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Determination of the Mass of Slow Particles by the Constant Sagitta Method (×).

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Summary. — A method is described in which the measurement of scattering of tracks at the end of their range is carried out on sets of cells of varying length, chosen so that the second difference of the sagitta remains constant. This permits the measurements to be made always at the optimum cell length and eliminates the inconveniences and inaccuracies of the methods in which the track is broken into several sections for measurement. The mass values of a series of K and J-particles have been determined, together with those of a group of protons which serves as calibration.

Introduction.

The measurement of the mass of particles by the scattering range method, introduced by GOLDSCHMIDT *et al.* (2), has regained importance in the last year since the K-particles were observed in the photographic emulsion (3).

(×) This method, developed independently at the laboratories of Brussels and Bombay was presented as a joint communication at the Congress on Cosmic Rays at Bagnères de Bigorre, July 1953 (1).

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(1) S. BISWAS, E. C. GEORGE and B. PETERS; C. DILWORTH, S. GOLDSACK and L. HIRSCHBERG: *Bagnères Conference*, in press.

(2) Y. GOLDSCHMIDT-CLERMONT, G. KING, H. MUIRHEAD and D. M. RITSON: *Proc. Phys. Soc.*, **61**, 183 (1948).

(3) C. O'CEALLAIGH: *Phil. Mag.*, **42**, 1032 (1951).

In this paper is described a convenient and accurate technique for making these measurements, by which the second difference of the sagitta of scattering is kept constant over the whole residual range.

A calibration has been made on a group of protons and the masses of 8 K-particles, 3 τ-particles and 3 J-particles are given.

Of these 4 K-particles were found in the Laboratory of Brussels, 4 K-particles, 1 τ and 3 J-particles were kindly lent by the laboratories of Genoa and of Milan and 2 τ's by the laboratory of Padova for measurement by this method.

The resolving power of the method is discussed.

Principle of the method.

The energy of a particle of mass M, chage Ze and residual range R can, when R is not too small, be written approximately

(1) $$E = 0.251\, Z^{1.16} M^{0.42} R^{0.58},$$

where E is in MeV, R in microns and M in units of proton mass.

The angle of scattering in a cell of t microns is given by

(2) $$\alpha = K_s Z t^{\frac{1}{2}} / p\beta \cdot 10^{-1} \quad \text{(degrees)},$$

where p is the momentum, β the velocity of the particle and K_s is the scattering « constant ».

In the non-relativistic approximation $p\beta = 2E$, and therefore

(3) $$\alpha = 0.199\, K_s t^{\frac{1}{2}} R^{-0.58} Z^{-0.16} M^{-0.42} .$$

Thus α varies rapidly with R towards the end of the range.

The existing techniques [2,4], consist in treating α as constant over a given portion of the track, and measuring it at constant cell length. This has the disadvantage:

a) of requiring either a fitting of $\bar{\alpha} - R$ curves or a weighting of several values of $\bar{\alpha}$ from several portions of track;

b) of rendering it impossible to maintain the measurement at strictly optimum cell-length over the sections of track in which α is supposed constant;

c) of requiring separate correction for noise level and separate cut-off of large angles in each section.

It seemed reasonable to try to vary the cell of measurement continuously along the track in such a way as to remain always at the optimum cell length. If at the same time the parameter of scattering remains constant, then the

(4) M. G. K. Menon and O. Rochat: *Phil. Mag.*, **42**, 1232 (1951).

calculation is considerably simplified. These two conditions can be fulfilled simultaneously only in the sagitta method.

From (3) the second difference D of the sagitta is given by

$$D = \alpha t = 0.00348\, K_s R^{-0.58} Z^{-0.16} M^{-0.42} t^{\frac{3}{2}}\ . \tag{4}$$

Since the noise level, ε, in the measurement of sagitta is practically independent of the cell-length for the short cells employed for slow particles, the signal to noise ratio D/ε is independent of t, if D is maintained constant. The cell-length t is chosen so that

$$t^{\frac{3}{2}} \propto K_s^{-1} R^{0.58} Z^{0.16} M^{0.42}\ . \tag{5}$$

Fig. 1 shows the variation of cell length in function of range for various particles, if K_s is supposed constant, for $D/\varepsilon \simeq 4$. Then, since D/ε is constant,

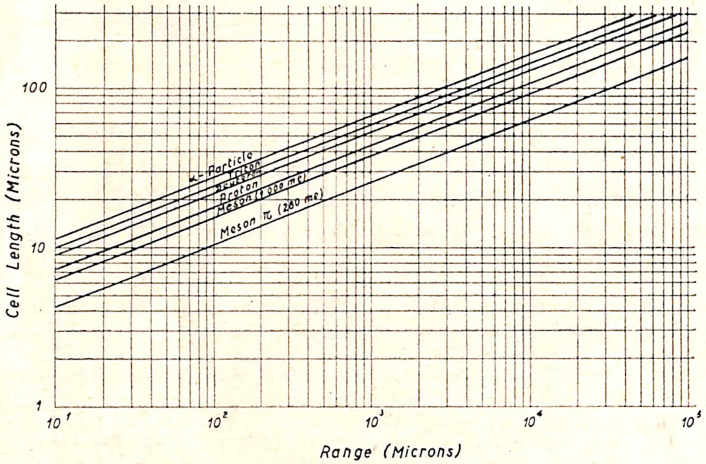

Fig. 1. – Relationship between Range and Cell Length to obtain constant Sagittal Scattering: $\bar{\delta} \propto t^{\frac{3}{2}} R^{-0.58} M^{-0.42} Z^{-1.16}$. Using the cell given here $D = 0.49$ microns (Arithmetic mean)

measurement can be made at optimum cell length over the whole track. Since D is constant, a straight mean can be taken and the mass M deduced either from the constants in equation (2) or by comparison with the mean sagitta $\bar{D}_N$ obtained on tracks of known particles:

$$M/M_N = (\bar{D}_N/\bar{D})^{2.38}\ . \tag{4}$$

The correction for noise and the cut-off of large angles can be applied over the whole track, since both corrections are made with respect to $\bar{D}$.

The standard error on the determination of mass is calculated taking the statistical error on $\overline{D}$ to be $0.75/\sqrt{N}$, with cut off at 4 times the mean angle of scattering and noise elimination between two cell lengths, N being the num-

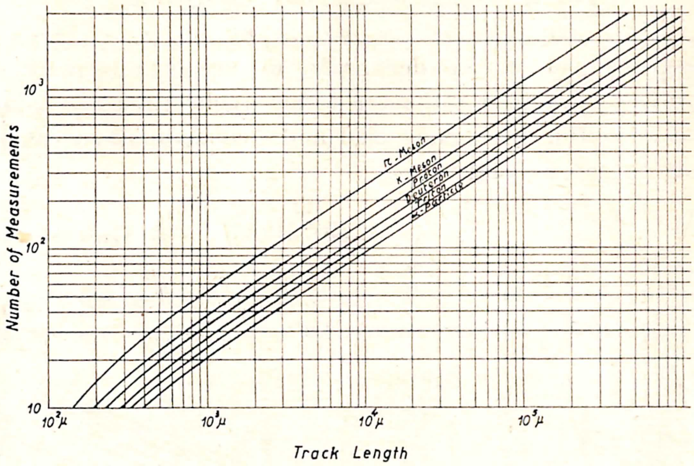

Fig. 2.

ber of independent measurements (see Fig. 2). In Fig. 3 is shown the percentage error on the mass in function of the length of track. The error being

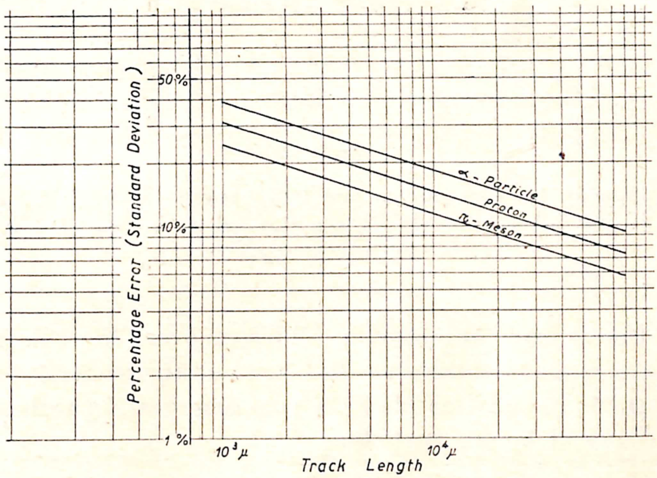

Fig. 3.

relatively small for lengths of more than 1 mm is taken symmetric here.

Variation of K_s of the range-energy exponent, and extension to relativistic velocities.

Equation (4) is not strictly correct, at low energies since the range-energy relation is not the simple power law of (1), and at high velocities for which $p\beta \neq 2E$. Thus the set of cells defined by (5) are not those for which D is truly constant. In addition, the scattering « constant » K_s is in reality a slowly varying function of R and t and not a constant as has been supposed in drawing the curves of Fig. 1.

There are two possible ways of taking into account these three effects. We can either construct a set of cells for which t is truly proportional to $(K_s/p\beta)^{-\frac{2}{3}}$ or, since the errors involved are small, use the simple sets of Fig. 1 and apply an overall correction to the resulting $\overline{D}$. We have employed the latter technique for two reasons. In the first place, improvements in the available knowledge of the range energy relation and the scattering constant, and possible slight changes in the constitution of the emulsion, do not make necessary a change in the cell-series used for measurements, but only in the constant used in the calculation.

Secondly, the lengths of cell for different particles at a given range are simply proportional to each other. It is therefore possible to make measurements, without prior assumptions about the mass, with a series of cells, say half those suitable for a π-meson (see Appendix II). The mean scattering is then calculated from the same measurements at increasing cell lengths, until the observed scattering is sufficiently greater than the noise level. Noise elimination can be carried out exactly as for mono-energetic tracks. For example a π-meson can be calculated at twice the fundamental cell, a K particle at three times and a proton at four times.

Using these cell sets we reduce the measured $\overline{D} = 1/N \sum_0^N D$, where N is the number of cells of measurement, to a standard D_s by applying a correction factor λ^{-1} which is a function of the range and of the mass determined. Values of λ^{-1} are given in Appendix I. With the values we have chosen, we find for protons

$$D_{sp} = 0.565 \, .$$

Thus to calculate the mass of an unknown particle we write

$$M/M_p = (D_{sp}/D_s)^{2.38} = (0.565/\lambda^{-1}\overline{D})^{2.38}$$

Experimental technique.

As in the normal sagitta method, the track is aligned parallel to the movement of the stage and readings taken of the position of the central grains in successive cells. It is more convenient in this case to displace the track at given values of the residual range R, rather than with the appropriate value of the cell length. Tables of residual ranges at which the measurements are to be made can be constructed for each type of particle (with a normally good micrometer screw and stage displacements can be made with a precision of 1 μ) or, if the simple uncorrected cell sets are to be used and noise eliminated between two cell lengths, a basic table such as that given in Appendix II for half π cell lengths can be used for the measurement of any track.

If the more rigorous sets of curves are used, preliminary measurements must be made to determine approximately the mass of the particle, and the set of ranges chosen accordingly (*).

It is important to keep the track closely parallel to the stage movement, as otherwise the second difference does not quite represent the scattering angle. The error can be shown to be negligible if the first difference δ_1 satisfies

$$\delta_1 \leqslant t/5$$

where t is the cell length.

This condition is rather stringent for π-mesons near the end of the range, and frequent reorientation is often necessary.

Steeply dipping tracks are not really suitable for these measurements. A correction must be applied if the dip is significant (cf. MENON and O'CEALLAIGH (5). If φ is the angle of dip, in the unshrunk emulsion the range is greater than the projected range by a factor $(\cos\varphi)^{-1}$ reducing the scattering by a factor $(\cos\varphi)^{0.58}$. The projected scattering is greater than the true scattering by the factor $(\cos\varphi)^{-\frac{3}{2}}$. In a uniformly dipping track the overall correction to the scattering is approximately a factor $(\cos\varphi)^{0.92}$ corresponding to a correction in the mass of $(\cos\varphi)^{-2.19}$.

These corrections are not serious in tracks long enough for useful measurement in a single emulsion, except, perhaps, at the end of the range, but in a stack of stripped emulsions a track may be steep and still be long enough for measurement.

Experimental results.

Measurements were made with the proton cell series on a group of tracks stopping in the emulsion of plates exposed in the Sardinia balloon flights of

(*) Such sets of ranges are being calculated at Göttingen for the proceedings of the Varenna Summer School (Società Italiana di Fisica, 1953).

(5) M. G. K. MENON and C. O'CEALLAIGH: *Phil. Mag.*, 44, 1291 (1953).

1952. The mass spectrum is shown in Fig. 4. The proton group is well enough separated for a mean value of its mass to be determined. This gives

$$M_p = 1\,790 \pm 40 \text{ m}_e ,$$

TABLE I.

Particle (*)	Range, μ	Mass, m_e	Error, m_e	$\frac{P_K}{P_J}$
K – M_1	2 000	1 270	290	$8.4 \cdot 10^1$
M_2	5 260	1 030	165	$3.0 \cdot 10^7$
M_4	1 470	1 360	340	$4.7 \cdot 10^0$
M_6	3 400	970	185	$2.7 \cdot 10^6$
B_1	3 000	800	160	$1.2 \cdot 10^8$
B_2	2 000	750	170	$4.9 \cdot 10^7$
B_3	1 400	985	255	$1.3 \cdot 10^3$
B_4	20 000	1 100	110	$1.0 \cdot 10^{16}$
J – M_1	16 000	2 210	260	$1.2 \cdot 10^{-7}$
M_2	1 250	2 340	680	$6.7 \cdot 10^{-2}$
M_3	900	2 330	770	$1.2 \cdot 10^{-1}$
τ – M_1	4 000	933	170	—
P_1	7 000	840	190	—
P_2	5 000	925	200	—

(*) The particles marked M are from Milan and Genoa, B from Brussels and P from Padova.

which is within a statistical deviation of the true value and gives us confidence in the method.

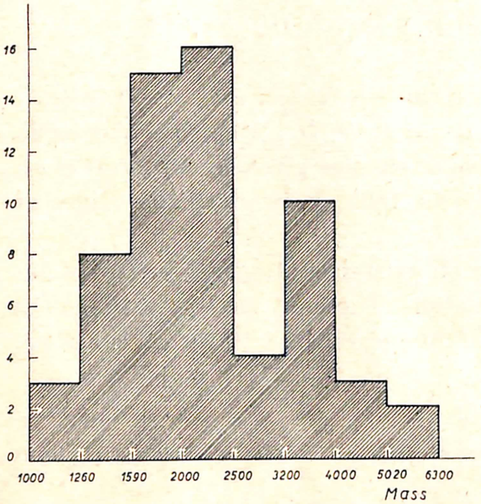

Fig. 4. – Distribution of mass of particles stopping in the emulsion.

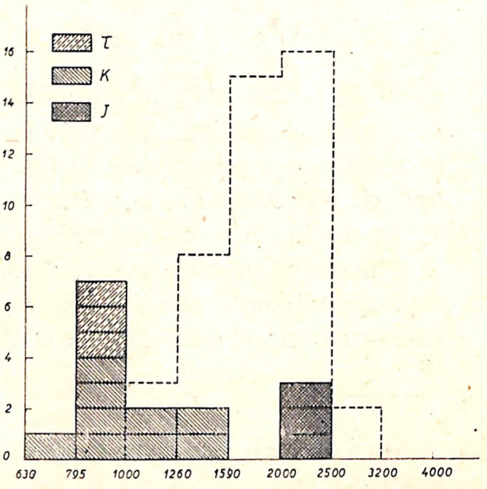

Fig. 5. – Distribution of mass of K-, τ-, and J-particles.

The mass values of a group of 8 K-particles, 3 τ's and 3 J's were also determined. The results are given in Table I and Fig. 5. The mean values obtained were:

$$M_K = 990 \pm 60 \text{ m}_e ,$$

$$M_\tau = 910 \pm 110 \text{ m}_e ,$$

$$M_J = 2\,260 \pm 210 \text{ m}_e .$$

The constancy of D was checked over the longest tracks by taking the means of 10 cells. The results are shown in Fig. 6.

Discussion.

The validity of the constant sagitta method seems to have been established. It should in principle be more precise than the preceding methods.

It is of interest to consider the resolving power of the method. We may take as examples two problems of current interest, the resolution of those K and J-particles which decay into fast secondaries, and the resolution of $K\rho$'s from protons (see below).

The former problem has been posed by BONETTI *et al.* (6). These two groups of particles differ considerably in mass, but on short tracks some overlap is possible. If we consider a track on which measurement of scattering-range on N cells gives a value $\overline{D}$ of the mean second difference, the ratio of the probability P_K of its being a K-particle to that P_J of its being a J-particle is

$$P_K/P_J = P_0\overline{D}_J/\overline{D}_K \exp\left[-\{(1-\overline{D}/\overline{D}_K)^2 - (1-\overline{D}/\overline{D}_J)^2\}\right]/2\varepsilon^2,$$

where P_0 is the ratio of the number of K to J-particles, and $\varepsilon = 0.75/\sqrt{N}$. It is evident that for $M = 1\,570$ m$_e$, $P_K/P_J = P_0\, D_J/D_K$, whatever the value of N. From the indication of the relative intensities given by BONETTI *et al.* (loc. cit.) at $M = 1\,570$ m$_e$, $P_K/P_J = 1.4$. In table I are given the values of P_K/P_J for the K and J-particles.

The second problem is that of the possible existence of negative K-particles captured without giving a nuclear disintegration. These, by analogy with the old phenomenological nomenclature of OCCHIALINI and POWELL (7) we can call $K\rho$'s.

(6) A. BONETTI, R. LEVI-SETTI, M. PANETTI and G. TOMASINI: Private communication.

(7) G. P. S. OCCHIALINI and C. F. POWELL: *Nature*, **162**, 168 (1948).

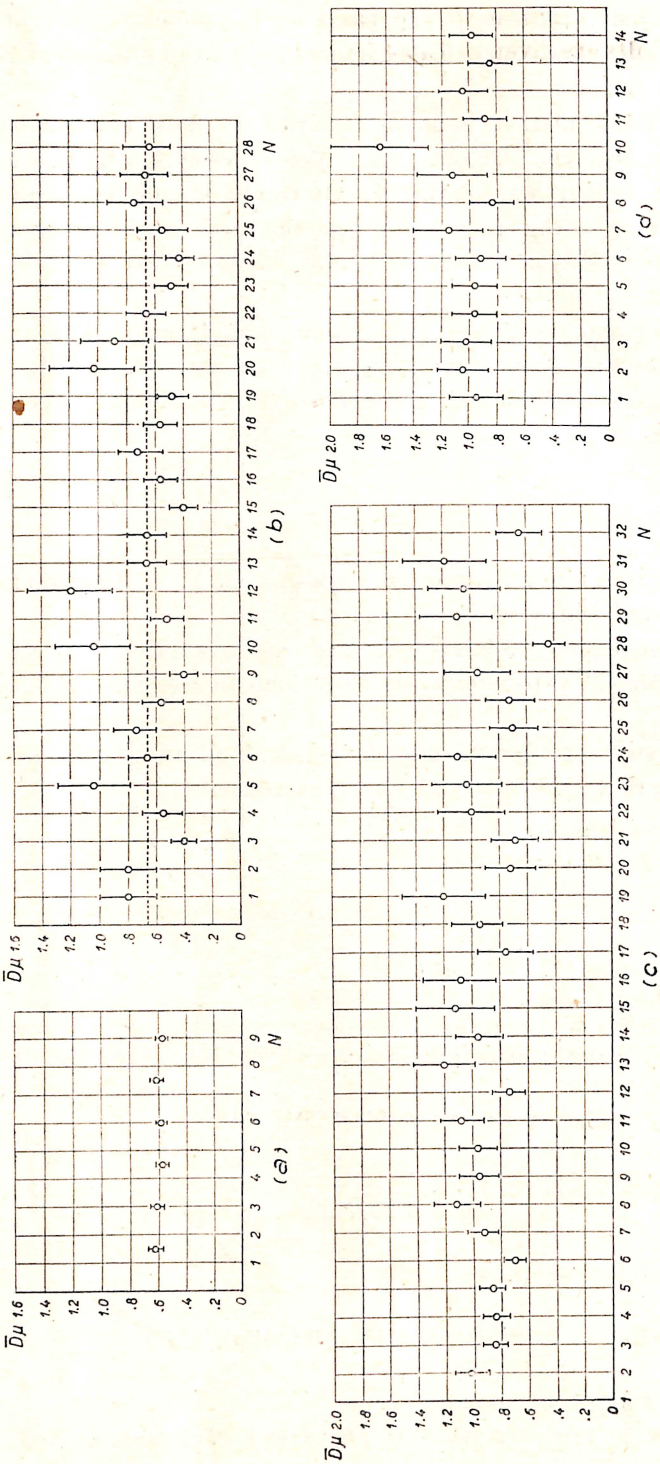

Fig. 6. – Constancy of D:

(a) Proton group;

(b) One long proton;

(c) Group of K-mesons;

(d) Group of τ-mesons.

The possibility of their existence is suggested by the small number of disintegrations produced by slow K-particles compared with the number of K-particles which decay ([8]).

The scattering range method is not necessarily the most efficient to apply to such a statistical problem. To obtain a sufficient resolution by this method, tracks of at least 2 cm length are required. On tracks of such length the ionization at the beginning of the track is low and the method of ionization range is quicker, a consideration of a certain weight when several hundred tracks must be measured.

When this paper was ready for publication, our attention was drawn to the article of HOLTEBEKK *et al.* ([9]), in which the variable cell length method had been developed independently and applied to discriminate between ^{3}He and ^{4}He fragments.

Acknowledgements.

We wish to express our gratitude to Professor G. P. S. OCCHIALINI who proposed and directed this work.

We thank our colleague D. HIRSCHBERG for his invaluable assistance, and Mrs. CORNIL and Miss VANDENCAMP for their help in the measurement and calculation.

We are very grateful to our friends of Genoa Milan and Padua, who lent us an important part of the material for measurement.

APPENDIX I.

We take as standard the constant

$$D_s = 0.199 \cdot 24 \cdot R^{-0.58} M^{-0.42} Z^{-0.16} t^{\frac{3}{2}} .$$

The measured value

$$D = K_s Z t^{\frac{3}{2}} / p\beta .$$

At each cell therefore we have

$$D_s / D = 0.199 \cdot 24 \cdot R^{-0.58} M^{-0.42} Z^{-0.16} p\beta / K_s Z$$

([8]) *Discussion of Bagnères Conference*, 1953.

([9]) T. HOLTEBEKK, N. ISACHSEN and S. O. SÖRENSEN: *Phil. Mag.*, **44**, 1037 (1953).

i.e.

$$\frac{D_s}{D} = \frac{24}{K_s} \frac{E/M}{0.251(R/MZ)^{0.58}} \frac{p\beta}{2E} = \delta_K \delta_{ER} \delta_{p\beta} \,.$$

The mean value of D is obtained by summing over the number N of cells of measurements

$$\overline{D} = D_s \sum_0^N (\delta_K \delta_{ER} \delta_{p\beta})^{-1}/N = \overline{\lambda}^{-1} D_s \,.$$

The correction factor $\delta_K = 24/K_s$ is determined from the values of K_s given by GOTTSTEIN *et al.* (10). $\delta_{ER} = E/M/0.251(R/MZ)^{0.58}$ is calculated from

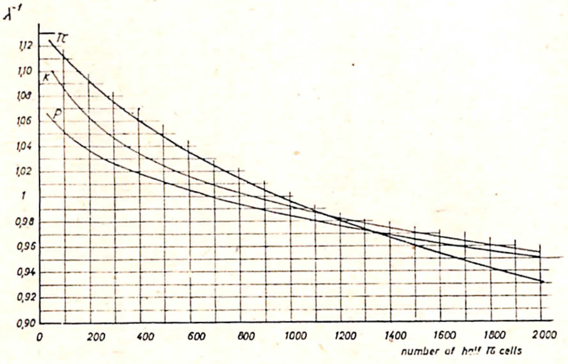

Fig. 7.

the range-energy curves for protons of ROTBLAT (11). The relativistic correction $\delta_{p\beta} = p\beta/2E = (1 + E/2Mc^2)(1 + E/Mc^2)^{-1}$.

Curves of $\delta^{-1} = (\delta_K \delta_{ER} \delta_{p\beta})^{-1}$ in function of the number of π cells in the track have been drawn for π-mesons, K-particles and protons and integrated numerically. The resultant values of λ^{-1} in function of N are given in fig. 7. These values are calculated for cell sets of t_π for π-mesons. $3t_\pi/2$ for K-particles and $2t_\pi$ for protons.

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(10) K. GOTTSTEIN, M. G. K. MENON, J, H. MULVEY, C. O'CEALLAIGH and O. ROCHAT: *Phil. Mag.*, **42**, 708 (1951), Curve with cut-off.

(11) J. ROTBLAT: *Nature*, **167**, 550 (1951).

Appendix II.

Table of ranges for half π cells.

No.	R	No.	R	No.	R	No.	R
1	48	51	372	101	907	151	1 616
2	52	52	381	102	920	152	1 632
3	56	53	390	103	932	153	1 648
4	60	54	399	104	945	154	1 664
5	64	55	408	105	958	155	1 680
6	69	56	417	106	971	156	1 696
7	73	57	426	107	984	157	1 712
8	78	58	436	108	997	158	1 728
9	82	59	445	109	1 010	159	1 744
10	87	60	455	110	1 023	160	1 760
11	92	61	464	111	1 036	161	1 776
12	97	62	474	112	1 050	162	1 793
13	102	63	483	113	1 063	163	1 809
14	108	64	494	114	1 077	164	1 826
15	113	65	504	115	1 090	165	1 842
16	119	66	514	116	1 104	166	1 859
17	124	67	524	117	1 117	167	1 875
18	130	68	534	118	1 131	168	1 892
19	136	69	544	119	1 144	169	1 909
20	142	70	554	120	1 158	170	1 926
21	148	71	564	121	1 172	171	1 943
22	154	72	575	122	1 186	172	1 960
23	160	73	585	123	1 200	173	1 977
24	167	74	596	124	1 214	174	1 995
25	173	75	606	125	1 228	175	2 012
26	180	76	617	126	1 242	176	2 030
27	186	77	627	127	1 256	177	2 047
28	193	78	638	128	1 271	178	2 065
29	200	79	649	129	1 285	179	2 082
30	207	80	660	130	1 300	180	2 100
31	214	81	671	131	1 314	181	2 117
32	221	82	682	132	1 329	182	2 135
33	228	83	692	133	1 343	183	2 153
34	236	84	704	134	1 358	184	2 171
35	243	85	715	135	1 372	185	2 189
36	251	86	727	136	1 387	186	2 207
37	258	87	738	137	1 402	187	2 225
38	266	88	750	138	1 417	188	2 243
39	273	89	761	139	1 432	189	2 261
40	281	90	773	140	1 447	190	2 279
41	289	91	785	141	1 462	191	2 297
42	297	92	797	142	1 477	192	2 315
43	305	93	809	143	1 492	193	2 333
44	313	94	821	144	1 507	194	2 352
45	321	95	833	145	1 522	195	2 370
46	330	96	845	146	1 538	196	2 389
47	338	97	857	147	1 553	197	2 407
48	347	98	870	148	1 569	198	2 426
49	355	99	882	149	1 584	199	2 444
50	364	100	895	150	1 600	200	2 463

Table of ranges for half π cells.

No.	R	No.	R	No.	R	No.	R
201	2 481	251	3 485	301	4 612	351	5 852
202	2 500	252	3 507	302	4 636	352	5 878
203	2 518	253	3 528	303	4 659	353	5 904
204	2 537	254	3 550	304	4 683	354	5 930
205	2 555	255	3 571	305	4 707	355	5 956
206	2 574	256	3 593	306	4 631	356	5 982
207	2 593	257	3 614	307	4 755	357	6 008
208	2 612	258	3 636	308	4 779	358	6 035
209	2 631	259	3 658	309	4 803	359	6 061
210	2 650	260	3 680	310	4 827	360	6 088
211	2 669	261	3 702	311	4 851	361	6 114
212	2 689	262	3 724	312	4 876	362	6 141
213	2 708	263	3 746	313	4 900	363	6 167
214	2 728	264	3 768	314	4 925	364	6 194
215	2 747	265	3 790	315	4 949	365	6 220
216	2 767	266	3 812	316	4 974	366	6 247
217	2 786	267	3 834	317	4 998	367	6 274
218	2 806	268	3 856	318	5 023	368	6 301
219	2 825	269	3 878	319	5 047	369	6 328
220	2 845	270	3 901	320	5 072	370	6 355
221	2 865	271	3 923	321	5 096	371	6 382
222	2 885	272	3 946	322	5 121	372	6 409
223	2 905	273	3 968	323	5 145	373	6 436
224	2 925	274	3 991	324	5 170	374	6 463
225	2 945	275	4 013	325	5 194	375	6 490
226	2 965	276	4 036	326	5 219	376	6 518
227	2 985	277	4 058	327	5 244	377	6 545
228	3 005	278	4 081	328	5 269	378	6 573
229	3 025	279	4 103	329	5 294	379	6 600
230	3 046	280	4 126	330	5 319	380	6 628
231	3 066	281	4 148	331	5 344	381	6 655
232	3 087	282	4 171	332	5 369	382	6 683
233	3 107	283	4 194	333	5 394	383	6 710
234	3 128	284	4 217	334	5 419	384	6 738
235	3 148	285	4 240	335	5 444	385	6 765
236	3 169	286	4 263	336	5 969	386	6 793
237	3 190	287	4 286	337	5 494	387	6 820
238	3 211	288	4 309	338	5 520	388	6 848
239	3 232	289	4 332	339	5 545	389	6 875
240	3 253	290	4 355	340	5 571	390	6 903
241	3 274	291	4 378	341	5 596	391	6 930
242	3 295	292	4 401	342	5 622	392	6 958
243	3 316	293	4 424	343	5 647	393	6 985
244	3 337	294	4 448	344	5 673	394	7 013
245	3 358	295	4 471	345	5 698	395	7 040
246	3 379	296	4 495	346	5 724	396	7 068
247	3 400	297	4 518	347	5 749	397	7 095
248	3 421	298	4 542	348	5 775	398	7 123
249	3 442	299	4 565	349	5 800	399	7 150
250	3 464	300	4 589	350	5 826	400	7 178

RIASSUNTO (*)

Si descrive un metodo col quale la misura dello scattering delle tracce a fine range si esegue su gruppi di celle di lunghezza variabile, scelte in modo che la seconda differenza della saetta rimanga costante. Ciò permette di eseguire la misura sempre alla lunghezza più favorevole della cella ed elimina le inesattezze e gli inconvenienti dei metodi in cui la traccia è divisa, per l'esecuzione della misura, in diverse sezioni. I valori delle masse di una serie di particelle K e J sono stati determinati, per confronto, assieme a quelli di un gruppo di protoni.

(*) *Traduzione a cura della Redazione.*