·994	2·115
1·2	1·756	1·934	2·095	2·237	2·369
1·4	1·926	2·124	2·301	2·461	2·608
1·6	2·083	2·300	2·492	2·667	2·830
1·8	2·231	2·464	2·673	2·863	3·038
2·0	2·369	2·619	2·844	3·051	3·239
2·2	2·498	2·765	3·006	3·224	3·426
2·4	2·621	2·905	3·159	3·392	3·606
2·6	2·737	3·037	3·306	3·552	3·778
2·8	2·847	3·161	3·446	3·703	3·941
3·0	2·950	3·280	3·578	3·850	4·101
3·2	3·048	3·395	3·706	3·990	4·251
3·4	3·141	3·503	3·828	4·123	4·398
3·6	3·230	3·606	3·944	4·254	4·537
3·8	3·314	3·705	4·057	4·377	4·673
4·0	3·394	3·800	4·164	4·497	4·802
4·2	3·471	3·890	4·268	4·611	4·929
4·4	3·544	3·977	4·368	4·723	5·050
4·6	3·613	4·061	4·463	4·830	5·168
4·8	3·679	4·140	4·555	4·933	5·281
5·0	3·742	4·217	4·644	5·033	5·392
5·5	3·877	4·394	4·852	5·269	5·652
6·0	4·014	4·555	5·041	5·484	5·893
6·5	4·128	4·699	5·214	5·683	6·116
7·0	4·227	4·830	5·372	5·867	6·324
7·5	4·315	4·947	5·516	6·038	6·517
8·0	4·391	5·053	5·649	6·194	6·696
8·5	4·457	5·147	5·770	6·338	6·863
9·0	4·513	5·231	5·879	6·471	7·018
9·5	4·561	5·306	5·979	6·594	7·162
10·0	4·601	5·373	6·070	6·708	7·297
12·0	4·688	5·561	6·352	7·076	7·747
14·0	4·683	5·648	6·526	7·331	8·076
16·0	4·605	5·655	6·613	7·494	8·311
18·0	4·468	5·597	6·629	7·580	8·464
20·0	4·284	5·485	6·586	7·604	8·551
25·0	3·666	5·024	6·278	7·444	8·534
30·0	2·881	4·372	5·757	7·051	8·264
35·0	1·981	3·587	5·084	6·488	7·810
40·0	0·994	2·700	4·297	5·798	7·217
45·0	−0·058	1·736	3·420	5·010	6·515
50·0	−1·165	0·710	2·474	4·142	5·726
60·0		−1·484	0·419	2·225	3·945
70·0			−1·800	0·124	1·962
80·0				−2·113	−0·172
90·0					−2·423

TABLE XII.

Electrons produced by Photons.

$\log_{10} Q_0^{(2)}(w_0, w; \zeta)$ $\epsilon = \log_{10}(w_0/w)$.

ζ	$\epsilon = 1{\cdot}0$	2·0	3·0	4·0	5·0
0·1					
0·2					
0·3					
0·4					
0·5			−0·079		
0·6			0·039		
0·7			0·146	0·263	
0·8			0·243	0·376	
0·9		0·141	0·332	0·480	
1·0		0·202	0·414	0·579	0·718
1·1		0·258	0·490	0·672	0·825
1·2		0·309	0·561	0·759	0·924
1·3		0·356	0·628	0·841	1·020
1·4	−0·028	0·399	0·691	0·920	1·112
1·5	−0·015	0·439	0·751	0·995	1·198
1·6	−0·005	0·477	0·807	1·065	1·283
1·7	0·003	0·511	0·860	1·133	1·362
1·8	0·009	0·543	0·910	1·199	1·440
1·9	0·012	0·572	0·958	1·260	1·513
2·0	0·014	0·599	1·003	1·320	1·584
2·2	0·012	0·648	1·087	1·431	1·719
2·4	0·005	0·689	1·164	1·534	1·844
2·6	−0·008	0·724	1·232	1·630	1·962
2·8	−0·025	0·754	1·295	1·719	2·072
3·0	−0·045	0·778	1·351	1·801	2·175
3·2	−0·069	0·798	1·403	1·877	2·272
3·4	−0·097	0·813	1·449	1·948	2·363
3·6	−0·126	0·825	1·491	2·013	2·449
3·8	−0·158	0·833	1·529	2·075	2·530
4·0	−0·191	0·837	1·562	2·131	2·606
4·2	−0·227	0·839	1·593	2·184	2·678
4·4	−0·263	0·838	1·619	2·234	2·746
4·6	−0·301	0·835	1·643	2·279	2·810
4·8	−0·341	0·828	1·663	2·321	2·871
5·0	−0·381	0·820	1·681	2·361	2·928
5·5	−0·486	0·791	1·714	2·446	3·057
6·0	−0·597	0·750	1·733	2·515	3·168
6·5	−0·711	0·701	1·740	2·569	3·264
7·0	−0·828	0·643	1·735	2·610	3·345
7·5	−0·948	0·578	1·721	2·640	3·413
8·0	−1·070	0·507	1·698	2·660	3·471
8·5	−1·195	0·431	1·667	2·670	3·517
9·0	−1·321	0·351	1·630	2·671	3·554
9·5	−1·499	0·266	1·586	2·665	3·582
10·0	−1·579	0·177	1·536	2·652	3·602
12·0	−2·110	−0·208	1·291	2·539	3·612
14·0	−2·656	−0·631	0·988	2·351	3·532
16·0		−1·081	0·641	2·105	3·384
18·0		−1·552	0·262	1·816	3·181
20·0		−2·039	−0·143	1·492	2·936
25·0			−1·239	0·567	2·180
30·0			−2·422	−0·474	1·279
35·0				−1·595	0·277
40·0				−2·776	−0·800
45·0					−1·935

TABLE XIIa.

Electrons produced by Photons (continued).

$\log_{10} Q_0^{(1)}$ $(w_0, w ; \zeta)$ $\epsilon = \log_{10} (w_0/w)$.

ζ	$\epsilon = 6{\cdot}0$	7·0	8·0	9·0	10·0
0·2					
0·4	−0·055				
0·6	0·288	0·359	0·418		
0·8	0·583	0·672	0·750	0·826	0·895
1·0	0·841	0·950	1·047	1·137	1·226
1·2	1·071	1·201	1·317	1·423	1·525
1·4	1·280	1·429	1·565	1·687	1·800
1·6	1·472	1·639	1·791	1·932	2·059
1·8	1·649	1·835	2·003	2·157	2·302
2·0	1·815	2·018	2·203	2·371	2·527
2·2	1·969	2·190	2·392	2·576	2·744
2·4	2·113	2·352	2·567	2·764	2·947
2·6	2·249	2·504	2·734	2·945	3·140
2·8	2·377	2·648	2·894	3·119	3·326
3·0	2·498	2·785	3·045	3·282	3·501
3·2	2·613	2·916	3·189	3·440	3·672
3·4	2·723	3·040	3·328	3·590	3·832
3·6	2·825	3·158	3·459	3·734	3·988
3·8	2·923	3·271	3·585	3·873	4·137
4·0	3·016	3·380	3·707	4·005	4·282
4·2	3·105	3·482	3·822	4·134	4·421
4·4	3·189	3·581	3·934	4·256	4·554
4·6	3·269	3·676	4·042	4·376	4·684
4·8	3·345	3·765	4·144	4·490	4·809
5·0	3·418	3·852	4·244	4·600	4·930
5·5	3·586	4·054	4·475	4·860	5·217
6·0	3·734	4·235	4·687	5·099	5·479
6·5	3·865	4·399	4·879	5·319	5·723
7·0	3·982	4·546	5·055	5·520	5·949
7·5	4·084	4·680	5·217	5·707	6·159
8·0	4·175	4·800	5·364	5·879	6·354
8·5	4·254	4·908	5·499	6·039	6·536
9·0	4·323	5·006	5·622	6·186	6·706
9·5	4·381	5·092	5·735	6·321	6·863
10·0	4·431	5·169	5·837	6·446	7·010
12·0	4·554	5·395	6·158	6·856	7·502
14·0	4·576	5·512	6·363	7·144	7·866
16·0	4·519	5·543	6·475	7·334	8·129
18·0	4·401	5·504	6·513	7·442	8·305
20·0	4·232	5·409	6·488	7·485	8·412
25·0	3·642	4·980	6·215	7·363	8·435
30·0	2·878	4·351	5·719	6·996	8·194
35·0	1·993	3·582	5·064	6·454	7·762
40·0	1·019	2·709	4·292	5·780	7·186
45·0	−0·024	1·757	3·428	5·005	6·498
50·0	−1·122	0·740	2·492	4·148	5·720
60·0		−1·440	0·452	2·248	3·958
70·0			−1·754	0·160	1·988
80·0				−2·066	−0·134
90·0					−2·376

TABLE XIII.

Photons produced by photons.

$\log_{10} P_0^{(2)}(w_0, w; \zeta)$ $\epsilon = \log_{10}(w_0/w)$.

ζ	$\epsilon = 1{\cdot}0$	2·0	3·0	4·0	5·0
0·1					
0·2					
0·3					
0·4					
0·5					
0·6					
0·7			0·458	0·647	0·737
0·8			0·528	0·721	0·842
0·9			0·594	0·794	0·941
1·0		0·399	0·657	0·864	1·037
1·1		0·437	0·717	0·940	1·127
1·2		0·474	0·774	1·013	1·214
1·3		0·508	0·829	1·084	1·297
1·4	0·067	0·541	0·882	1·152	1·378
1·5	0·072	0·572	0·932	1·216	1·455
1·6	0·076	0·602	0·979	1·279	1·529
1·7	0·079	0·629	1·025	1·339	1·601
1·8	0·81	0·655	1·069	1·396	1·669
1·9	0·082	0·679	1·110	1·451	1·737
2·0	0·082	0·702	1·150	1·504	1·800
2·2	0·079	0·743	1·224	1·604	1·923
2·4	0·073	0·779	1·291	1·698	2·037
2·6	0·063	0·810	1·353	1·785	2·145
2·8	0·051	0·836	1·409	1·865	2·246
3·0	0·035	0·858	1·461	1·940	2·342
3·2	0·017	0·877	1·508	2·010	2·432
3·4	−0·003	0·892	1·550	2·076	2·517
3·6	−0·025	0·904	1·589	2·137	2·596
3·8	−0·050	0·912	1·624	2·194	2·672
4·0	−0·076	0·918	1·656	2·247	2·743
4·2	−0·104	0·921	1·685	2·296	2·810
4·4	−0·133	0·922	1·710	2·343	2·875
4·6	−0·164	0·921	1·733	2·386	2·936
4·8	−0·196	0·917	1·753	2·426	2·992
5·0	−0·229	0·912	1·770	2·463	3·046
5·5	−0·317	0·889	1·804	2·544	3·169
6·0	−0·411	0·857	1·825	2·611	3·275
6·5	−0·510	0·815	1·834	2·664	3·368
7·0	−0·613	0·766	1·833	2·705	3·446
7·5	−0·720	0·710	1·822	2·735	3·513
8·0	−0·830	0·648	1·804	2·756	3·569
8·5	−0·943	0·580	1·778	2·768	3·615
9·0	−1·059	0·508	1·745	2·772	3·652
9·5	−1·177	0·431	1·707	2·769	3·680
10·0	−1·297	0·350	1·663	2·759	3·702
12·0	−1·793	−0·006	1·439	2·660	3·719
14·0	−2·311	−0·402	1·157	2·487	3·649
16·0	−2·844	−0·829	0·831	2·257	3·513
18·0		−1·280	0·470	1·984	3·323
20·0		−1·749	0·082	1·674	3·090
25·0			−0·978	0·783	2·364
30·0			−2·129	−0·228	1·490
35·0				−1·323	0·512
40·0				−2·482	−0·543
45·0					−1·657

TABLE XIIIa.

Photons produced by Photons (continued).

$\log_{10} P_0^{(2)}(w_0, w; \zeta)$ $\epsilon = \log_{10}(w_0/w)$.

ζ	$\epsilon = 6{\cdot}0$	7·0	8·0	9·0	10·0
0·2					
0·4					
0·6			0·901	0·972	1·035
0·8			1·185	1·276	1·361
1·0	1·185	1·319	1·444	1·553	1·658
1·2	1·387	1·542	1·683	1·809	1·926
1·4	1·573	1·748	1·905	2·050	2·179
1·6	1·746	1·939	2·111	2·271	2·420
1·8	1·907	2·117	2·305	2·479	2·640
2·0	2·058	2·285	2·490	2·679	2·851
2·2	2·200	2·443	2·665	2·866	3·054
2·4	2·333	2·594	2·828	3·043	3·242
2·6	2·458	2·736	2·985	3·215	3·426
2·8	2·577	2·870	3·136	3·376	3·598
3·0	2·689	2·998	3·276	3·530	3·766
3·2	2·797	3·121	3·412	3·680	3·926
3·4	2·899	3·237	3·543	3·821	4·079
3·6	2·994	3·348	3·667	3·959	4·228
3·8	3·086	3·455	3·787	4·089	4·369
4·0	3·174	3·556	3·901	4·216	4·508
4·2	3·258	3·653	4·011	4·338	4·639
4·4	3·337	3·747	4·118	4·455	4·768
4·6	3·412	3·836	4·218	4·569	4·890
4·8	3·485	3·922	4·317	4·677	5·011
5·0	3·554	4·005	4·411	4·784	5·126
5·5	3·713	4·196	4·633	5·032	5·400
6·0	3·855	4·371	4·835	5·260	5·653
6·5	3·981	4·527	5·021	5·471	5·889
7·0	4·093	4·669	5·190	5·667	6·107
7·5	4·192	4·798	5·345	5·847	6·310
8·0	4·280	4·914	5·488	6·013	6·499
8·5	4·357	5·019	5·619	6·167	6·675
9·0	4·424	5·113	5·739	6·310	6·839
9·5	4·482	5·198	5·847	6·442	6·992
10·0	4·531	5·273	5·947	6·564	7·135
12·0	4·656	5·497	6·262	6·965	7·616
14·0	4·683	5·615	6·466	7·248	7·973
16·0	4·635	5·651	6·580	7·437	8·233
18·0	4·525	5·619	6·622	7·548	8·410
20·0	4·367	5·531	6·602	7·593	8·518
25·0	3·802	5·123	6·345	7·484	8·550
30·0	3·063	4·516	5·869	7·133	8·323
35·0	2·201	3·768	5·233	6·608	7·905
40·0	1·248	2·915	4·478	5·951	7·344
45·0	0·223	1·980	3·632	5·192	6·671
50·0	−0·857	0·980	2·711	4·350	5·908
60·0		−1·170	0·701	2·478	4·173
70·0			−1·480	0·415	2·227
80·0				−1·789	0·126
90·0					−2·097

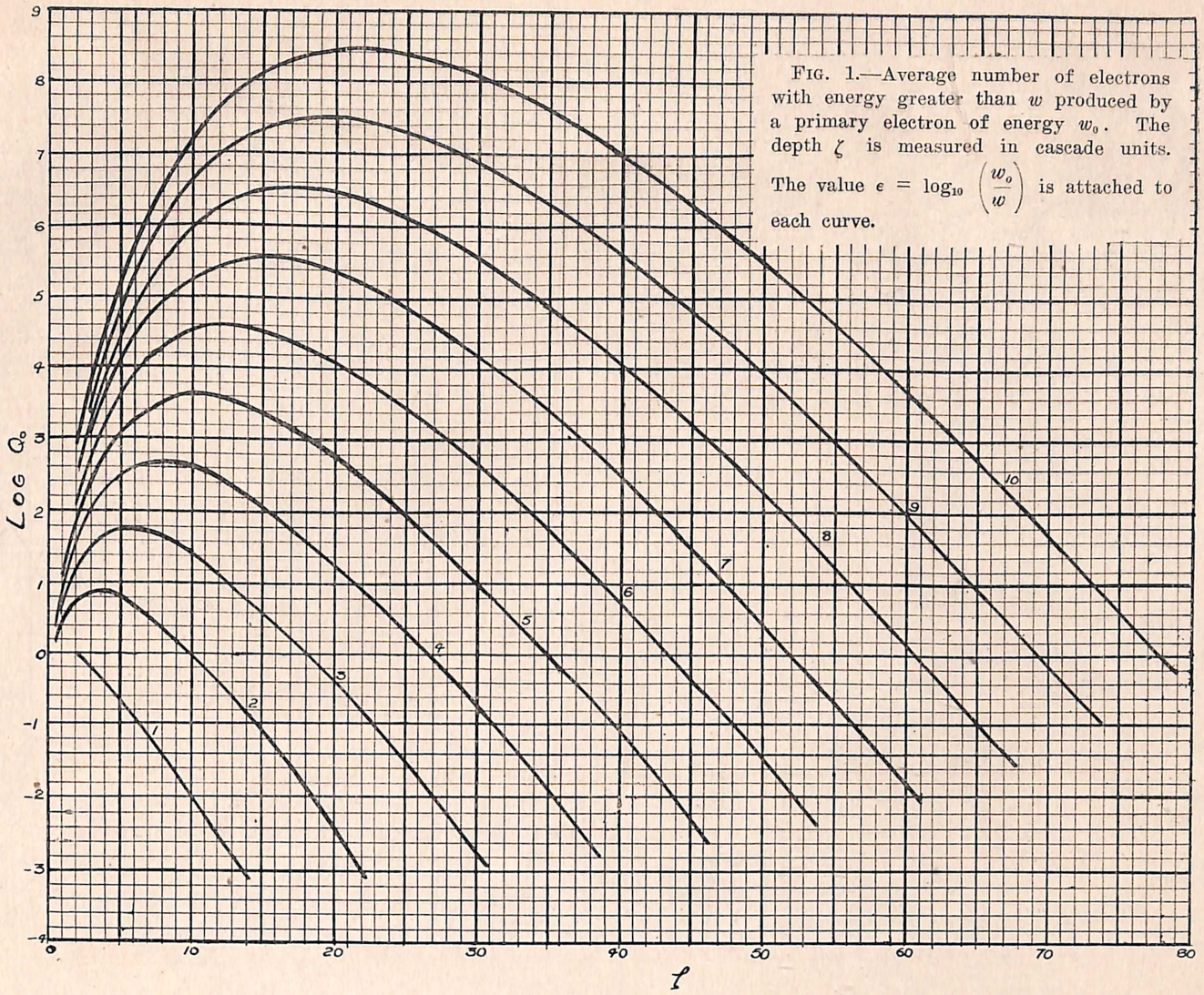

FIG. 1.—Average number of electrons with energy greater than w produced by a primary electron of energy w_0. The depth ζ is measured in cascade units. The value $\epsilon = \log_{10}\left(\frac{w_0}{w}\right)$ is attached to each curve.

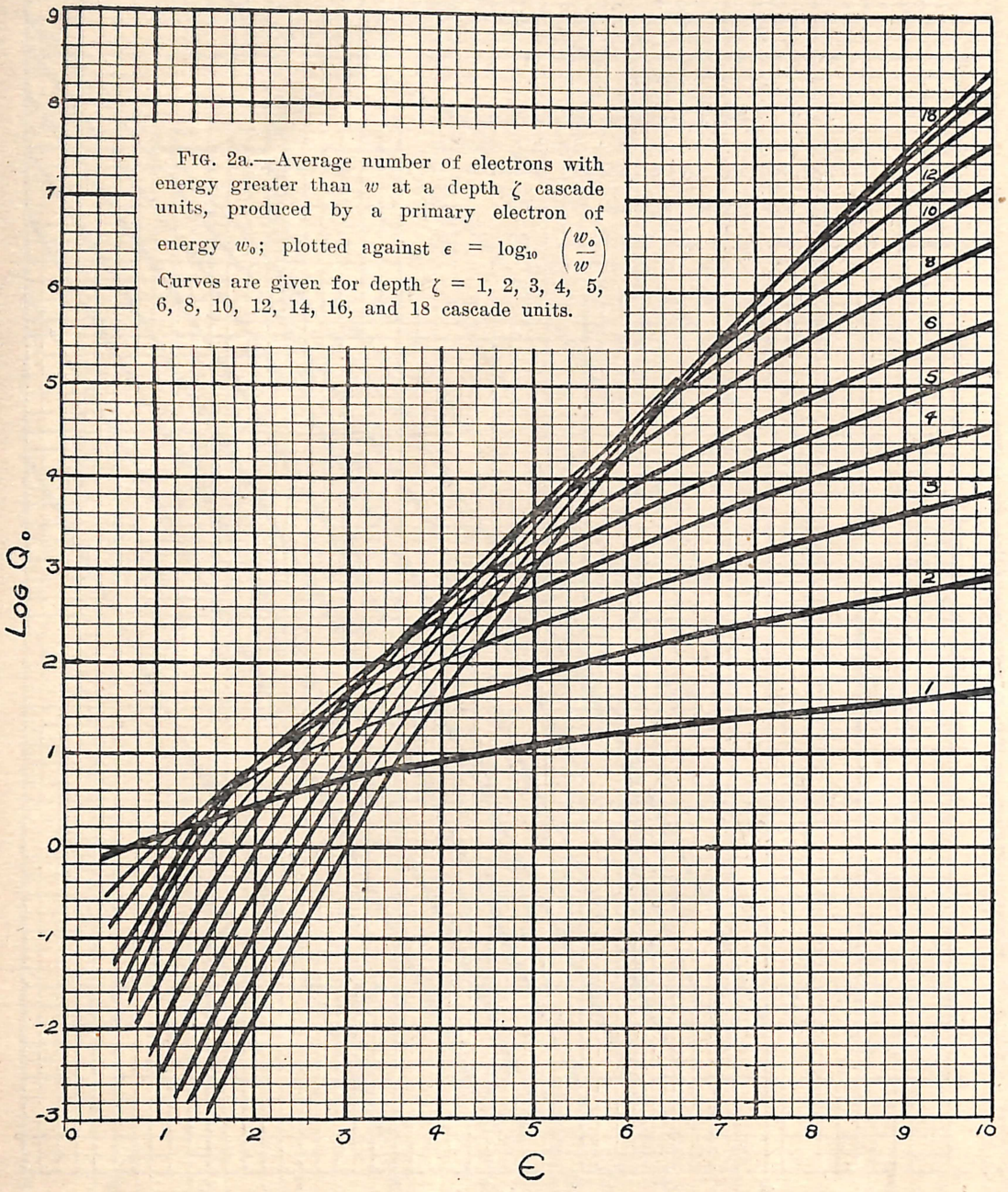

FIG. 2a.—Average number of electrons with energy greater than w at a depth ζ cascade units, produced by a primary electron of energy w_0; plotted against $\epsilon = \log_{10}\left(\frac{w_0}{w}\right)$. Curves are given for depth ζ = 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, and 18 cascade units.

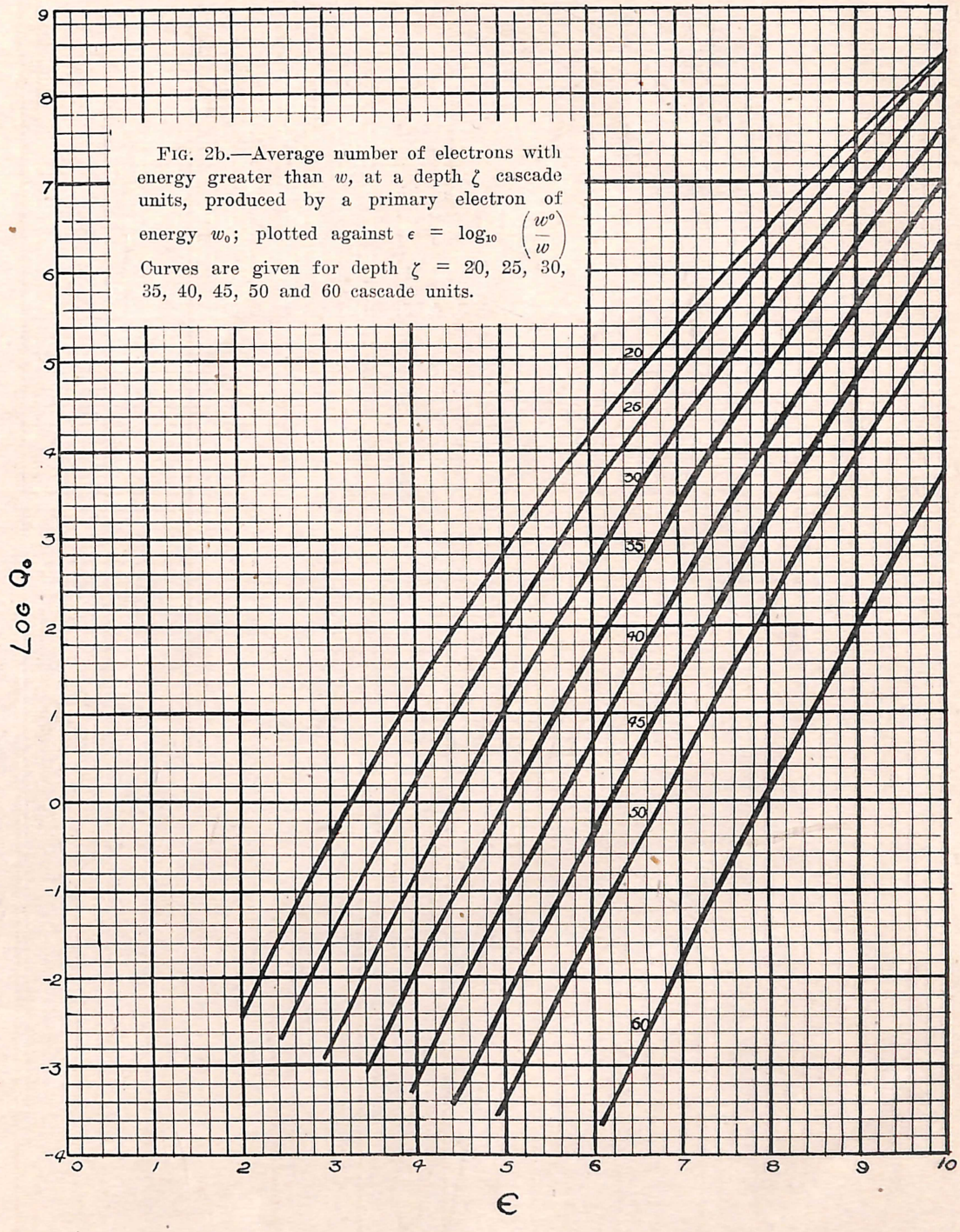

FIG. 2b.—Average number of electrons with energy greater than w, at a depth ζ cascade units, produced by a primary electron of energy w_0; plotted against $\epsilon = \log_{10}\left(\frac{w^0}{w}\right)$ Curves are given for depth ζ = 20, 25, 30, 35, 40, 45, 50 and 60 cascade units.

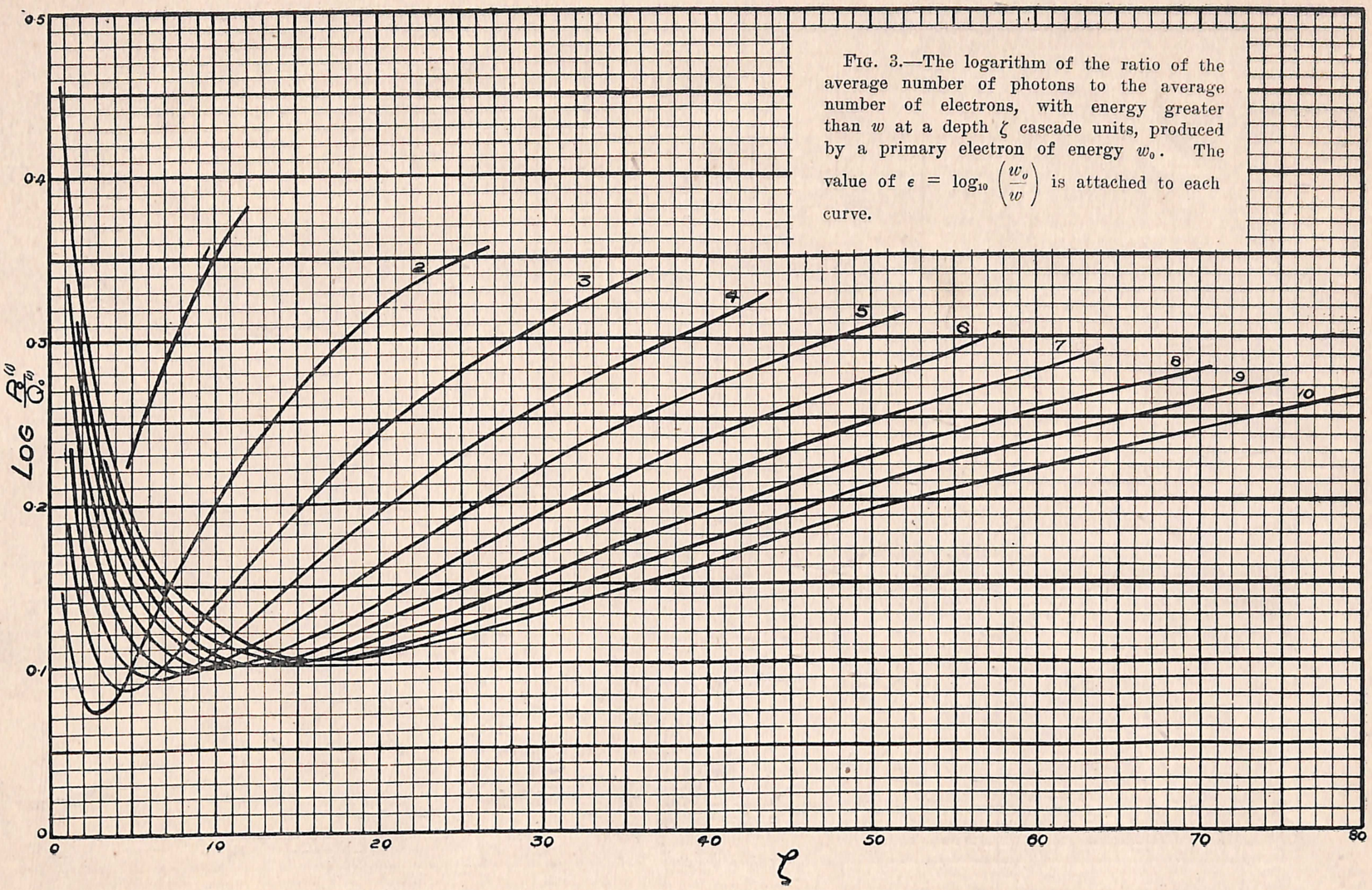

FIG. 3.—The logarithm of the ratio of the average number of photons to the average number of electrons, with energy greater than w at a depth ζ cascade units, produced by a primary electron of energy w_0. The value of $\epsilon = \log_{10}\left(\frac{w_0}{w}\right)$ is attached to each curve.

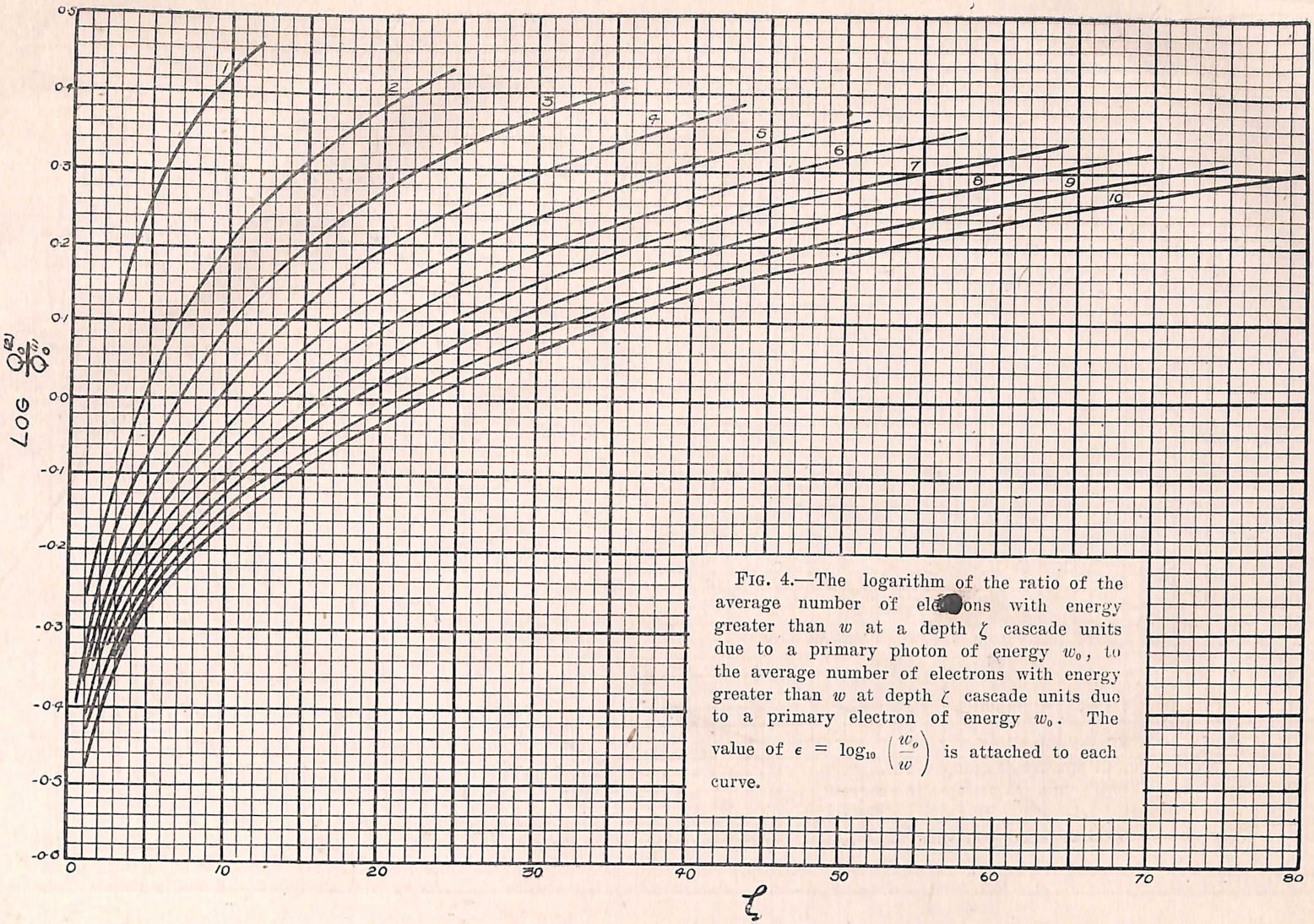

FIG. 4.—The logarithm of the ratio of the average number of electrons with energy greater than w at a depth ζ cascade units due to a primary photon of energy w_0, to the average number of electrons with energy greater than w at depth ζ cascade units due to a primary electron of energy w_0. The value of $\epsilon = \log_{10}\left(\frac{w_o}{w}\right)$ is attached to each curve.

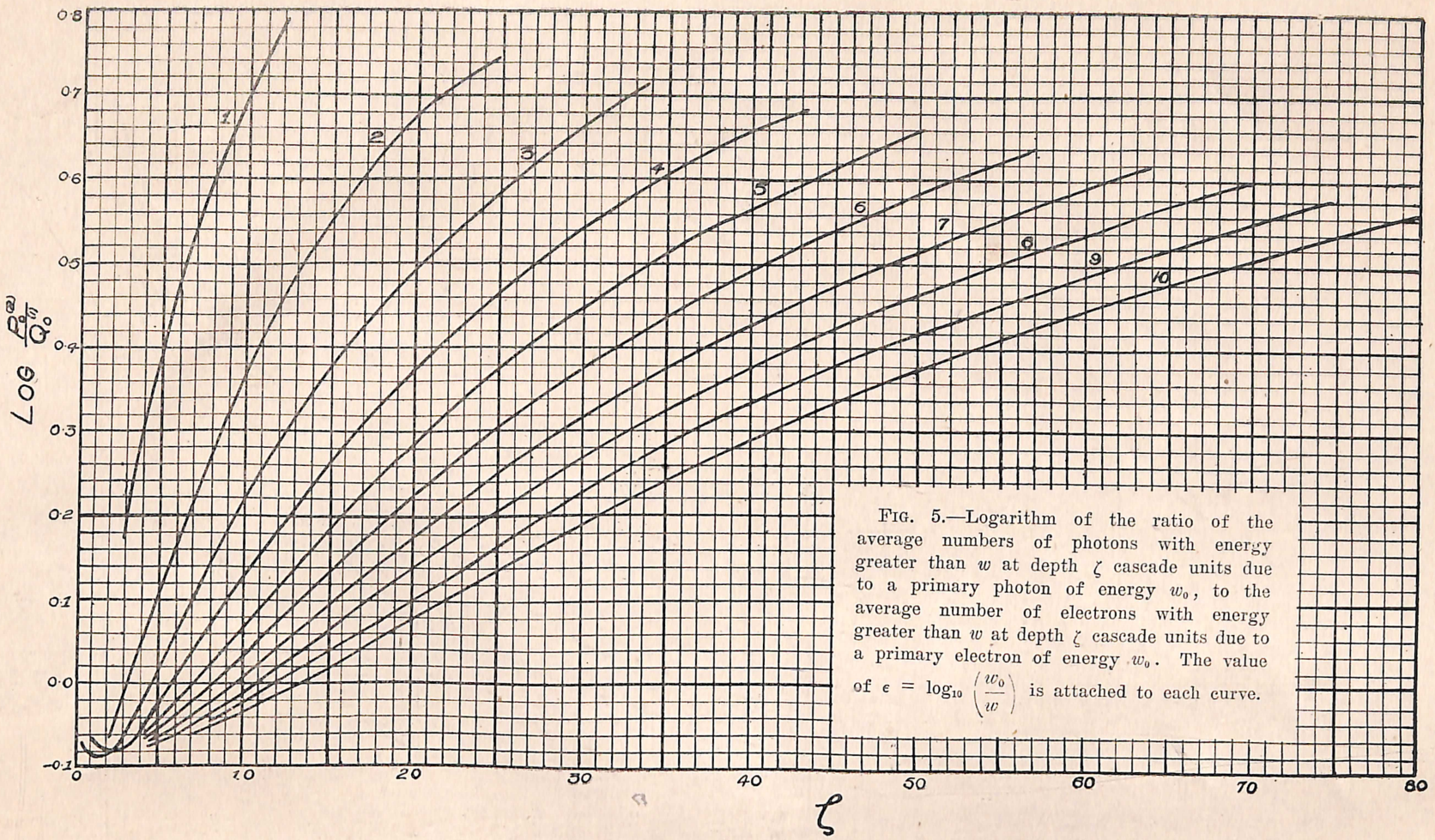

FIG. 5.—Logarithm of the ratio of the average numbers of photons with energy greater than w at depth ζ cascade units due to a primary photon of energy w_0, to the average number of electrons with energy greater than w at depth ζ cascade units due to a primary electron of energy w_0. The value of $\epsilon = \log_{10}\left(\frac{w_0}{w}\right)$ is attached to each curve.