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On the Development of the Nucleon Component of the Cosmic Radiation in Air

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ABSTRACT. The development of the nucleon component in air is considered in terms of a model which is an extension of that proposed by Heitler and Jánossy. The numerical results obtained are compared with experiment, and good agreement is found.

§ 1. INTRODUCTION

In recent years it has become increasingly evident that the development of the nucleon component of the cosmic radiation must be described in terms of some cascade process. This fact has been clearly brought out both by direct experiments and by indirect experiments, such as those on the variation with altitude and latitude of the slow and fast neutron component of the cosmic radiation and of the star component (Bernardini *et al.* 1949, Simpson *et al.* 1948, 1949, 1950). A theory for such a process was suggested tentatively by Heitler and Jánossy (1949) and applied to the high energy region (energies above 5×10^9 ev.). It was found that the high energy phenomena could be satisfactorily described in these terms. An interesting feature of the above work was the insensitivity of the results to the detailed mechanism of the model used. We have accordingly felt justified in investigating the application of this model to the interpretation of the experimental results on low energy nucleons. Mathematical investigation of the nucleon cascade was given by one of us (H.M.) recently.

§ 2. FORMULATION OF THE MODEL

Although little is known about the cross sections for nuclear processes with energies in excess of 2×10^8 ev., the following features of such processes are currently assumed:

(*a*) The basic process of energy loss takes place in individual nucleon–nucleon collisions each of which leads to the production of one secondary nucleon. In addition such processes will lead to meson production in a fraction of the cases in the energy range 2×10^8 to 2×10^9 ev., and in the majority of cases above 2×10^9 ev.

(*b*) The cross section for nucleon–nucleon interactions is known experimentally to fall off with roughly a $1/E$ dependence (E is the energy) up to energies of the order of 2×10^8 ev. Theoretically it is thought that the cross section will continue to decrease until processes of meson production become important, and it will then rise again to roughly the geometrical cross section. Extrapolation of present experimental values for the interaction cross section leads to an absorption mean free path for 2×10^8 ev. neutrons of the order of 130 gm/cm^2 in air. The absorption mean free path for nucleons in the 10^9 ev. range is also roughly 130 gm/cm^2. From cosmic-ray experiments an absorption mean free path of 135 gm/cm^2 in air is obtained for the nucleon component at sea level. If the nucleons in the range of

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energies 2×10^8 to 2×10^9 ev. had a substantially lower absorption mean free path, such nucleons would tend to predominate in the nucleon component at sea level, and this would then have an absorption mean free path greater than the observed one of 135 gm/cm^2 in air. It seems, therefore, that the increase in the cross section for meson production at energies above 2×10^8 ev. counterbalances the decrease in the cross section for elastic collisions, the absorption mean free path thus remaining constant.

(*c*) The interaction mean free path for nucleons with nuclei is roughly geometrical. Experimental justification for this is provided by the Berkeley experiments for nucleons with energies up to 3×10^8 ev., by the Bristol group experiments for nucleons with energies up to 10^9 ev., and from penetrating shower experiments for energies in the 10^9 ev. range.

Accordingly the main features which must be incorporated in a theory of the absorption of the nucleon component would appear to be a multiplication ratio of 2, i.e. every nucleon–nucleon interaction giving rise to a secondary nucleon, a constant absorption mean free path of 130 gm/cm^2, and a constant interaction mean free path of 65 gm/cm^2 in air.

The cross section adopted below incorporates the necessary features outlined above. The cross section is defined as follows. In a process in which a nucleon of energy E_0 loses energy, and also gives rise to a recoil nucleon, it is assumed that the probability for a collision to occur is given by

$$w(E_0\,; E_1; E_2)\, dE_1\, dE_2 = w\left(\frac{E_1}{E_0}, \frac{E_2}{E_0}\right) \frac{dE_1\, dE_2}{E_0^2} = \sigma \epsilon_2^{\beta}(1-\epsilon_1)^{\nu}\, d\epsilon_1\, d\epsilon_2 \quad \ldots\ldots (1)$$

where we have taken $\epsilon_i = E_i/E_0$ with $i=1,\ 2,\ w(\epsilon_1, \epsilon_2) = \sigma\beta\epsilon_2^{\beta}(1-\epsilon_1)^{\nu}$ and $\sigma = 15$, $\beta = 2$ and $\nu = 1$. The above cross section was suggested tentatively by Heitler and Jánossy (1949) and investigated analytically by Messel (1950). This model gives a loss of energy out of the nucleon component. (At high energies this energy will appear predominantly as mesons, and at energies of 10^8 ev. partly as mesons and partly as ionization losses. At lower energies the ionization losses are too large to be covered by this term, and it will be in this sense that we shall talk of solutions neglecting ionization losses.)

§ 3. EVALUATION AND NUMERICAL RESULTS

Neglecting ionization losses, the methods of solution of a cascade of the above type have been given in full previously (Heitler and Jánossy 1949, Jánossy 1950, Messel 1950, Jánossy and Messel 1950). We are interested physically in solutions for nucleons with energies of the order of 10^8 ev. For nucleons with energies of the order of 10^8 ev. the rate of loss of energy by ionization will be of the same order as that in nuclear collision processes, and thus the neglect of ionization losses is not justified.

In analogy with the similar problem encountered for the low energies in the electron–photon cascade, we shall make the approximation that the number of nucleons above 10^8 ev. is given by the formula neglecting ionization loss, in which the limit of 10^8 ev. is replaced by (10^8 ev. + E_0). In the electron–photon cascade E_0 is roughly the energy loss by ionization per cascade length. (Note that in this approximation we do not attempt to differentiate between numbers of protons and neutrons. It is obvious that low energy protons will be preferentially removed from the cascade.)

The comparisons which we shall make with experiment are insensitive to the choice of minimum energy and, therefore, we do not consider that the large uncertainties inherent in the above approximation are serious.

The results of the papers quoted previously give for the number of particles $N(E/E_c, \theta)$ with energies greater than E at a depth θ in inhomogeneous matter, that is, matter in which the nucleons are grouped in nuclei.

$$N\left(\frac{E}{E_c}, \theta\right) = \frac{1}{2\pi i}\int_{s_0-i\infty}^{s_0+i\infty}\left(\frac{E}{E_c}\right)^{-s}\frac{\gamma}{s(\gamma-s)}\exp - \theta f(d_A\alpha_s)\,ds \qquad \gamma > s_0 \quad \ldots\ldots(2)$$

with

$$N\left(\frac{E}{E_c}, 0\right) = \begin{cases}\left(\frac{E}{E_c}\right)^{-\gamma} & E > E_c\\ 1 & E < E_c\end{cases} \quad \ldots\ldots(3)$$

E_c is the cut-off energy and $\gamma = 1{\cdot}7$.

$$\alpha_s = \int_0^\infty\int_0^\infty (1-\epsilon_1{}^s-\epsilon_2{}^s)\,w(\epsilon_1,\epsilon_2)\,d\epsilon_1 d\epsilon_2 \quad \ldots\ldots(4)$$

$$f(t) = 1 - 2\,\frac{1-(1+t)e^{-t}}{t^2}, \quad \ldots\ldots(5)$$

and d_A is the average number of collisions which a nucleon suffers along a nuclear diameter.

If we denote the range of nuclear forces by R_k, then d_A is given by

$$d_A = 1{\cdot}5A^{1/3}\left\{\frac{R_k}{1{\cdot}37\times 10^{-13}}\right\}^2, \quad \ldots\ldots(6)$$

with A the atomic weight.

For air d_A was taken to be equal to 2·41. The depth θ is actually $\theta = \bar{\theta}n\Phi_A$, where $\bar{\theta}$ is the depth in gm/cm^2 and $n\Phi_A$ is the reciprocal cross section. We have evaluated curves giving the numbers of nucleons with energies above 10^8 and 2×10^9 ev. for the incident power law spectrum given above. We chose values of the cut-off energy E_c of 2×19^9 ev. and 15×10^9 ev. respectively, corresponding roughly to the latitude cut-offs for northern latitude, and the geomagnetic equator.

The 10^8 ev. curve is calculated from the above formulae using a value of E equal to $2{\cdot}66\times 10^8$ ev. To approximate the effects of ionization loss this value should be $2{\cdot}3\times 10^8$ ev. from the considerations given above. However, the use of a value of $2{\cdot}66\times 10^8$ ev. which bears the same ratio to 2×10^9 ev. as does 2×10^9 ev. to 15×10^9 ev. saves a considerable amount of computation, without appreciably changing the results.

The experiments with which we shall compare the above data are all of a 'non-directional' variety, and accordingly it is necessary to apply an inverse Gross transformation integrating over all directions of incidence. The results are given in Figures 1 and 2.

§ 4. COMPARISON OF RESULTS WITH EXPERIMENT

The results can be compared with the altitude and latitude dependence of the slow and fast neutron component, of the star component, and the burst-producing component. We shall discuss each of these in turn.

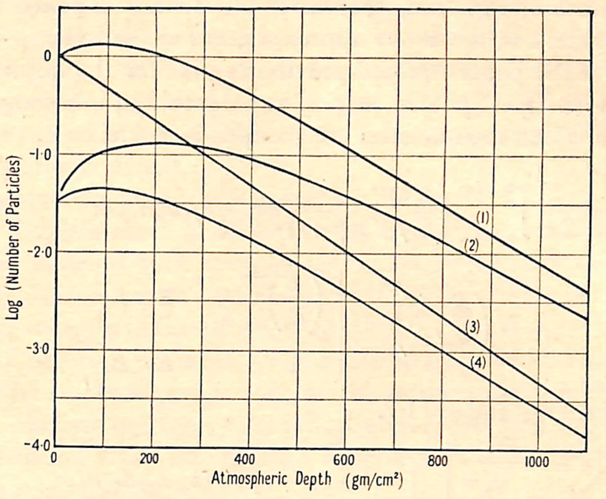

Figure 1. Plot of the logarithm of the vertical intensity of particles with energies greater than E due to a primary power law spectrum given by (3), against atmospheric depth in gm/cm^2.

(1) $E=10^8$ ev., $E_c=2\times 10^9$ ev., the cut-off energy at 'northern latitudes'.
(2) $E=10^8$ ev., $E_c=15\times 10^9$ ev., the cut-off energy at the geomagnetic equator.
(3) $E=2\times 10^9$ ev., $E_c=2\times 10^9$ ev.
(4) $E=2\times 10^9$ ev., $E_c=15\times 10^9$ ev.

The curves are normalized to an intensity of one at northern latitudes and to log $(2/15)^{1\cdot 7}$ at the geomagnetic equator.

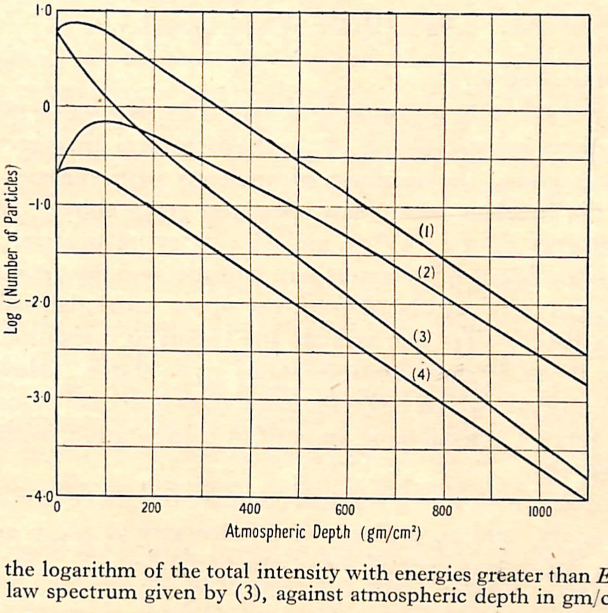

Figure 2. Plot of the logarithm of the total intensity with energies greater than E due to a primary power law spectrum given by (3), against atmospheric depth in gm/cm^2.

(1) $E=10^8$ ev., $E_c=2\times 10^9$ ev.
(2) $E=10^8$ ev., $E_c=15\times 10^9$ ev.
(3) $E=2\times 10^9$ ev., $E_c=2\times 10^9$ ev.
(4) $E=2\times 10^9$ ev., $E_c=15\times 10^9$ ev.

The curves are normalized to an intensity of log 2π at northern latitudes and to log $2\pi\,(2/15)^{1\cdot 7}$ at the geomagnetic equator.

(*a*) *Altitude Dependence of Bursts, Photographic Plate Stars and Slow and Fast Neutrons.*

The close connection between the bursts observed in an ionization chamber and the star production observed in a photographic plate has been adequately discussed by Rossi and Williams (1947), and it is clear that the phenomenon being observed is almost certainly the same in both cases.

The question of the production of stars and neutrons has been widely discussed by many authors, and from these it would appear that the following views are probably correct.

(i) The excitation energy necessary for the production of a star with three or more prongs is generally greater than 10^8 ev. This corresponds to our lower limit of 10^8 ev. for nucleons.

(ii) The model we have outlined is not valid for nucleon energies below 10^8 ev., due both to the rapid rise in cross section and to the fact that such nucleons are

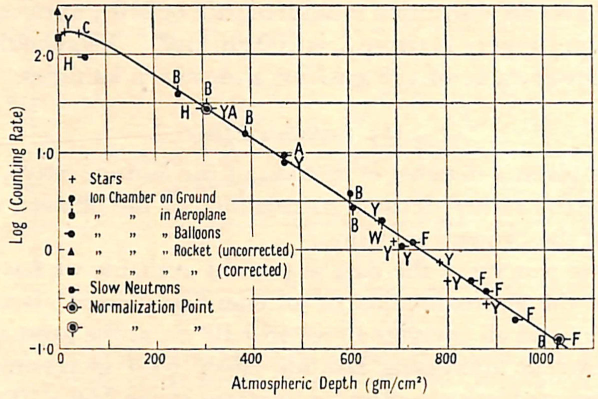

Figure 3. Theoretical curve obtained by us and the experimental results normalized by Rossi. Points from star data by Camerini *et al.* and Yagoda have also been added. The theoretical curve has been normalized to agree with the data at 650 gm/cm^2.

A, Agnew *et al.*; B, Bridge; W, Williams; H, Hulsizer;
Y, Yuan *et al.*; F, Fünfer; +Y, Yagoda; C, Camerini *et al.*

produced predominantly by evaporation from excited nuclei. Due to the origin in excited nuclei, the number of such nucleons will, however, be directly related to the intensity of the star component at an equivalent altitude. (The interpretation at the highest altitudes is complicated by the fact that nucleons are emitted isotropically in ‘ evaporation ’ processes, and that the resultant slow neutrons arising from this flux can further diffuse a considerable distance through the atmosphere. This leads, as is well known, to the occurrence, both theoretically and experimentally, of a maximum in intensity of the neutron component, which has no connection with the cascade maximum suggested by our model.)

In Figure 3 we have transcribed collected experimental results for bursts and neutron intensities given in the Rossi review article (1948) for variation in intensity throughout the atmosphere, and have normalized our theoretical curve to give agreement at 650 gm/cm^2 depth. We have in addition added later photographic plate data on stars of Yagoda (1949) and Camerini *et al.* (1949). The results of Bernardini *et al.* (1948, 1949) are represented accurately by an exponential absorption between sea level and 25,000 metres with a mean free path of

$135 \pm 4\,gm/cm^2$, in excellent agreement with the results of the two authors mentioned above. As far as can be judged, the change in intensity with atmospheric depth of these results is represented accurately by our theoretical curve. (It was pointed out by Rossi (1948) that these results could in no way be fitted by an inverse Gross transform of an exponential, and it was suggested that this deviation was due to some type of transition effect at the top of the atmosphere.)

It is not possible to state from the observed experimental data whether or not the small maximum at $20\,gm/cm^2$ depth actually occurs. In this connection we note from Figure 2, curve 2, the very much more marked maximum that should occur at the geomagnetic equator.

Another interesting feature shown in Figure 2, curve 2, is the departure of the curve from exponential behaviour down to depths of several hundred grammes. Systematic measurements made on slow and fast neutron intensities and burst production by Simpson *et al.* (1948, 1949, 1950) at the geomagnetic equator do in fact show such an effect. These measurements at atmospheric depths between 200 and $400\,gm/cm^2$ give an absorption mean free path of $210 \pm 8\,gm/cm^2$, whereas the slope expected from Figure 2, curve 2, is $200\,gm/cm^2$. This should be compared with the much lower value of $135\,gm/cm^2$ at northern latitudes.

(*b*) *Latitude Effect*

We have not given a detailed evaluation of the latitude effect, as it would be necessary to use the rather detailed calculations of the allowed energies at the various angles of the primary radiation.

In view of the paucity of the data at present available we have considered it sufficient to use the approximation of all energies allowed above 2×10^9 ev. at northern latitudes and all energies above 15×10^9 ev. at the geomagnetic equator. Our model shows an increasing latitude effect with increasing altitude. At 30,000 ft. the predicted latitude effect for neutrons above $2{\cdot}66 \times 10^8$ ev. is 5·0. The observed value is 3·5. The predicted value at $550\,gm/cm^2$ depth is, according to photographic plate measurements of Yagoda (1949), of the order of 3·5, whereas the theoretical value obtained by us is 3·0. In view of the approximation we have had to make for the incident primary spectrum at the geomagnetic equator and the uncertainty of the factor 1·7 occurring in the power law spectrum, we consider the agreement with our model as satisfactory.

(*c*) *Change of Star Type with Height*

Recent work (Camerini *et al.* 1949, Salant *et al.* 1948, Yagoda 1949) has made it certain that there is a slow but definite change in the star prong distribution between balloon altitudes and sea level. At sea level there is an increase in the proportion of small stars and a decrease by a factor of 2 in the proportion of ' hard ' stars (Brown *et al.* 1949, Camerini *et al.* 1949), i.e. stars associated with the production of particles with minimum ionization. The most complete data at present available are those of Camerini *et al.* (1949) at 70,000 ft. A significant feature of these data is the high proportion (three-quarters) of stars due to neutral primaries and of small stars not associated with the production of minimum ionization particles. This is in qualitative agreement with our model, which predicts a very rapid increase of secondaries in the range 10^8 to 2×10^9 ev. From Figure 2, curves 1 and 3, it is seen that at 70,000 ft. already two-thirds of the nucleons with energies above 10^8 ev. are in the range 10^8 to 2×10^9 ev. This ratio has increased to the order of nine-tenths at 3,000 metres, a change which might well account for the observed experimental differences. Assuming that only one-tenth of the particles with energies between 10^8 and 2×10^9 ev. are capable of forming ' hard ' stars (corresponding to a cross section for meson production of the order of 10^{-27} cm^2), and that one half of the particles with energies above 2×10^9 ev. are capable of forming such stars, we can in fact account quantitatively for the change observed.

For interest we have plotted in Figure 4, using the above results, values calculated for the change in the ratio of ' hard ' stars to total star population with variation in altitude for both northern latitude and the geomagnetic equator. Theoretically there should be practically no difference between these ratios in passing from northern latitudes to the geomagnetic equator, even up to the highest altitudes obtainable in balloon flights. This is due to the rapid rate at which equilibrium is established between the low and high energy nucleons. No data are at present available on these points, and confirmation would be of interest.

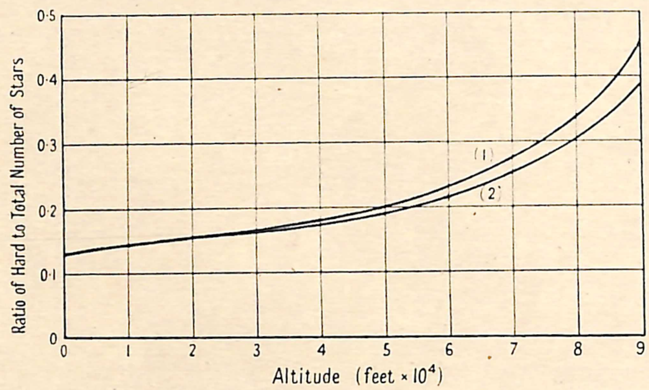

Figure 4. The ratio of the number of ' hard ' stars to the total number of stars plotted against the altitude, in feet, for cut-off energies : (1) 15×10^9 ev., (2) 2×10^9 ev.

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