K. Greisen. Cosmic Rays under ground. Phys. Rev. 73, 521, 1948

For the number of π-mesons of a particular energy as a function of depth, one therefore obtains the equation

$$\frac{du}{dt} = Ae^{-t} - \pi\left[1 + (B/t)\right]$$

where Ae^{-t} is prop. to the number of primary rays of sufficiently high energy. $B = 8.10^5\ \mu / c\tau_0 E$, μ = rest energy, τ_0 proper life time, E = energy of the π-meson. t is measured in units of the mean free path. From this one may calculate the number of ordinary mesons of a particular energy:

$$m = \int_0^{10} \frac{B}{t}\, r\, dt \approx \frac{B}{B+1} A$$

Thus one obtains from the intermediate step in creation of mesons a new factor $\frac{B}{B+1}$ in the spectrum.

For $B \gg 1$ there is no change, while for $B \ll 1$, this factor is just B, which is inversely proportional to the energy.

The transition (experim.) occurs at $\sim 8.10^{10}$ eV, where B must be ~ 1. So we find $\tau_0 = 6.10^{-8}$ sec.

~~m = B.P. ...~~

$$B = \frac{C}{E}$$

$$m = \frac{C/E}{C/E+1} \cdot A = \frac{A}{1 + E/C}$$

$$M = \int_E^{\infty} m\, dE = \int_E^{\infty} \frac{A\, dE}{E^{3}(1 + E/C)} = \ln\left(\frac{E}{C} + 1\right)$$