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ABSTRACT

A new method is developed for determining the mass of particles coming to rest in nuclear emulsions. Multiple coulomb scattering is measured with cells whose lengths vary along the trajectory in such a manner as to compensate for the momentum loss of the particle and keep the mean deviation between adjacent cells constant over the entire track. It is shown that this procedure is more convenient and inherently more accurate than that based on scattering measurements with constant cellsize. The method has been applied to *K*-mesons which at the end of their range either decay into a single charged relativistic particle (K^+-mesons) or into 3π-mesons (τ-mesons) or give rise to capture stars (K^--mesons). The results are within experimental error consistent with the assumption that the mass of these three classes of particles are identical and equal to the well established mass of the τ-mesons. The average mass of a group of 9 long *K*-mesons determined with our scattering procedure is $M_K = 974 \pm 42 m_e$.

I. INTRODUCTION

THE mass of particles which come to rest in photographic emulsions can be obtained by measuring as a function of residual range, either the variation of ionization or the variation of multiple scattering along their track. The choice of the method which will give highest accuracy in a particular case depends on the mass of the particle, the orientation and length of the track available for measurement and the sensitivity of the emulsion.

A measurement of ionization can be obtained from the photometric density of the track, from the number of developed grains per unit track length (grain density) or from the proportion of the track length which is free of developed silver grains (integrated gap length). Of these three methods for comparing ionization, the measurement of grain density is by far the most accurate when applicable and has been used in most of the earlier work to determine the mass of particles stopping in emulsions. The reason why mass determination from grain density *versus* range is no longer used extensively is two fold:

(*a*) With the development of more sensitive emulsions it has become possible and, for most experiments, desirable to employ emulsions which are sensitive to particles of minimum ionization. In such emulsions the grain density begins to saturate when the velocity of a singly charged particle falls below $v = \cdot 4$ c. Accurate grain density measurements require, therefore, that the track of the particle is observable over an appreciable length such that in at least part of the trajectory the particle's velocity exceeds $v = 0 \cdot 4\, c$.

(*b*) Interest has shifted to particles of rest mass considerably larger than that of π-mesons. The saturation effect extends, therefore, over a large part of the residual range. For instance, accurate grain density measurements on particles of mass 1,000 electron masses or larger require observable residual ranges of more than $1 \cdot 6$ cm. while the available track length is usually much shorter. Even if sufficient track length is available, there is the danger that some inelastic scattering which is unobservable in the long saturated portion of the track, introduces a large error in the mass value obtained from grain density *versus* residual range.

Comparative ionization measurements by photometric density or integrated gap length on the other hand are applicable in sensitive emulsions to tracks of particles with small residual range and large rest mass. These methods promise to gain in importance in the near future, since recent technical improvements have increased their accuracy.[1, 2, 3]

However, the method which is most frequently used at present for obtaining the mass of stopping particles consists of measuring multiple coulomb scattering of tracks and residual range. A serious difficulty in this method as used in the past arises from the fact that the rapid energy loss of slow particles produces a rapid variation of the scattering values with range. The available track length must therefore be sub-divided into sections for which the scattering value is determined separately and these results of necessarily low statistical accuracy have to be fitted by trial and error to the best fitting of a series of scattering *versus* range curves representing various mass values. A modification of this procedure has been adopted by Menon and Rochat[4] who relate the scattering value obtained for half of the available track length to an empirically determined function of the particle mass and range.

All methods so far employed have however in common the use of a constant cell length to determine scattering in limited sections of the track.

Apart from the inaccuracy involved in relating scattering values of comparatively low statistical accuracy obtained from various sections of the track to the particle mass, there are other disadvantages, arising from the necessity of sub-dividing the available track length.

(*a*) Correction for large angle single scattering is usually made by omitting from the averaging process all deflections which exceed the average deflection by a certain factor (usually all readings larger than four times average are rejected). This cut-off value varies with residual range and must be determined separately on the basis of the limited number of measurements available in each of the sections into which the track was divided. Such a procedure for large angle scattering correction introduces therefore appreciable fluctuations.

(*b*) The optimum cell length is obtained by the compromise that the cell must be small enough to permit a large number of individual measurements yet large enough so that the scattering between cells is sufficiently high above the noise level. When the scattering increases due to the particle's slowing down, this optimum cell size decreases. A cell size which is optimum at the high energy end of a section of track is larger than optimum at the low energy end, resulting in a waste of information.

These disadvantages can be overcome if instead of using cells of constant size along parts of the track one uses a set of cell sizes which vary continuously in such a way as to keep the mean deflection per cell constant along the trajectory.* The mean deflection obtained with such a set of cell sizes is therefore independent of particle energy and only a function of mass. The set of cell sizes can be chosen such as to yield for a given noise level, optimum statistical accuracy for all parts of the track and the cut-off value for the single scattering correction can now be determined on the basis of deflections averaged over the entire available track length.

In Section II this method is described in detail. In Section III it is shown that once a set of cell lengths has been calculated which make the mean deviation independent of range for tracks produced by particles of a given mass, the same set of cells will give a mean deflection independent of range when applied to tracks produced by a particle of different mass. Similarly it can

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* A very similar scattering method has been worked out simultaneously and independently by C. Dilworth, S. Goldsack and L. Hirschberg at Bruxelles. An outline of the method was presented by Mrs. Dilworth-Occhialini as a joint paper of the Bruxelles and the Bombay groups at the Cosmic Ray Conference at Bagnères de Bigorre, July 1953.

be applied to the projection into the plane of the emulsion of a steeply dipping track. A simple relation is derived giving the mass of the particle as a function of the mean deviation and the dip angle of its track.

In Section IV, we discuss noise corrections and also the choice of the schemes which will give optimum statistical accuracy. We also estimate the error in mass values obtained by this method.

In Section V, we apply the method to tracks of known particles and determine their mass. Here it appears that within statistical accuracy correct mass values are obtained and in agreement with our calculations the experimental mass values are independent of the particular scheme which is used.

Finally in Section VI, we apply our method to *K*-mesons coming to rest in an emulsion block detector and compare the results with mass values obtained by other methods and in other laboratories.

II. Outline of a Scattering Measurement Using Variable Cell Length

We have modified Fowler's[5] " co-ordinate method of scattering determination " in the following way.

Based on the semi-theoretical relation between range in emulsion and energy and the relation between scattering, velocity and cell length, we have calculated for particles of a given mass the cell length required such that at a given residual range the average absolute value of the second difference† $\bar{D}$ has a certain predetermined value Δ. We then obtain a relation between cell length and range and can use this relation to find along the track a set of points (whose separation increases in the direction away from the stopping point), which when used as the end points of successive cell intervals give an average second difference whose value is constant along the track and equal to Δ.

Such a set of increasing cell lengths calculated for a given value of Δ and for a given particle mass M we shall call a " scattering scheme " and designate it as " M_Δ scheme ". We shall show in Section III, that if such a scattering scheme is applied to the projection into the plane of the emulsion of a track produced by a particle of mass $M' \neq M$, traversing the emulsion

† The second difference D and the cell length t are defined as follows: Let P_i, P_{i+1}, P_{i+2}, etc., represent individual points on a track and let y_i be the perpendicular distance between point P_i and an arbitrary line which is approximately parallel to the track, then the second difference D_i is defined by $D_i = y_i - 2y_{i+1} + 2y_{i+2}$ and the cell length t_i by the distance between P_i and P_{i+1}.

making an angle θ with the emulsion surface, the resulting average second difference Δ' (M, Δ, M$'$, θ) is to a high degree of accuracy independent of residual range and can be represented by a simple function of the variables. The mass M$'$ can, therefore, be easily determined by applying an arbitrary scattering scheme to any track ending in the emulsion.

The measured quantity Δ' must be interpreted as the average absolute value of the second differences (obtained from measurements on the projected track) after correction has been made for noise and for large angle single scattering. The noise correction is treated in Section IV. The large angle scattering correction is made by omitting from the averaging process all those second differences which exceed four times the average value of all the remaining second differences measured along the track.

III. CALCULATION OF SCATTERING SCHEMES AND THE RELATION BETWEEN SCATTERING VALUES AND PARTICLE MASS

In order to calculate the cell length required such that when measuring scattering on the track of a particle of mass M and residual range R, one obtains on the average a value for the second difference whose absolute magnitude is Δ, one needs a relation between energy and residual range in emulsion as well as a relation connecting the multiple scattering with particle mass and velocity and with the cell length employed.

(*a*) *The Range-Energy Relation.*—For protons of energy up to 35 Mev we have used the available experimental information on ranges in photographic emulsions as quoted by Beiser[6] (we have decreased all his ranges by 1% to take into account the slightly greater stopping power of Ilford G-5 emulsions). The experimental values were plotted on the graph giving the range-energy relation for protons in Air, Aluminium and Lead which was published by the Princeton group and is based on the calculations of Smith.[7] When plotted in such a way it becomes apparent that the ratio $r = \dfrac{R^2_{emulsion}}{R_{alum} \times R_{lead}}$ approaches a constant value as the proton energy increases. We have made the assumption that "r" remains constant for proton energies higher than those for which emulsion data are available and have used this extrapolation up to residual ranges of 10 cm. in emulsion.

The range-energy curve for singly charged particles of mass M different from the proton mass M_p is then easily obtained from the relation

$$R_M(v) = \frac{M}{M_p} R_p(v) \qquad (1)$$

where v is the particle velocity and R its range, (*b*) *The Scattering Formula.*—For the relation between the mean absolute value of the second difference Δ, the particle mass M and velocity βc and the cell length t we have used a formula based on the theory of Williams[8] and calculated for emulsions by Voyvodic and Pickup.[9] This formula may be written

$$\frac{M\Delta}{m_e} = 0{\cdot}96 \sqrt{\frac{2}{3}}\, \frac{1{\cdot}006\, t^{3/2} \sqrt{1-\beta^2}}{0{\cdot}511 \times 57{\cdot}3\, \beta^2} \left[1{\cdot}45 + 0{\cdot}8 \left\{ ln \frac{0{\cdot}94\, t}{\beta^2 + 0{\cdot}3} \right\}\right] \quad (2)$$

(here the factor 0·96 represents the effect of smoothing out scattering since the position of the track is measured by observing the position of a small number of grains rather than that of a single grain. The factor $\sqrt{\frac{2}{3}}$ arises because the second difference is determined using successive sagittas rather than tangents. The factor 0·511 converts the rest energy of the electron $m_e c^2$ into *M*eV. Δ and t are measured in microns.)

The numerical factor 1·006 represents the "scattering constant" derived theoretically for the case that no correction is made for large angle single scattering when measuring Δ. It can be shown that for the particular choice of cell lengths as a function of range which keep Δ constant, the introduction of a large angle cut-off corresponds simply to a reduction in the scattering constant. Although we employ the customary cut-off procedure, we have calculated our schemes with the theoretical constant 1·006 and later introduced a correction to the scattering constant on the basis of the calibration measurements on the masses of known particles (see Section V).

(*c*) *Relation Between Cell Length* t *and Residual Range R for Constant M and* Δ.—Using the range energy relation for protons in elmusion, together with equations 1 and 2, we can now for any value of M and Δ calculate the required cell length t as a function of residual range. The corresponding graphs for $\Delta = 1\,\mu$ and $\Delta = 1{\cdot}6\,\mu$ are shown in Fig. 1 using for M the mass of the proton and of the π-meson respectively. From these graphs one can now easily construct the scattering schemes $P_{1\cdot0}$ and $\pi_{1\cdot6}$. Starting at a small but arbitrary residual range R_0 one finds from Fig. 1, the corresponding value t_0, next one finds the value of t_1 corresponding to the residual range $R_0 + t_0$ and the value t_2 corresponding to the residual range $R_0 + t_0 + t_1$, etc. The scattering schemes $P_{1\cdot0}$ and $\pi_{1\cdot6}$ obtained in this manner for protons and π-mesons are given in the Appendix. When for instance the cell lengths t listed in the $P_{1\cdot0}$ scheme are used for measuring scattering on flat proton tracks we expect to find for any of the tracks and for any part of the tracks the same mean absolute value for the second difference, namely 1·0 microns.

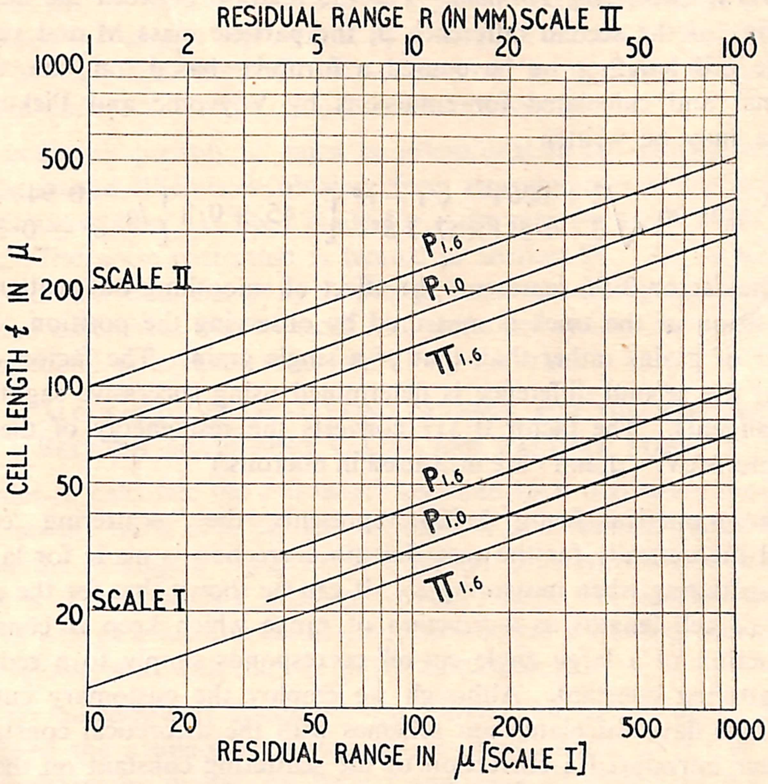

FIG. 1. The variation of the cell-length, t with Residual Range R for constant value of the mean absolute second difference, $\triangle$. The curves correspond to $\triangle = 1 \cdot 6\ \mu$ for protons and π-mesons and to $\triangle = 1 \cdot 0\ \mu$ for protons.

(*d*) *The Application of a Scattering Scheme to Dipping Tracks.*—In ordinary nuclear plates no reliable mass measurements can be made on tracks which make an appreciable angle with the plane of the emulsion, because the available track length is always short. If one employs, however, an emulsion block detector[10] tracks with appreciable dip when traced through many contiguous emulsion sheets often provide sufficient track length for mass determinations. For this reason the usefulness of the scattering method proposed here will be greatly extended if it can be applied to mass determinations on dipping tracks.

Consider for instance a proton track which in the undeveloped emulsion makes an angle θ with the plane of the emulsion and let us calculate from the range-energy relation and equations (1) and (2), the mean second difference $\bar{D}(\theta, R)$ which we should obtain by using at a residual range $R_i = R^{proj}$ sec θ not the cell length t_i obtainable from the graph in Fig. 1, but the increased value $t_i' = t_i \sec\theta$. The value $\bar{D}(\theta, R)$ so calculated represents then the mean second difference which we should measure if we simply applied our scattering scheme to the projection into the plane of the emulsion of a uniformly dipping track. [The measured value of $\bar{D}(\theta, R)$ is not affected by a subsequent uniform shrinkage of the emulsion thickness.] In Table I the ratio, $\bar{D}(\theta, R)/\Delta$ is shown for various values of R and θ using the $\pi_{1 \cdot 6}$ scheme on a π-meson track. One sees that $D(\theta, R)$ is very nearly independent of R and this independence also holds for other scattering schemes. Its dependence on θ can in the range $0 < \theta < 60°$ be represented by the empirical formula.

$$\bar{D}(\theta) = \Delta (\sec\theta)^{0 \cdot 975} \tag{3}$$

Equation 3 permits then to obtain the mean absolute value of the second difference for a flat track from measurements carried out on the projection of a uniformly dipping track. (If the dip angle is different in different sections of the track, the sections must be treated separately.)

TABLE I

$D(\theta, R)/\Delta$ as a Function of Range for Various Values of the Dip Angle θ and for the $\pi_{1 \cdot 6}$ Scattering Scheme

Range / Dip Angle θ	1 mm.	2 mm.	5 mm.	1 cm.	2 cm.	5 cm.	10 cm.
0°	1·000	1·000	1·000	1·000	1·000	1·000	1·000
15°	1·035	1·030	1·030	1·030	1·030	1·035	1·035
30°	1·150	1·150	1·145	1·145	1·145	1·150	1·150
45°	1·405	1·405	1·405	1·405	1·400	1·405	1·405

(*e*) *Application of Scattering Schemes to Particles of Unkown Mass.*—Using again the range-energy relation and equations (1) and (2), we calculate now as a function of residual range the mean absolute second difference $\bar{D}(M', R)$ which we should obtain if a certain scattering scheme M_Δ is applied to a flat track produced by a particle of mass $M' \neq M$. We find that in the range interval of ·1 to 10 cm., if the particle's mass M′ is nearly equal to the scheme mass M, the variation of $\bar{D}(M', R)$ with range is less than 1%. As M′ becomes much smaller than M, the variation of $\bar{D}(M', R)$ with range increases, but even when $M' \simeq \frac{1}{6} M$ as in the case of a π-meson measured with the proton scheme, the variation of $\bar{D}$ (M′, R) with range does not exceed 3% of the mean value‡. We can, therefore, neglect this slight dependence on range and plot for any scheme $M_\triangle$ the mean absolute second difference $\bar{D}$ (M′) as a function of particle mass (M′). This graph is shown in Fig. 2. The curves can be represented by the relation

$$\bar{D}(M') = \Delta\left(\frac{M}{M'}\right)^{0.439} \tag{4}$$

If we designate by Δ' the mean absolute second difference obtained on a track of dip angle θ using the scattering scheme $M_\triangle$ we obtain by combining

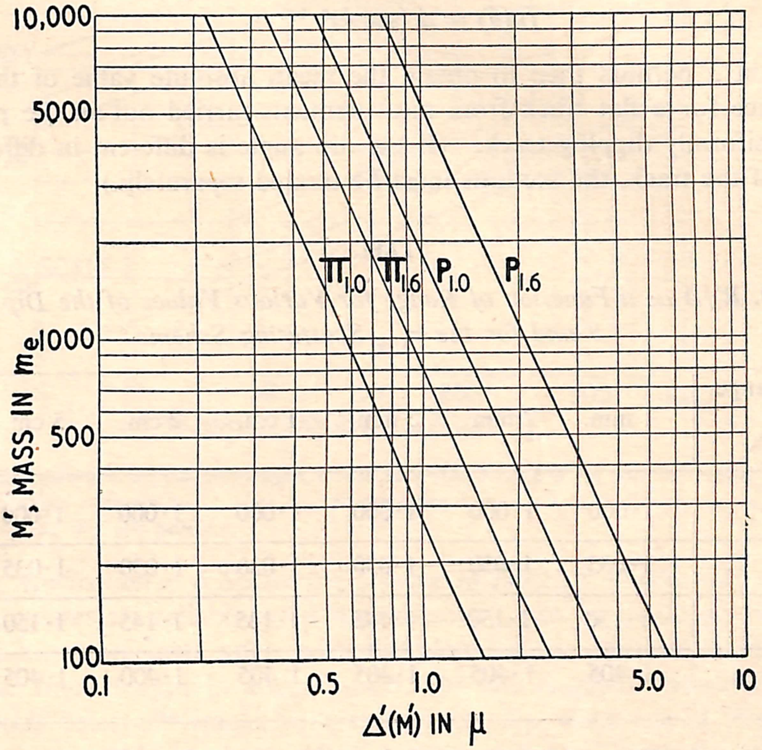

FIG. 2. The mean absolute second difference $\triangle'$ (M′) is shown as a function of particle mass M′ when measured with $P_{1\cdot 6}$, $\pi_{1\cdot 6}$, $P_{1\cdot 0}$ and $\pi_{1\cdot 0}$ scattering schemes.

the results of paragraphs (*d*) and (*e*) the mass M′ of the particle responsible for the track

$$M' = M\left(\frac{\Delta}{\Delta'}\right)^{2.276} (\sec\theta)^{2.221} \tag{5'}$$

‡ If the range-energy curve is approximated by a single power law the variation is much larger and reaches about 10% for π-mesons scattered according to a proton scheme. The deviation from a power law in the range-energy relation tends to decrease the variation of $\bar{D}$ (M′, R) with range.

IV. Noise Correction and Errors in the Mass Determination

The statistical accuracy of the measured scattering value and therefore of the particle's mass depends for a track of given length on the number of available cells and, therefore, on the choice of the scattering scheme. The choice is essentially limited by the condition that the measured scattering value must be significantly larger than the noise level. The noise arises from the fact that the centres of individual grains are not located exactly on the trajectory and that the determination of the exact location of these centres involves a small reading error.

The former factor is the most important source of noise, which should, therefore, have a magnitude of order or half the diameter of the average grain. In the region where the method discussed here is useful (particle range < 10 cm.) the cell lengths will never get so large as to make the noise in the movement of the microscope stage a significant contribution to the noise level; the spurious scattering due to noise should, therefore, be independent of cell length. Since the position of the track is determined by fitting a hairline in the eyepiece through a short section of track, the spurious scattering may be expected to increase slightly with decreasing grain density. We shall show, however, in this section that it is advantageous to chose the scattering scheme in such a way that the noise correction becomes very small and, therefore, the small variations of noise along the track can be neglected.

When measuring the spurious scattering on tracks of singly and doubly charged particles which, because they produce very energetic nuclear interactions, are known to have very high energy, we find that the noise has a value of $0 \cdot 16\,\mu$ and varies only slightly with cell length and track density. We shall assume that the noise has a gaussian distribution and a mean absolute value of $0 \cdot 16\,\mu$ for all our scattering measurements. We shall also assume that the *true* second differences due to multiple coulomb scattering and therefore also the *measured* second differences (which include noise) have a gaussian distribution. On these assumptions one can show that if Δ, D, ϵ represent the mean absolute values of the true second differences, the measured second differences and the noise respectively, Δ is given by

$$\Delta = (D^2 - \epsilon^2)^{\frac{1}{2}}. \qquad (6)$$

For a track of finite length the contribution of the noise to the scattering value is not the noise measured on infinitely long tracks but will fluctuate around this value. If n readings are taken the standard deviation in the mean absolute values D and ϵ are $\frac{0\cdot 76}{\sqrt{n}}$ D and $\frac{0\cdot 76}{\sqrt{n}}$ ϵ* respectively (Kendall[11]).

The standard deviation for Δ becomes, therefore,

$$\delta\Delta = \frac{0\cdot 76}{\sqrt{n}}\left(\frac{D^4 + \epsilon^4}{D^2 - \epsilon^2}\right)^{\frac{1}{2}}. \tag{7}$$

Since n is inversely proportional to the cell length, and, therefore, proportional to $\Delta^{-\frac{2}{3}}$ we can write equation (7) as

$$\frac{\delta\Delta}{\Delta} = \sigma\left[\frac{D^4 + \epsilon^4}{(D^2 - \epsilon^2)^{5/3}}\right]^{\frac{1}{2}} \tag{7 a}$$

where σ is a constant.

The quantity $\frac{\delta\Delta}{\Delta}$ has a flat minimum at $\Delta = 2\cdot 40\ \epsilon$ and this minimum value is

$$\left(\frac{\delta\Delta}{\Delta}\right)_{min} = \frac{0\cdot 90^{**}}{\sqrt{n_0}}. \tag{8}$$

The optimum scattering scheme is, therefore, one in which the second difference is $2\cdot 40$ times the average noise and the corresponding error in the mass determination (from equations 5′ and 8) is

$$\left(\frac{\delta M}{M}\right)_{min} = \frac{2\cdot 05}{\sqrt{n_0}} \tag{9}$$

n_0 being the number of independent cells available for this scheme and track length. If, however, we decide to use a scheme in which the second difference has two and a half times its value at optimum, the percentage error in the mass value increases only slightly because the increase due to reduction in the number of available cells by the factor $(2\cdot 5)^{\frac{2}{3}}$ is partly compensated

* This is not strictly true because the noise due to the displacement of the centre of a small group of grains from the true trajectory will affect three successive readings of second differences. We believe, however, that the simplified relations which we use here are sufficiently accurate for our purpose.

** An estimate of the standard deviation lying between $\frac{.8}{\sqrt{n}}$ and $\frac{.9}{\sqrt{n}}$ has been in use in most laboratories. They have however, been applied even in such cases where the second difference Δ was much smaller than its optimum value of $2.4\ \epsilon$. For instance if the cell lengths are chosen so small as to give $\Delta \approx \epsilon$ the error according to equation 7 becomes $\frac{\delta\Delta}{\Delta} = \frac{1.7}{\sqrt{n}}$ or twice as large as the estimate based on the usual convention. We believe, therefore, that our estimates of error are more conservative than those which have been in use until now.

by the reduced effect of noise fluctuations on the final value. One obtains for $\Delta = 6\,\epsilon$ a percentage error in the mass

$$\frac{\delta M}{M} = \frac{1 \cdot 78}{\sqrt{n}} = \frac{2 \cdot 41}{\sqrt{n_0}} = 1 \cdot 17 \left(\frac{\delta M}{M}\right)_{min}. \qquad (10)$$

Thus the error is only increased by 17% over its lowest possible value if we use a scattering scheme in which the second difference is six times the noise level. With such a scheme the noise correction to the measured second difference D amounts to only 1·4%. If we use such a scheme the small variations of noise along the track and small differences between the values obtained by different observers become unimportant. Therefore, we believe that in practice the most reliable and accurate mass values are obtained by choosing a scattering scheme in which the second differences are from four to six times larger than the noise level ($0 \cdot 6\,\mu \lesssim \Delta \lesssim 1\,\mu$).

V. Application of Scattering Schemes to Particles of Known Mass

We have applied the $\pi_{1 \cdot 6}$ scheme to long tracks of known π-mesons and obtained about 900 values of the second difference D. The distribution of the D values is shown in Fig. 3, together with the Gaussian curve

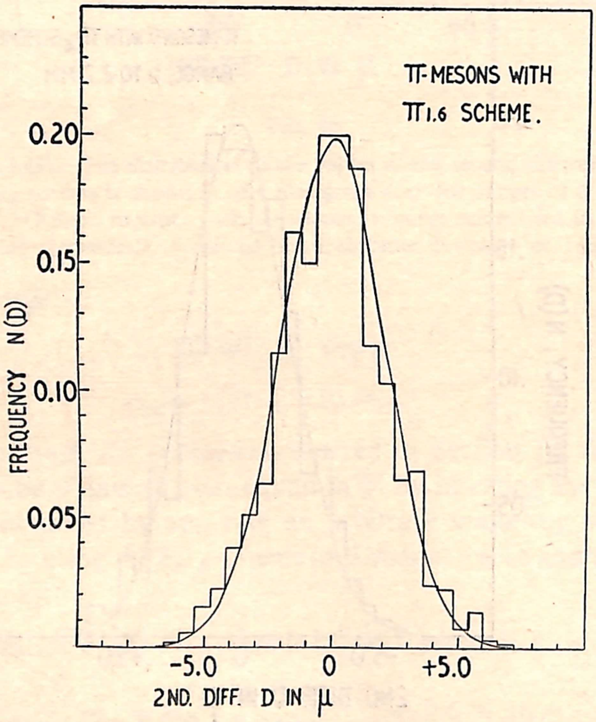

Fig. 3. The distribution of the values of the second difference D for π-mesons measured with $\pi_{1 \cdot 6}$, scheme is shown in the histogram together with the Gaussian curve normalised to the same area and corresponding to the theoretically predicted mean absolute deviation of $1 \cdot 6\mu$.

normalized to the same area and corresponding to the theoretically predicted mean absolute deviation of 1·6 microns. The average of the 900 experimental D values (after correction for the noise $\epsilon = 0\cdot 16\,\mu$ and a small correction for dip) is

$$\frac{\Sigma \mid D_i \mid}{n} = \Delta = 1\cdot 630 \pm \cdot 0415\,\mu$$

and the corresponding value of the mass is

$$m_\pi = 264 \pm 17\, m_e \text{ (electron masses)}$$

In order to exhibit the fact that the experimental D value distribution is independent of range as predicted by theory we show in Figs. 4*a* and 4*b* separately the distribution of D values from the last 2·7 mm. of track length and from those parts of the tracks where the residual range exceeded 2·7 mm. The curves shown in Figs. 4*a* and 4*b* are again the expected Gaussian distribution curves.

We have also applied the $P_{1\cdot 6}$ scheme to long proton tracks and obtained about 800 D values.

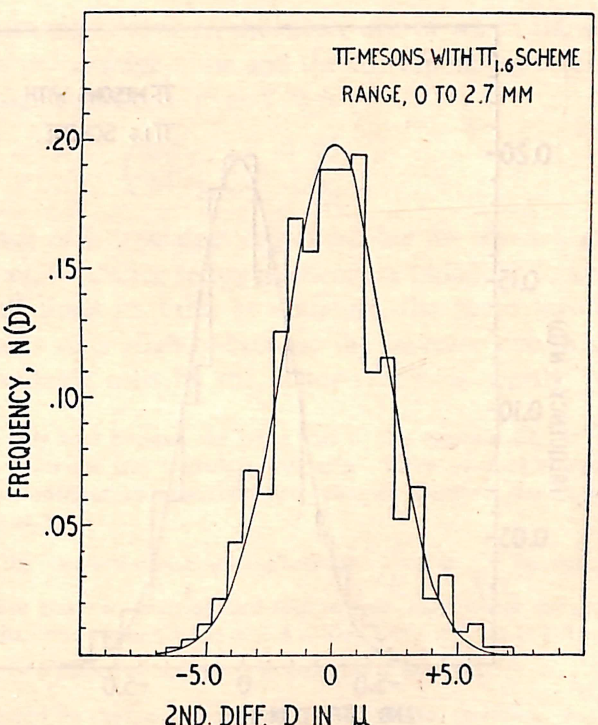

FIG. 4*a*

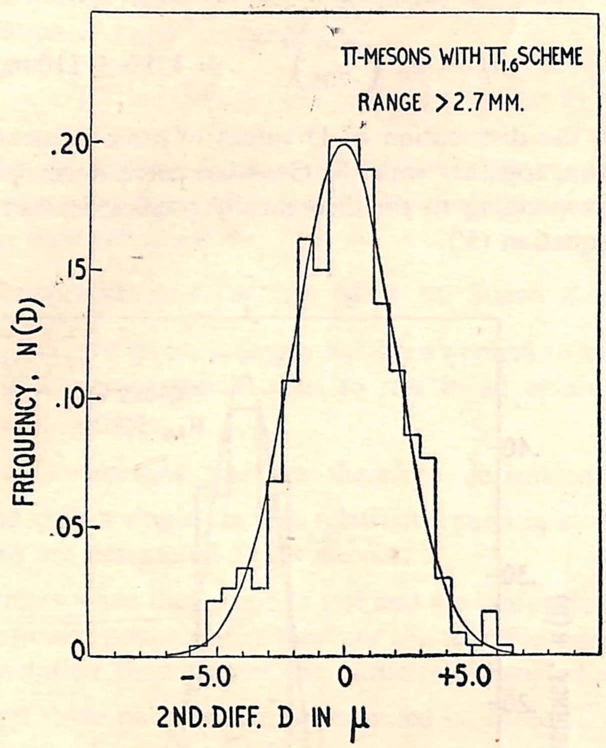

FIG. 4*b*

FIGS. 4 (*a*) and (*b*). The distribution of the values of the second difference D for π-mesons measured with $\pi_{1\cdot 6}$ scheme is shown in the histogram 4(*a*) for ranges of 0 to 2·7mm. and n 4 (*b*) for ranges > 2·7 mm., together with the Gaussian curve normalised to the same area and corresponding to the theoretically predicted mean absolute deviation of 1·6μ.

Their average is

$$\Delta = 1\cdot 602 \pm \cdot 046\,\mu$$

$$m_p = 1831 \pm 120\, m_e.$$

In order to test our conclusion derived in Section III that the mass of a particle may be obtained from equation 5′ by inserting the average second difference Δ' obtained by applying an arbitrary scattering scheme we have determined Δ' by using the $P_{1\cdot 0}$ scheme on π-meson tracks and the $\pi_{1\cdot 6}$ schemes on proton tracks.

The application of the $P_{1\cdot 0}$ scheme to π-mesons gave $\Delta' = 2\cdot 339 \pm \cdot 061\,\mu$

$$m_\pi = m_p \left(\frac{1\cdot 0}{2\cdot 339}\right)^{2.276} = 266 \pm 16\, m_e.$$

The application of the $\pi_{1\cdot 6}$ scheme to protons gave $\Delta' = 0\cdot706 \pm 0\cdot019\,\mu$

$$m_p = m_\pi \left(\frac{1\cdot6}{\cdot706}\right)^{2.276} = 1780 \pm 110\, m_e.$$

In Fig. 5 the distribution of D values of protons measured with $\pi_{1\cdot 6}$ scheme is shown, together with the Gaussian curve normalised to the same area and corresponding to the theoretically predicted mean absolute value of Δ', from equation (5′).

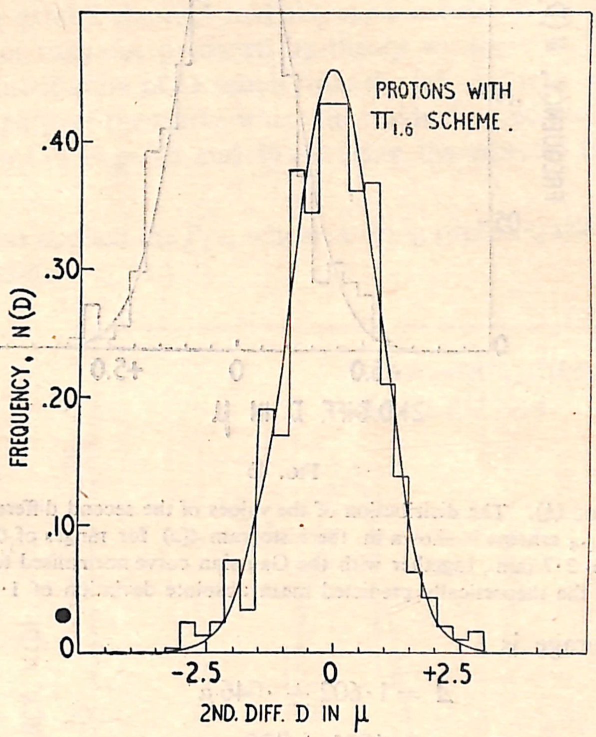

FIG. 5. The distribution of the values of the second difference D′ for protons measured with $\pi_{1\cdot 6}$ scheme is shown in the histogram together with the Gaussian curve normalised to the same area and corresponding to the theoretically predicted mean absolute value of $\Delta' = .696\mu$ (from equation 5′).

These results then, verify the conclusions derived in the previous section that Δ' is independent of range and that formula 5′ is valid. They also establish that the stopping power of G-5 emulsions and the scattering constant for emulsions which we have used implicitly in our scattering schemes are approximately correct. A slightly better fit with the known masses of our calibration particles is obtained if we change the scattering constant (equation 2) from $K = 1{\cdot}006$ to $K = 1{\cdot}020 \pm {\cdot}013$. Correspondingly equation 5′ must be changed to

$$M' = (1{\cdot}03 \pm {\cdot}03)\, M \left(\frac{\Delta}{\Delta'}\right)^{2.276} (\sec\theta)^{2.221}. \qquad (5)$$

This implies that in addition to the statistical errors our mass values derived with these schemes may be subject to a 3% systematic error until more accurate calibration data are available.

VI. Determinations on the Mass of Slow *K*-Mesons

The method of variable cell length has been applied to mass determinations on 9 long *K*-mesons which come to rest in an emulsion block. Of these 9 particles,

2 decay into 3 π-mesons and are therefore designated as τ-mesons.

5 decay and emit a single charged relativistic particle at the end of their range and are designated as K^+-mesons.

2 produce stars when they come to rest and are designated as K^--mesons (it is, however, possible that they are the negative counterpart of the τ-mesons rather than that of the particles designated as K^+-mesons).

The data on these particles are summarized in Table II.

Column 1 describes the nature of the particle.

Column 2 describes the nuclear event in which the particle originates (The notation is that introduced by Brown *et al.*[12]).

Column 3 gives the observed range and

Column 4 gives the mean dip angle.

Columns 5, 6 and 7 give the mass values obtained by our method and with three different scattering schemes. ($K_{1\cdot0}$ denotes a scheme calculated for particles of mass $M = 1000\, m_e$).

Column 8 gives the mean mass value from different schemes.

Column 9 the mass value obtained from grain density *versus* range.

In each case the mass value derived from the track of a given particle by applying different scattering schemes are consistent with each other. If we assume that the 9 particles listed in Table II form a homogeneous group as far as their masses are concerned we obtain as a weighted average from scattering *versus* residual range.

$$M_K = 974 \pm 42\, m_e.$$

TABLE II

Data on K-mesons

Particle	Origin	Range (cm.)	Dip	Mass from Scattering Schemes (m_e)				Mass from G. d vs. R
				$\pi_{(1\cdot6)}$	$P_{(1\cdot0)}$	$K_{(1\cdot0)}$	Mean	
1 K^+ ..	20+5n	1·37	0°	875±140	1100±190	930±145	955±140	950±100
2 K^+ ..	14+2n	1·13	·5°	885±115	1020±195	1040±175	980±155	1120±100
3 K^+ ..	Outside of Block	5·41	9·5°	930±95	1085±125	1060±115	1025±95	1115±100
4 K^+ ..	1+2n	3·20	1° to 9°	700±85	850±115	690±85	740±85	980±100
5 K^+ ..	8+?α	1·33	4°	1070±170	1075±190	875±145	1005±170	865±100
3 K^- ..	4+6n	2·49	2°	980±120	1065±150	..	1015±120	975±100
4 K^- ..	Outside of Block	2·85	3°	770±95	960±140	..	850±95	1320±100
1 + ..	6+0p	0·665	12°	1175±230	1225±280	..	1195±230	970±100
3 τ^+ ..	13+10α	0·745	29–43°	1485±315	1535±375	..	1505±315	..
Mean ..							974±42	1035±35

The mass values obtained for 7 of the 9 particles agree with the mean value within one standard deviation and the remaining two mass values lie within two standard deviations.***

We shall now compare our results with those obtained in other laboratories. Upto the time of the Cosmic Ray Conference at Bagnéres de Bigorre in July 1953 a total of 41 scattering measurements on K^+-mesons have been reported by various other laboratories; 25 of these measurements were carried out on tracks of length greater than 2 mm. and will be compared with our results.

The average obtained on three tracks each, by the groups at Brussels, Milan, Paris and Rochester are in agreement with each other. The weighted mean mass of these 12 particles is

$$M_{K^+} = 974 \pm 29\, m_e$$

in excellent agreement with our own value.

The 13 particles measured by the Bristol group give a weighted average mass of

$$M_{K^+} = 1120 \pm 43\, m_e$$

which lies higher by about four standard deviations. The discrepancy could be either due to a slight systematic difference in the measuring procedures, but probably it should be considered as an indication that there exists another type of K^+-meson with somewhat greater mass.

It seems, in any case, very likely that at least a large fraction of K^+-mesons have a mass close to 975 m_e.

The mass of the τ-mesons can be determined from an analysis of its decay products more accurately than from direct measurements. The weighted mean mass for 11 τ-mesons reported by various laboratories upto July 1953 is given by Amaldi *et al.*[13] as

$$M_\tau = 980 \pm 4{\cdot}4\, m_e$$

While similar mass determinations on 3 τ-mesons in our own laboratory[14] gave

$$M_\tau = 976 \pm 2{\cdot}0\, m_e.$$

*** We have no explanation for the high mass value obtained for particle 3 τ (the mass derived from the energy of its decay products is 975 m_e). Its track is very steep and traverses. 8 emulsions such that only ~ 900 μ of track length is available for scattering in each emulsions On the other hand scattering measurements on an equally steep proton track gave no anomalous results.

We conclude that within experimental error the masses of K^+, -K^- -and τ-mesons are identical and close to about 975 m_e, although the existence of some K-mesons with somewhat higher mass cannot be ruled out.

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APPENDIX I

(A) $\pi_{1\cdot 6}$ SCATTERING SCHEME

No.	Range R (μ)	Cell length t (μ)	No.	Range R (μ)	Cell length t (μ)
1	10	11	36	1597	70
	21	15		1667	71
	36	18		1738	72
	54	21		1810	73
	75	24		1883	74
6	99	26	41	1957	75
	125	28		2032	76
	153	30		2108	77
	183	33		2185	78
	216	35		2263	79
11	251	36	46	2342	80
	287	38		2422	81
	325	40		2503	82
	365	41		2585	83
	406	43		2668	84
16	449	45	51	2752	85
	494	46		2837	86
	540	48		2923	86
	588	49		3009	87
	637	50		3096	88
21	687	52	56	3184	89
	739	53		3273	90
	792	54		3363	91
	846	56		3454	92
	902	57		3546	93
26	959	58	61	3636	94
	1017	60		3730	94
	1077	61		3824	95
	1138	62		3919	96
	1200	63		4015	97
31	1263	64	66	4112	98
	1327	66		4210	98
	1393	67		4308	99
	1460	68		4407	100
	1528	69		4507	100

No.	Range R (μ)	Cell No. t (μ)	No.	Range R (μ)	Cell length t (μ)
71	4607	101	96	7343	119
	4708	101		7462	120
	4809	102		7582	120
	4911	102		7702	121
	5013	103		7823	121
76	5116	104	101	7944	122
	5220	104		8066	122
	5324	105		8188	123
	5429	106		8311	124
	5535	107		8435	125
81	5642	109	106	8560	126
	5751	110		8686	127
	5861	110		8813	127
	5971	111		8940	128
	6082	112		9068	129
86	6194	112	111	9197	130
	6306	112		9327	130
	6418	113		9457	130
	6531	113		9587	131
	6644	114		9718	131
91	6758	115	116	9849	132
	6873	116		9981	132
	6989	117		10113	133
	7106	118			
	7224	119			

(B) $P_{1\cdot 0}$ SCATTERING SCHEME

1	40	22	11	418	53
	62	27		471	55
	89	30		526	57
	119	34		583	60
	153	37		643	62
6	190	40	16	705	64
	230	43		769	66
	273	46		835	68
	319	48		903	70
	367	51		973	72

No.	Range R (μ)	Cell length t μ)	No.	Range R (μ)	Cell length t (μ)
21	1045	74	61	5216	132
	1119	76		5348	133
	1195	78		5481	134
	1273	80		5615	136
	1353	81		5751	138
26	1434	83	66	5889	139
	1517	85		6028	140
	1602	86		6168	141
	1688	88		6309	142
	1776	90		6451	142
31	1866	91	71	6593	143
	1957	93		6736	145
	2050	95		6881	146
	2145	96		7027	148
	2241	98		7175	150
36	2339	100	76	7325	150
	2439	100		7475	151
	2539	101		7626	152
	2640	102		7778	153
	2742	104		7931	154
41	2846	105	81	8085	155
	2951	106		8240	156
	3057	108		8396	157
	3165	110		8553	159
	3275	112		8712	160
46	3387	113	86	8872	161
	3500	114		9033	162
	3614	115		9195	162
	3729	116		9357	163
	3845	118		9520	164
51	3963	120	91	9684	165
	4083	121		9849	167
	4204	122		10016	168
	4326	123			
	4449	124			
56	4573	126			
	4699	127			
	4826	129			
	4955	130			
	5085	131			